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# A framework for the characterization of the transient handling responses of non-linear vehicles

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**Abstract:** The paper attempts to provide an integrated framework for the objective assessment of the transient handling responses of non-linear vehicles. The ultimate aim of the proposed framework is the characterization of such responses as neutral or under/over-steering. A new methodology is employed, which has previously been applied for the classification of the transient response of linear vehicles. The success of the proposed method is judged based on its robustness, the consistency of the results, and their practical implications. Furthermore, the results are compared with the findings of traditional approaches for the characterization of the steady-state and transient handling behaviour. The corresponding discussion reveals the agreement between the approaches, but also highlights the slightly different definition of the neutrally steering vehicle, as perceived by the proposed method.

**Keywords:** transient, handling, yaw moment, tyre force, measure, non-linear

## 1 INTRODUCTION

The characterization of the steady-state handling response is a more-or-less straightforward task, facilitated by the definition of the under-steer coefficient in the case of linear vehicle models [1], or the handling diagram in non-linear cases [1–4]. On the contrary, the assessment of the transient response imposes certain difficulties. The adoption of a linear bicycle model enables the analytical calculation of the transient response to simple steering inputs such as step/ramp or sinusoidal steering. The roots of the corresponding characteristic equation dictate whether the response is expected to be oscillatory or aperiodic, and whether the motion is stable or unstable. It can be easily proven that in the linear case, only the response of an over-steering vehicle can become unstable [1, 2], whereas an under-steering vehicle is always stable. Consequently, if a linear vehicle exhibits an unstable

behaviour one can safely classify it as over-steering. The judgement is less certain when the conditions are such that the response of a vehicle is stable. Then, a system identification approach might result in the accurate estimation of the vehicle parameters, and subsequently the calculation of the under-steer coefficient for the final characterization of its behaviour. In any case, this might prove a tedious task, which cannot serve as part of a robust, well-defined, assessment procedure. The problems with the assessment of transient handling behaviour are discussed thoroughly in reference [5], where the failure of the roots of the characteristic equation to provide a definitive picture is demonstrated through a practical example. The situation becomes worse when non-linearities are taken into account. These non-linearities are primarily related to the build-up of tyre forces during the course of a manoeuvre. Pacejka [3] investigates the stability of a vehicle with non-linear tyres at high lateral accelerations, with the tyres operating well within the non-linear range. According to his analysis, two separate conditions are required for the vehicle response to be deemed stable, as opposed to the one and only condition which ensures the stability of a linear vehicle response. The first condition is related

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to the slope of the steer-angle with respect to the curvature of path under constant forward velocity and can be seen as equivalent to the stability condition of a linear vehicle model [3]. In particular, this first condition is violated when the steering needs to change direction (i.e. when the vehicle is over-steering). The remaining, second condition for the vehicle stability, deserves further attention, as it reveals a possible void within the existing theory. This condition may be violated when the tyre characteristics show a peak in the side force followed by a downward slope [3]. Certainly, this might be the case with a neutral steer vehicle, characterized by an even weight distribution between front and rear axles and non-linear tyres, exhibiting a distinct force peak. Apparently, such a vehicle might violate this second stated condition, and thus become unstable. However, such an instability does not imply an over-steering response. In any case, the existing theory does not include any provisions for the classification of such responses. Especially, with non-linear vehicles, the explicit characterization of any transient response, particularly responses which do not lead to instabilities, cannot be performed with confidence. Consequently, as the vehicle models in use become increasingly complex, incorporating many sources of non-linearity, the assessment of their response becomes more descriptive, than precise and quantitative [6].

The proposed framework outlined here attempts to provide a means for the explicit assessment of the response of a vehicle during the course of any possible transient manoeuvre. The underlying principles of the method have already been presented for the linear case [5], whereas the present work focuses towards the influence of non-linear tyre behaviour. The findings of the method are related, throughout the paper, to the implications of traditional steady-state handling assessment methods, such as the handling diagram [1–4].

## 2 METHOD FOR ASSESSMENT OF VEHICLE HANDLING BEHAVIOUR

A detailed description of the proposed assessment procedure can be found in reference [5]. Although the work in reference [5] deals with responses generated by linear vehicle models, the method includes provision for non-linear responses/vehicles and can be used in such cases. A brief overview of the method is provided here.

The assessment of the instantaneous handling behaviour throughout the course of a transient manoeuvre is performed using two related measures, namely the dynamic and neutral kinematic normalized yaw impulses, denoted by  $\Gamma_d$  and  $\Gamma_{kn}$  respectively. The definition of the dynamic normalized impulse is

given as

$$\Gamma_d = \frac{|a| \sum_{i=1,2} F_{yi} \cdot dt - |b| \sum_{i=3,4} F_{yi} \cdot dt}{|a| \sum_{i=1,2} F_{yi} \cdot dt + |b| \sum_{i=3,4} F_{yi} \cdot dt} \quad (1)$$

where each lateral force in equation (1) represents the projection of the net tyre force of a possibly steered and braking tyre on the  $y$ -axis of the SAE vehicle frame of reference [7].

Considering a linear bicycle model, the above relation can be simplified and expressed as merely a function of kinematic quantities and a parameter  $\rho$  [5]

$$\Gamma_d = \Gamma_k = \frac{U\delta - V(1 - \rho) - r(|a| + \rho|b|)}{U\delta - V(1 + \rho) - r(|a| - \rho|b|)} \quad (2)$$

where  $\Gamma_k$  denotes the kinematic normalized yaw impulse and parameter  $\rho$  is defined as

$$\rho = \frac{|b| C_r}{|a| C_f} \quad (3)$$

If, in addition, a neutral vehicle is assumed,  $\Gamma_k$  can be simplified further, resulting in the neutral kinematic normalized yaw impulse as

$$\Gamma_k = \Gamma_{kn} = \frac{U\delta - rl}{U\delta - 2V - r(|a| - |b|)} \quad (4)$$

The basis of the handling assessment is the axiomatic declaration that a linear neutral bicycle model, without phase lags in the generation of the tyre forces, behaves neutrally under any possible steady-state or transient operating conditions. Note that  $\Gamma_{kn}$ , when calculated for a neutral linear vehicle during the course of a transient manoeuvre will always equate to  $\Gamma_d$ .

The assessment of the response of a given vehicle is based on the initial assumption that the vehicle is neutral-steer (i.e. it resembles the behaviour of the linear neutral bicycle model). This assumption is checked by calculating  $\Gamma_{kn}$  for the assumed neutral response and subsequently comparing it with the dynamic yaw impulse  $\Gamma_d$ , as calculated by equation (1). If  $\Gamma_{kn}$  is found to be equal to  $\Gamma_d$ , then the response is characterized as neutral as long as the equality holds true [5]. In the event that  $\Gamma_d$  becomes smaller than  $\Gamma_{kn}$ , the response is characterized as one of under-steering [5], whereas if  $\Gamma_d$  is found to be greater than  $\Gamma_{kn}$ , the response is classified as over-steering.

The comparison between  $\Gamma_d$  and  $\Gamma_{kn}$  is performed by employing two different ratios, denoted  $Q$  and  $Q_s$ , respectively, each one characterized by its own strengths and weaknesses [5]. The  $Q$  ratio is provided

by the following relation

$$Q = \frac{\Gamma_d}{\Gamma_{kn}} \quad (5)$$

Obviously, for  $Q = 1$  the vehicle behaves neutrally. If  $Q < 1$  it exhibits under-steering, whereas  $Q > 1$  indicates over-steering. The usefulness of the  $Q$  ratio as a handling performance index deteriorates under certain conditions, as a result of  $\Gamma_{kn}$  and/or  $\Gamma_d$  becoming negative [5]. This observation has led to the definition of the alternative  $Q_s$  ratio, as follows

$$Q_s = \frac{(\Gamma_d - \Gamma_{kn})B_1}{|\Gamma_d - \Gamma_{kn}| + B_2} \quad (6)$$

where  $B_1$  and  $B_2$  are Boolean expressions yielding 1 (true) or 0 (false) according to the following relations

$$B_1 = \begin{cases} 1 & \text{for } |Q - 1| > \varepsilon \\ 0 & \text{for } |Q - 1| \leq \varepsilon \end{cases} \quad (7)$$

$$B_2 = \begin{cases} 1 & \text{for } |Q - 1| \leq \varepsilon \\ 0 & \text{for } |Q - 1| > \varepsilon \end{cases} \quad (8)$$

Parameter  $\varepsilon$  represents the 'neutral margin', dictating the maximum relative difference allowed between  $\Gamma_d$  and  $\Gamma_{kn}$ , before classifying a transient response as non-neutral.

As long as  $|Q - 1| = |(\Gamma_d - \Gamma_{kn})/\Gamma_{kn}| \leq \varepsilon$ , the vehicle is considered neutral, with  $B_1 = 0$ ,  $B_2 = 1$ , and  $Q_s = 0$ . On the other hand, if:  $|Q - 1| = |(\Gamma_d - \Gamma_{kn})/\Gamma_{kn}| \leq \varepsilon$ , the difference between  $\Gamma_d$  and  $\Gamma_{kn}$  is such that the vehicle cannot be characterized as neutral. Under such conditions:  $B_1 = 1$ ,  $B_2 = 0$ , while  $Q_s$  represents the sign of the difference between  $\Gamma_d$  and  $\Gamma_{kn}$ . If:  $Q_s = -1$ , then the vehicle exhibits under-steer, whereas if:  $Q_s = 1$  it would over-steer.

As is shown later, the  $Q$  ratio provides a more integrated picture of the transient handling response. It allows a direct quantitative comparison of different responses. Alternatively, the  $Q_s$  ratio can only determine whether a manoeuvre is neutral, under-, or over-steering. Overall, the two measures should be used in a complementary fashion, the latter being most useful when the definition of the former deteriorates [5].

### 3 VEHICLE AND TYRE MODEL

The method described in the previous section does not involve any assumptions regarding the complexity of the prescribed vehicle model. Ultimately, the method can be applied experimentally to measured responses of real vehicles. Nevertheless, a simple, two degree-of-freedom bicycle model equipped with non-linear tyres is employed for the purpose of the analysis. The main reason behind this choice is to enable the establishment of a direct relationship

between the observed transient behaviour and the tyre characteristics. Furthermore, the construction of the corresponding steady-state handling diagrams is made straightforward and can be performed without the tedious requirement of running a series of steady-state handling simulations.

Assuming a constant forward speed  $U$  and sufficiently small steer angles, the equations of motion for the lateral and yaw degrees of freedom of the model become [8]

$$\sum F_y = Y_f(a_f) + Y_r(a_r) = m \left( \frac{dV}{dt} + U \cdot r \right) \quad (9)$$

$$\sum M_z = |a|Y_f(a_f) - |b|Y_r(a_r) = I_{zz} \left( \frac{dr}{dt} \right) \quad (10)$$

where  $Y_f$  and  $Y_r$  denote the front and rear lateral tyre forces as non-linear functions of the corresponding slip angles.

If the vehicle is assumed to be front-steered and the slip angles  $a_f$  and  $a_r$  are sufficiently small, the following relations apply [1]

$$a_f = \delta - \frac{V + |a|r}{U} \quad (11)$$

$$a_r = -\frac{V - |b|r}{U} \quad (12)$$

The non-linear lateral tyre forces  $Y_f$  and  $Y_r$  are calculated using a version of the Magic Formula tyre model, as described in reference [9]. The general form of the formula is

$$Y_{f,r}(a) = y(x) + S_V \quad (13)$$

with

$$y(x) = D \sin[C \arctan\{Bx - E(Bx - \arctan Bx)\}] \quad (14)$$

and

$$x = \tan a + S_H \quad (15)$$

The primary parameters of the Magic Formula, namely  $B$ ,  $C$ ,  $D$ ,  $E$ ,  $S_H$ , and  $S_V$ , appear as functions of the normal load,  $F_z$ , the camber angle,  $\gamma$ , and a number of secondary constants [9]. A set of experimentally obtained secondary constants [10] is employed as a starting point for both front and rear tyres. The force functions are then simplified by assuming zero vertical/horizontal shifts ( $S_V = S_H = 0$ ) and zero camber angle ( $\gamma = 0$ ). Finally, in order to manipulate the shape of the characteristic curves, the peak value  $D$ , the stiffness factor  $B$ , and the shape factor  $C$  were altered accordingly.

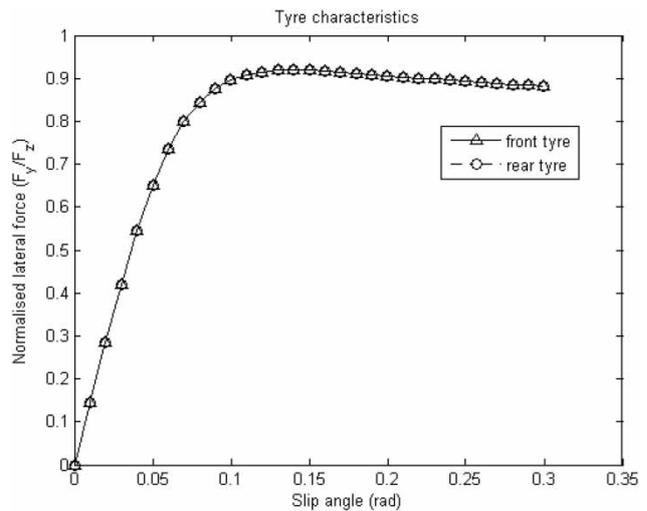
Equations of vehicle motion (9) and (10) are solved numerically, taking into account the geometric relations in (11), (12), and the lateral tyre forces as predicted by equations (13) to (15), also taking into account the aforementioned simplifications/assumptions. Prior to any simulation, the corresponding normalized tyre characteristics are obtained by dividing the actual lateral force characteristics with the axle weight, as dictated by the front/rear weight distribution. The steady-state handling diagram is then drafted and finally, the actual vehicle responses are used for the calculation of the  $Q$  and  $Q_s$  ratios, described in section 2.

#### 4 APPLICATION OF THE METHOD

The method is demonstrated through a series of carefully selected test cases. The parameters of the vehicle chosen for the analysis are listed in Table 1. The weight distribution is deliberately made even between the front and rear axles, in order to be able to achieve a neutral handling behaviour, provided that the tyre forces are non-linearly dependent on the vertical weight. This rather interesting case of a vehicle with even weight distribution and identical front-rear tyre characteristics is first discussed. The normalized front and rear tyre characteristics are shown in Fig. 1, whereas Fig. 2 illustrates the resulting steady-state handling diagram, which, as expected, consists of a straight line. As the forward speed increases from zero, the required steer-angle to negotiate a curve of given radius does not change (i.e. the steering gradient with respect to the normalized centripetal acceleration ( $a_y/g$ ) is zero and the handling curve reduces to a straight line). The normalized centripetal acceleration increases (arrow pointing upwards in Fig. 2) up to the maximum value permitted by the tyre characteristics. Any attempt to increase the forward speed beyond this point results in instability, with the vehicle operating within the region marked by the downward pointing arrow, shown in Fig. 2. Although the existing theory predicts this type of instability [3], the overall response is still characterized as neutral steer. From the outset, it was anticipated that such a behaviour, as exhibited by the non-linear neutral vehicle, would not resemble that of a linear neutral vehicle. The steady-state handling diagram of the linear neutral vehicle consists of a continuously increasing straight line [1] (i.e. the centripetal acceleration increases indefinitely with forward velocity, when the steer-angle and path

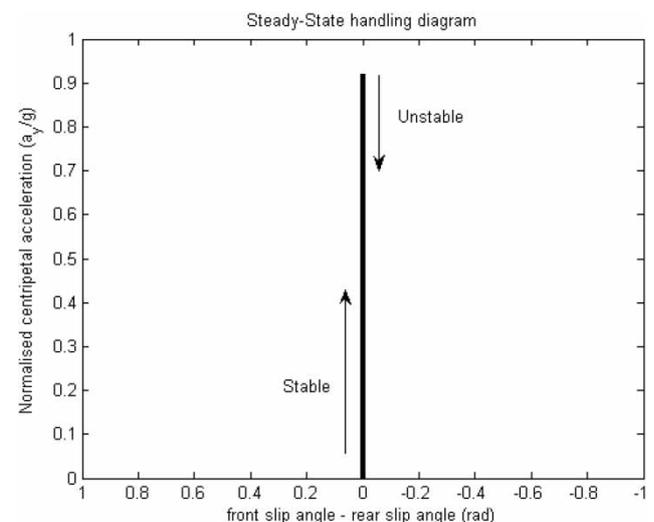
**Table 1** Vehicle parameters

$I_{zz}$ (kg m <sup>2</sup> )	$m$ (kg)	$a$ (m)	$b$ (m)
2e3	1.3e3	1.25	1.25

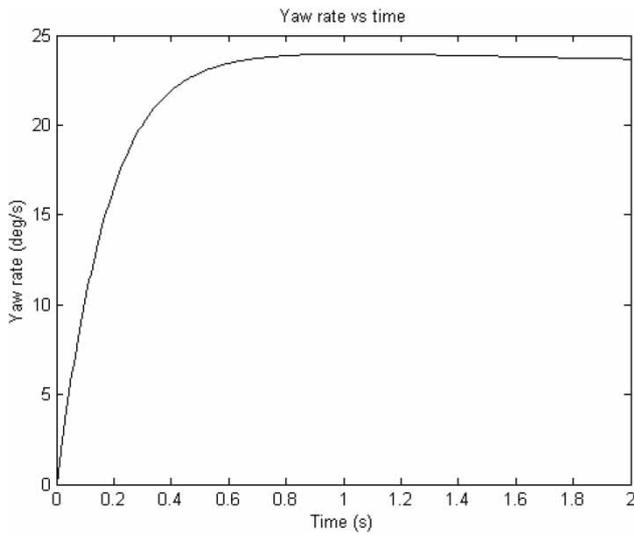


**Fig. 1** First test case: identical front-rear tyre characteristics

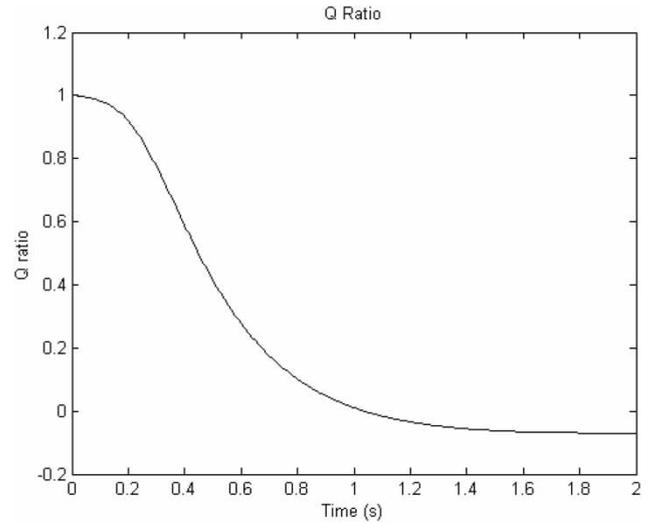
curvature remain constant). This type of neutrality would be perceived by a hypothetical driver as the ability to increase the forward speed without adjusting for the steer-angle, while remaining on the same path. The steady-state behaviour of the two vehicles, linear and non-linear, would be indistinguishable up to the point where the non-linear tyres reach their peak force generating ability. Beyond this point, the non-linear vehicle would become unstable. It might be argued that this instability is of an under-steering type, since the centripetal acceleration and, therefore, the yaw rate are limited by the peak tyre force. Still, this fact cannot be signified by the handling diagram. The question to be answered is whether it is sensible, in terms of handling assessment, to accept the existence of an unstable neutral behaviour, or, instead,



**Fig. 2** First test case: the steady-state handling diagram for a neutral-steer, non-linear vehicle



**Fig. 3** First test case: yaw rate response of the vehicle to a step-steer input of 0.04 rad at 30 m/s forward speed

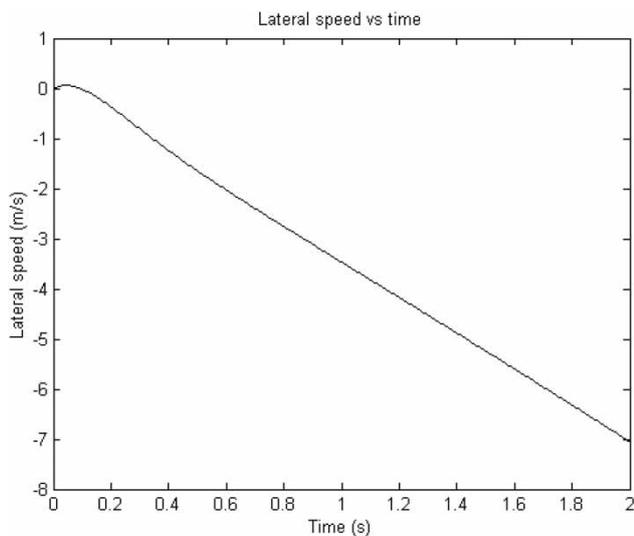


**Fig. 5** First test case: Q ratio response of the vehicle to a step-steer input of 0.04 rad at 30 m/s forward speed

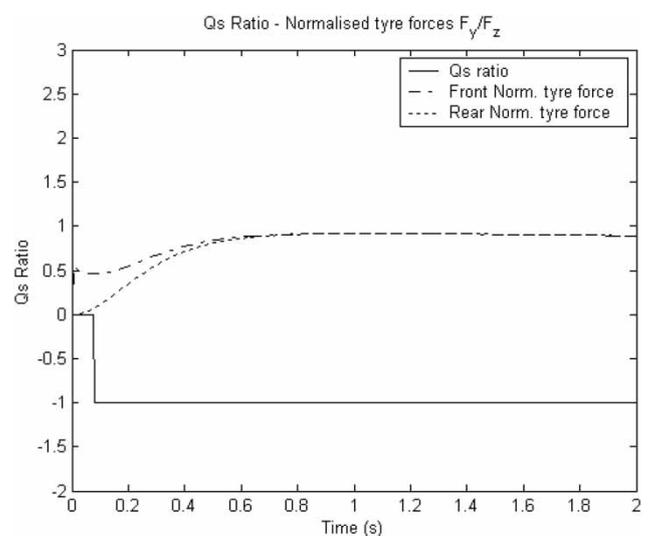
to distinguish between these two types of neutral vehicles.

At this point, the response of the non-linear neutral vehicle to a transient manoeuvre might shed some light on the situation. In particular, the vehicle is subjected to a step-steer input of 0.04 rad, at a forward speed of 30 m/s. Figures 3 and 4 show the yaw-rate and lateral velocity responses, respectively. The Q ratio is calculated using equation (5) at every instant of time during the course of the manoeuvre, and shown in Fig. 5. The  $Q_s$  ratio is depicted in Fig. 6, calculated using equation (6), with the neutral margin  $\varepsilon$  set equal

to 0.01. To support the analysis, Fig. 6 also includes the normalized tyre forces calculated by dividing the actual force with the corresponding axle weight. Both Q and  $Q_s$  ratios suggest that only at the very beginning of the manoeuvre, the transient response of the non-linear vehicle can be considered as neutral. The duration of this initial neutral behaviour is strongly related to the value of the neutral margin  $\varepsilon$ , which dictates the point up to which  $\Gamma_d$  might be considered to equate to  $\Gamma_{kn}$ . Beyond this point, the rate of the lateral force build-up significantly alters the handling behaviour, which is classified as under-steering. For the non-linear vehicle, the lateral force increases



**Fig. 4** First test case: lateral velocity response of the vehicle to a step-steer input of 0.04 rad at 30 m/s forward speed



**Fig. 6** First test case:  $Q_s$  ratio response of the vehicle to a step-steer input of 0.04 rad at 30 m/s forward speed

with a decreasing rate and this is reflected in the normalized yaw impulse  $\Gamma_d$ , which is consistently smaller than that of the assumed neutral vehicle  $\Gamma_{kn}$  (refer to section 2 and reference [5]). It should be emphasized that the transition from neutral to under-steering behaviour is not marked by the peak tyre force, as shown clearly in Fig. 6, where the normalized tyre forces are plotted together with the  $Q_s$  ratio. It is the difference between  $\Gamma_d$  and  $\Gamma_{kn}$  that indicates the transition from neutral to under-steering, always estimated within the accuracy specified by the neutral margin  $\varepsilon$ .

The important conclusion to be drawn from this first case study can be summarized in the following observation. Whereas the steady-state behaviour of the linear and non-linear neutral vehicles is almost indistinguishable, at least up to the point where the tyre forces reach their peak values (that is, if a peak exists at all), the transient response is continuously dependent on the slope of the tyre characteristics and differs considerably between the linear and non-linear neutral vehicles. This difference is successfully captured and quantified by the proposed method of handling assessment. Finally, the  $Q$  and  $Q_s$  ratios have led to the explicit characterization of the unstable part of the response as under-steering, in agreement with the rather descriptive argument stated earlier in this section.

The second test-case involves the vehicle used previously, this time equipped with tyres exhibiting the characteristics illustrated in Fig. 7. The tyre characteristics are such that an over-steering response would be expected at low slip angles, whereas at higher side-slips the rear tyre forces become dominant and the response is anticipated to shift towards under-steer.

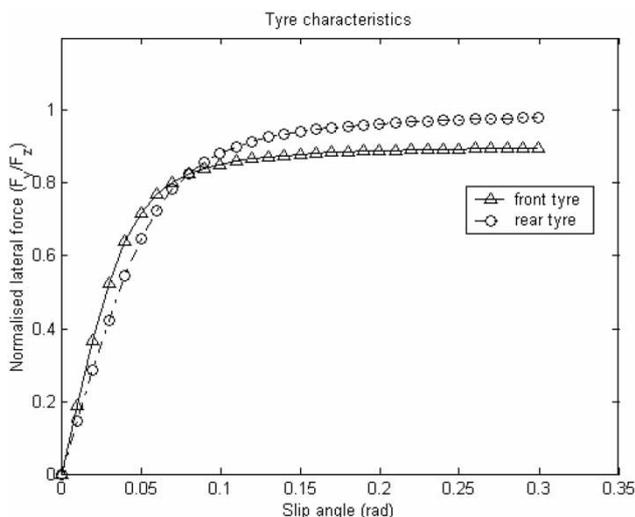


Fig. 7 Second test case: tyre characteristics

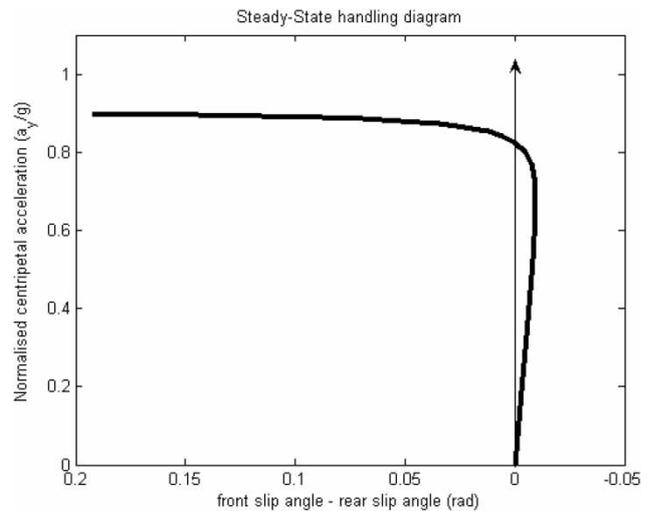


Fig. 8 Second test case: the steady-state handling diagram

It will be shown that this sequence is captured precisely by the proposed method during the response to a step-steer input of 0.04 rad at 30 m/s, which is severe enough to ensure that the vehicle switches between the two possible handling states. Before proceeding to discuss the actual transient response, the steady-state handling diagram in Fig. 8 shows the initial over-steering tendency, which changes gradually to one of under-steer. The transient response is evaluated using the  $Q$  and  $Q_s$  ratios, depicted in Figs 9 and 10, respectively. In Fig. 9, the  $Q$  ratio is initially predicted larger than unity, a fact which, not only points to the initial over-steering behaviour of the vehicle, but also quantifies the exact amount of over-steer,

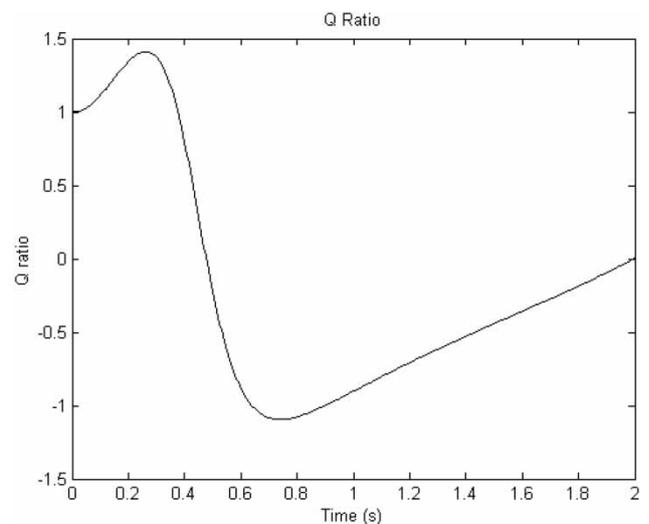
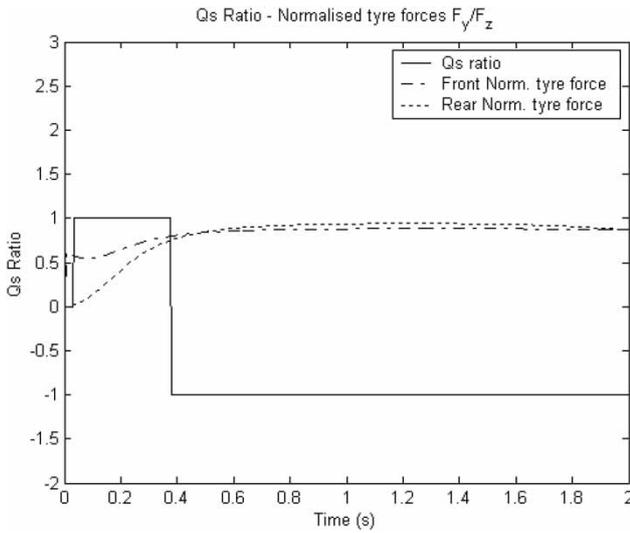


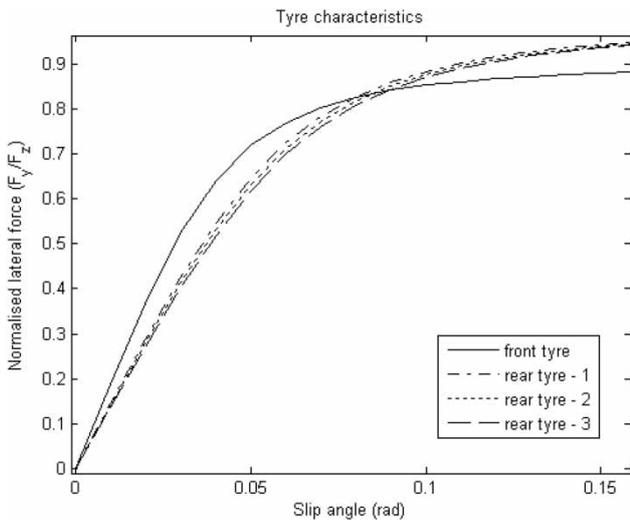
Fig. 9 Second test case:  $Q$  ratio response of the vehicle to a step-steer input of 0.04 rad at 30 m/s forward speed



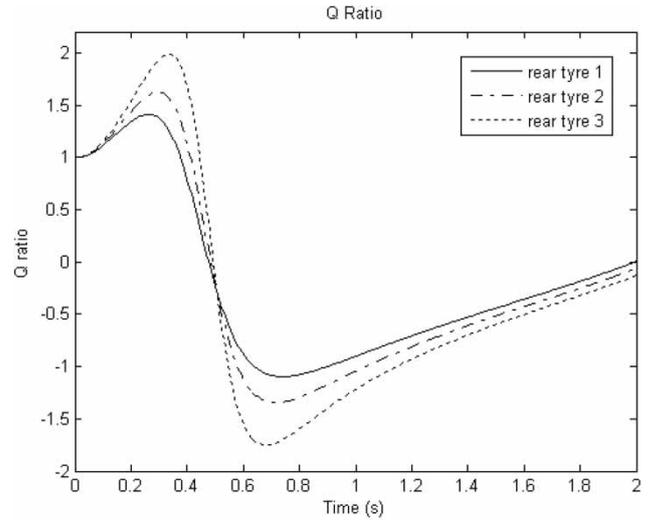
**Fig. 10** Second test case:  $Q_s$  ratio response of the vehicle to a step-steer input of 0.04 rad at 30 m/s forward speed

through the actual value of the ratio. Accordingly, the  $Q_s$  ratio is predicted as unity, explicitly indicating over-steer. As the response progresses, the anticipated switch from over to under-steering is clearly captured by both these.

The usefulness of the  $Q$  ratio is demonstrated further by considering the responses of three different vehicle setups to the same step-steer input of 0.04 rad at 30 m/s. The vehicle details remain the same, while the tyre characteristics are shown in Fig. 11. The front tyre characteristic is identical in all the setups, while the initial slope (the cornering stiffness) of the third rear tyre is 3 per cent smaller than that of the second



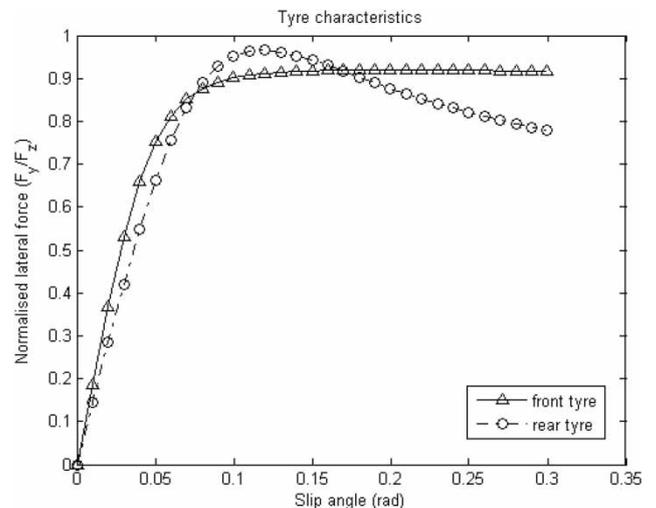
**Fig. 11** Third test case: tyre characteristics corresponding to three different vehicle set-ups with the front tyres remaining the same



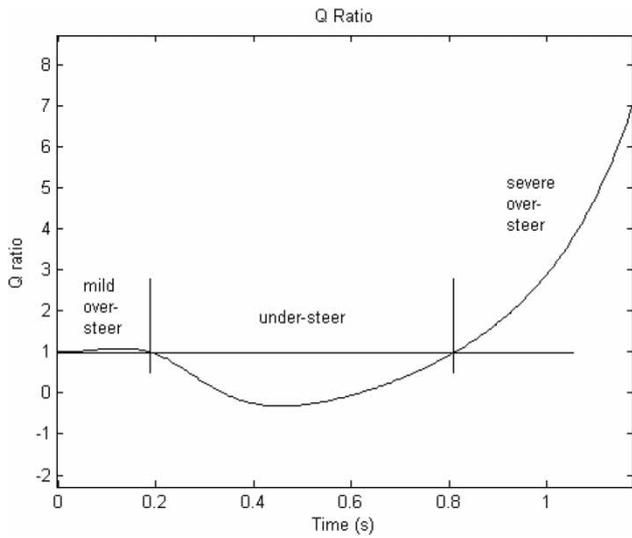
**Fig. 12** Third test case:  $Q$  ratio response of the three different vehicles to a step-steer input of 0.04 rad at 30 m/s forward speed

one, which, in turn, is 3 per cent smaller than that of the first. The resulting  $Q$  ratios are provided in Fig. 12, which shows the exact variation of the amounts of over/under-steer during the course of the transient manoeuvre.

To conclude the step-steer test cases, the rather peculiar tyre characteristics illustrated in Fig. 13 are considered. Based on this, the response is anticipated to be over-steering initially, then tend to under-steer and finally revert to over-steer. To ensure that the vehicle achieves the full sequence of handling behaviours, the forward speed is increased to 60 m/s, the step-steer input remaining equal to 0.04 rad. It is clear that both the  $Q$  and  $Q_s$  ratios confirm these predictions,



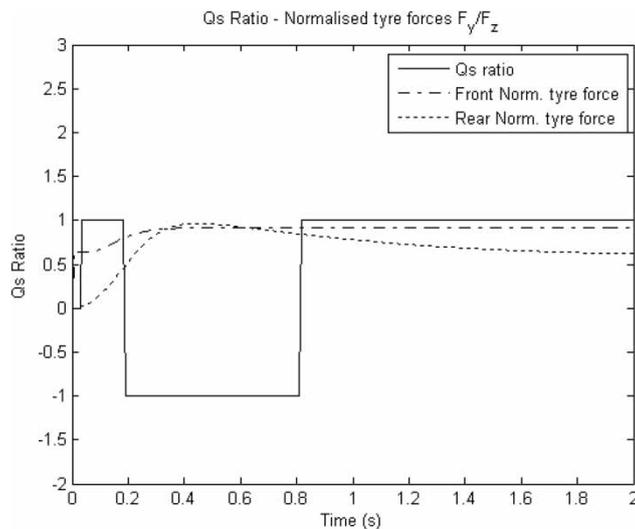
**Fig. 13** Fourth test case: tyre characteristics



**Fig. 14** Fourth test case:  $Q$  ratio response of the vehicle to a step-steer input of 0.04 rad at 60 m/s forward speed

showing a mild over-steering tendency in the beginning of the manoeuvre (Figs 14 and 15). Thereupon, the behaviour switches to under-steer and finally, the vehicle undergoes excessive over-steer, entering an unstable mode of operation.

At this point it is essential to clarify the relation between the steady-state handling behaviour as expressed by the handling diagram and the predictions of the proposed method for the assessment of the vehicle transient behaviour. In the preceding analysis of all test cases, extensive reference was made to the agreement between the handling diagram and the transient assessment method. It should



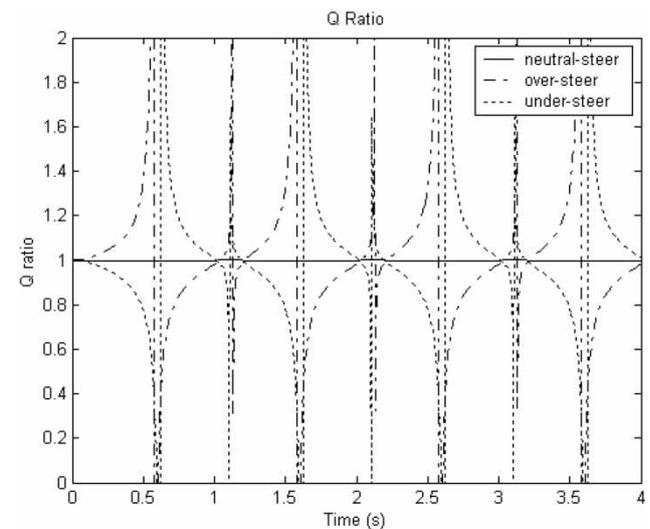
**Fig. 15** Fourth test case:  $Q_s$  ratio response of the vehicle to a step-steer input of 0.04 rad at 60 m/s forward speed

be emphasized that this agreement does not enable the assessment of the transient behaviour by means of the handling diagram, a process which eventually reduces to the calculation of the normalized tyre forces per axle. The fundamental relation upon which the handling diagram relies is given as [3]

$$\frac{F_{yf}}{mg(b/l)} = \frac{F_{yr}}{mg(a/l)} = \frac{a_y}{g} \tag{16}$$

These relationships apply only to steady-state conditions. Therefore, the calculation and comparison of the normalized tyre forces cannot be employed as a general method for the assessment of a transient handling manoeuvre. Indeed, the reference to the handling diagrams is aimed only to show the agreement between the findings of the proposed transient handling assessment method and the expected tendencies.

Hitherto, the step-steer manoeuvre has dominated the analysis. Similar results have been found in the case of a ramp-steer manoeuvre. However, as a final test case, the somewhat more interesting response to a sinusoidal steering input is presented. Three hypothetical vehicles are subjected to a sinusoidal steering with amplitude equal to 0.02 rad and a frequency of 0.5 Hz. The forward speed in all cases is equal to 15 m/s and the neutral margin  $\epsilon$  is kept equal to 0.01. The first vehicle is neutral, according to steady-state terminology, characterized by the tyre curves shown in Fig. 1. The second vehicle is made to over-steer, according to the steady-state terminology by reducing the initial slope of the cornering curve of the rear tyre by 5 per cent. Finally,



**Fig. 16** Fifth test case:  $Q$  ratio response to a sinusoidal steering input of a neutral-, over- and under-steering vehicle

the third vehicle is made to under-steer by increasing the cornering stiffness of the rear tyres by 5 per cent. Figure 16 shows the  $Q$  ratio for all three vehicles. The deterioration of the  $Q$  ratio as a handling index under such operating conditions immediately becomes evident. For example, the over-steering vehicle (dash-dotted line) shows a  $Q$  ratio increasing rapidly until it changes its sign abruptly. This is the result of the neutral kinematic yaw impulse (denominator in equation (5)) reducing steadily and finally becoming negative. As a result, the  $Q$  ratio yields sensible results only up to  $\sim 0.5$  s after the initiation of the steering input. It is interesting to note, though, that the neutral vehicle (solid line) exhibits a  $Q$  ratio equal to unity throughout the manoeuvre. Magnification of a small area around the solid line, as depicted in Fig. 17, shows that, in reality, the  $Q$  ratio of the neutral vehicle oscillates periodically. This is expected, since, according to the theory presented in section 2,  $\Gamma_d = \Gamma_{kn}$  at any instant only for the linear neutral vehicle. The forward speed and maximum steer-angle are deliberately made relatively low in order to ensure operation in the almost linear range of the tyres. However, the slight non-linearity of the tyre curves near the origin shows itself through the  $Q$  ratio observed.

Figures 18 and 19 depict the robust  $Q_s$  ratio for the over- and under-steering vehicles, respectively. The corresponding  $Q_s$  ratio for the neutral vehicle has not been included. By observing the maximum fluctuation of the  $Q$  ratio as in Fig. 17 and considering the fact that  $\varepsilon = 0.01$ , relation (6) yields a  $Q_s$  ratio equal to unity throughout the manoeuvre. According to Figs 18 and 19 the responses of both the over- and under-steering vehicles change suddenly from over-

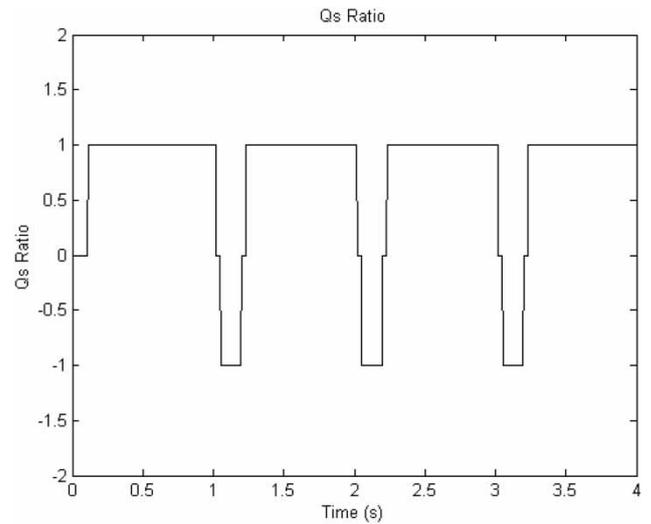


Fig. 18 Fifth test case:  $Q_s$  ratio response to a sinusoidal steering input of an over-steering vehicle

to under-steering and vice-versa. This odd prediction is easily explained considering, for the sake of argument, the response of the under-steering vehicle, depicted in Fig. 19. According to the  $Q_s$  ratio, the vehicle is predicted to under-steer up to approximately half a period of the excitation. During that time, the steer-angle is positive. Because the vehicle under-steers, towards the end of the half period it possesses a less 'steered' state than the state it would have possessed if it were neutral. As the steering starts to become negative, the less 'steered' vehicle is ready to steer to the opposite direction more than it would if it were neutral (i.e. the vehicle temporarily over-steers). After a short period, equilibrium is re-established and

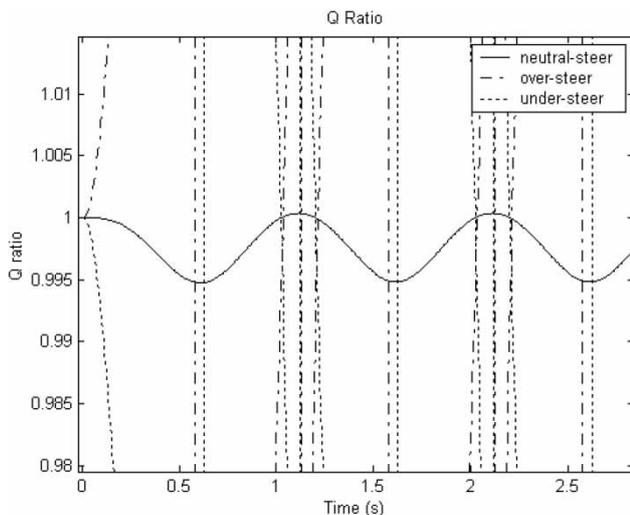


Fig. 17 Fifth test case: detail from Fig. 16 showing the oscillatory nature of the  $Q$  ratio for the neutral non-linear vehicle

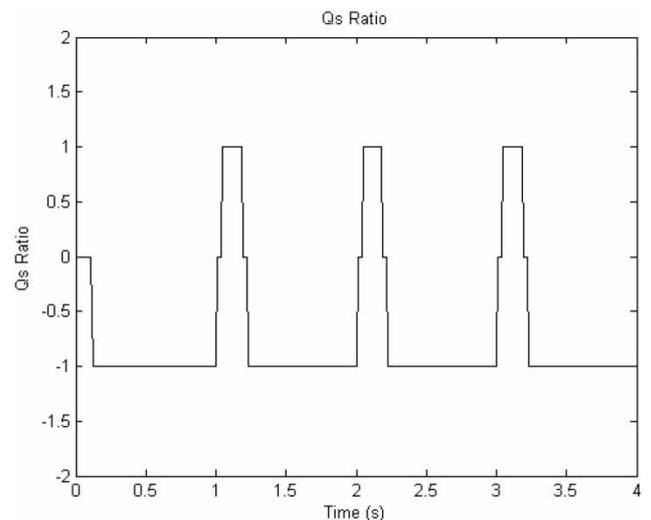


Fig. 19 Fifth test case:  $Q_s$  ratio response to a sinusoidal steering input of an under-steering vehicle

the vehicle exhibits under-steering behaviour for the rest of the negative steering, until, approximately, the end of a full steering cycle. A similar analysis applies to the over-steering vehicle. An important observation is that the sudden changes in the responses of both non-neutral vehicles cover a small fragment of the full steering cycle. Otherwise, most of the time during the cycle, the characterization of the transient response of the vehicles through the  $Q_s$  ratio agrees with their steady-state classification as over- or under-steering.

Finally, it should be mentioned that the prediction of over-steering regimes in the operation of an under-steering vehicle and vice-versa might prove to agree with drivers' perception. For instance, it is generally experienced that, when attempting a slalom manoeuvre with an over-steering vehicle, additional steering is required during the change in the direction of steer (change of sign), in order to achieve the required path. This additional steering might be perceived as the result of a temporarily under-steering behaviour. However, this is only a preliminary observation and any definite relation between the predictions of the proposed framework and driving experience requires experimentation and possibly statistical analysis of the view of a large number of drivers.

## 5 COMMENTS ON THE METHODOLOGY

Throughout the previous analysis, frequent references were made to the stability/instability of vehicles. The present framework does not intend to investigate such issues. These references aim primarily to highlight some traditional notions. For example, the fact that only an over-steering vehicle might show instability in its yaw motion, or, in other cases to point out less traditional findings, for instance the fact that a neutral non-linear vehicle might become unstable in terms of its lateral speed. These predictions are well documented and theoretical approaches for the assessment of vehicle stability can be found in references [1–4], or indeed in early works such as in reference [11].

When only transient handling data is available, some of these notions might work in reverse: a continuously increasing yaw rate points towards over-steer. However, when the motion is stable, it is difficult to draw conclusions. Considering the best scenario, one might try to identify the system parameters from the test data and then construct a virtual steady-state test for the determination of whether the vehicle is under/over or neutral-steering, in steady-state terms. In the most likely case of a non-linear vehicle, the handling diagram might show a transition between different behaviours. Having

arrived this far, it is impossible to work backwards, i.e. to argue that at instant  $t_0$  during the course of the transient manoeuvre the vehicle was actually operating in its over/under or neutral-steering regime.

This is the aim of the proposed methodology. The neutral-, over-, and under-steering behaviours under transient conditions have been defined in reference [5] and are described in section 2 of the present paper, in a way similar to the axiomatic definition of the same behaviours under steady-state conditions. The steady-state definitions have proven to be sensible, as, for example, an over-steering vehicle tends to reduce the radius of curvature of its path as the forward speed increases during cornering and therefore requires the reduction of steer-angle to remain on the same path. The judgement is not as straight-forward under transient conditions. In all cases, the yaw rate increases as a result of a moment imbalance between the front and rear ends of the vehicle. The present methodology proposes the use of the neutral kinematic yaw impulse as the benchmark, which signifies the neutral behaviour. It is subsequently observed that a vehicle with a greater dynamic yaw impulse exhibits a greater normalized yaw moment which is related to a greater contribution of the front tyres as opposed to that of the rear ones. Compared to the steady-state situation, the focus is now moved from the balance of front/rear slip angles to the balance of the normalized front/rear yaw moments and the behaviour is deemed to be over-steering when this balance tends to steer the vehicle further than in the assumed neutral case. A similar logic applies to the under-steering case.

The usefulness of the approach is enhanced by the fact that the transient results show agreement with the transitions in behaviour predicted by the steady-state handling diagrams. Therefore, with a single set of transient handling data it is possible to: (a) quantify the vehicle's handling behaviour and relate it to the influence of factors such as the normalized tyre characteristics and (b) form a view about the general characteristics of the vehicle.

A practical example might clarify the situation further: a highly non-linear vehicle is assumed performing a lap on a racing track. The vehicle never becomes unstable and there is no steady-state test available for obtaining a handling diagram. Even if there were a steady-state test, the corresponding conditions would probably not be similar to the track conditions. By calculating the  $Q$  and  $Q_s$  ratios, it is possible to obtain a picture of the relative contribution of the front and rear ends of the vehicle during the lap. This information can be used in various ways for the improvement of the handling performance.

## 6 CONCLUDING REMARKS

The handling assessment method introduced in reference [5] has been successfully applied for the characterization of transient handling responses of non-linear vehicles. The results of the method are found to be in accord with the existing steady-state theory and the method appears to be capable of capturing the effects of various combinations of highly non-linear tyre characteristics. The main aim of the method is to explicitly characterize the instantaneous response during the course of any single transient manoeuvre, without relying on the comparison of a number of different responses, irrespective of whether the response is stable or unstable. The fundamental strengths and weaknesses of the method have been discussed in detail in reference [5], while the present work concentrates on the interpretation of the results from the viewpoint of quantification of the influence of tyre non-linearities. Undoubtedly, the next step would be experimental verification of the method. In this respect, the successful indirect calculation of front and rear tyre forces, using the corresponding accelerations and inertias, would enable experimental application of the method without the need for sophisticated force-measuring wheel hubs. Furthermore, the choice of the neutral margin  $\varepsilon$  could probably be related to the subjective assessment of the response by the driver, resulting in an assessment framework with direct practical implications, which might subsequently be used for the optimization of transient handling behaviour.

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## APPENDIX

### Notation

$a$	distance of the centre of gravity from the front axle
$a_f$	front slip angle
$a_r$	rear slip angle
$a_y$	centripetal acceleration
$b$	distance of the centre of gravity from the rear axle
$B$	stiffness factor (magic formula)
$B_1, B_2$	boolean expressions
$C$	shape factor (magic formula)
$D$	peak value (magic formula)
$E$	curvature factor (magic formula)
$F_{yi}$	lateral force at the $i$ th corner of a vehicle
$F_z$	normal load on tyres
$g$	acceleration of gravity
$I_{zz}$	yaw moment of inertia
$l$	wheelbase
$m$	mass of the vehicle
$Q, Q_s$	ratios
$r$	yaw rate
$S_H$	horizontal shift (magic formula)
$S_V$	vertical shift (magic formula)
$U$	forward velocity
$V$	lateral velocity
$\Gamma_d$	dynamic normalized yaw impulse
$\Gamma_k$	kinematic normalized yaw impulse
$\Gamma_{kn}$	neutral kinematic normalized yaw impulse
$\delta$	steer angle
$\varepsilon$	neutral margin
$\rho$	parameter
<i>Subscripts</i>	
$f$	indicates the front of the vehicle
$r$	indicates the rear of the vehicle
$y$	lateral direction
$z$	normal direction