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Short Communication

Rate of shear of an ultrasonic oscillating rod viscosity probe

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ABSTRACT

Ultrasonic oscillating rod probes have recently been used by researchers to measure viscosity and/or density in fluids. However, in order to use such probes to characterise the rheological properties of fluids, it is necessary to define the shear rate produced by the probe. This paper proposes an analytical solution to estimate the shear rate of ultrasonic oscillating rod viscosity probes and a method to measure their maximum operational shear rate. A relationship is developed which relates the torsional surface velocity of an oscillating cylindrical rigid body to the rate of shear in its vicinity. The surface displacement and torsional surface velocity of a torsional probe of length 1000 mm and diameter 1 mm were measured over the frequency range from 525 to 700 kHz using a laser interferometer and the maximum shear rate estimated. The reported work provides the basis for characterising shear rate for such probes, enabling their application for rheological investigations.

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1. Introduction

The rheological characteristics of fluids influence many aspects of their performance, such as pumpability, droplet breakup in spray drying, emulsion formation, flow into moulds, formability and so forth, and qualities such as stability during and after processing. Hence, quantitative knowledge of the rheological properties of liquids and slurries is a crucial requirement for process design, process control, optimisation and the production of consistent quality products in many process environments such as petro- and speciality-chemicals, oil/gas, food, pharmaceutical, pulp, paper and nuclear. Among the rheological properties of fluids, viscosity is an integral component of a large number of quality control procedures in the processing of complex liquids [1].

Monitoring viscosity on-line and in-situ provides real-time data that can be used to optimise the process and support product quality. Viscosity can be expressed as the proportionality of the force (the shear stress) to the relative rate of movement (the rate of shear strain or shear rate). This proportionality can be independent of shear rate, in the case of Newtonian fluids, or vary with shear rate for non-Newtonian fluids. It is common to present viscosity as a function of shear rate in conventional methods of viscosity measurement. Hence, knowledge of the *shear rate* is of paramount importance in the study of fluids and their rheological characteristics such as viscosity.

Ultrasonic oscillating rod viscosity probes have been used to measure viscosity in liquids by many researchers [2–10]. This type of viscosity probe is robust and contains no moving parts and also does not represent hygiene risks. The instrument is of low cost so that it can be disposable after use. Therefore, there is a great interest for such an instrument in industrial applications where knowledge of its characteristics such as shearing rate can be beneficial.

In operation a rod, usually of metal, is immersed in the liquid to be tested. Transducers excite ultrasonic torsional (twisting) waves at one end of the rod, and these travel along the rod and are reflected back to the transducer either from the end of the rod or from an embedded discontinuity part way down. The echoes thus received are amplified and processed to extract the propagation velocity and/or the attenuation of the waves in the rod. Both velocity and attenuation are affected by coupling between the motions of the rod surface associated with the propagating wave in the rod and motions in the liquid in contact with the rod. Thus, depending on the wave mode, either the attenuation or the velocity can be used to calculate the viscosity of the surrounding liquid.

Viscosity measurements made with ultrasonic oscillating rod viscosity probes have been reported in a number of works. When testing general purpose Cannon hydrocarbon oil viscosity standards (Cannon Instrument Company, State College, PA, USA) with an ultrasonic oscillating rod operating in the 50–100 kHz range Kim and Bau [2], Costley et al. [5] and Vogt et al. [8] showed that the measurements agreed well with viscosity measurements using conventional rheometers. However, Ai and Lange [9] did not find good agreement in the higher frequency range 300–400 kHz for

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Nomenclature

\mathbf{V}	shear wave velocity field	ω	angular frequency
Φ	vector potential	η	dynamic viscosity
k	wave number	ρ	density
ν	kinematic viscosity	$\dot{\gamma}_{max}$	maximum shear rate

Cannon polybutene viscosity standards. Similarly, Shepard et al. [7] were unsuccessful in measuring general purpose Brookfield silicone fluid viscosity standards (Brookfield Engineering Laboratories Inc., Middleboro, Massachusetts, USA) in the frequency range 50–100 kHz and a varying shearing rate with an estimated effective shear rate about 25 s^{-1} ; they related the inaccurate results to non-Newtonian behaviour of such liquids under relatively low and varying shear rate. Successful comparisons were reported by Shepard et al. [7] and Vogt et al. [8] for simpler liquids such as glycerol/water mixtures. An example of high temperature operation was demonstrated by Costley et al. [5] who measured the viscosity of molten glass; the shear rate was not defined. Recently, Rabani et al. [10] showed the oscillating rod viscosity probe to work well when applied to Newtonian fluids; however, anomalous behaviour was observed in the measurement of the viscosity of engine, gear and silicone oils.

The measured viscosities of these materials appeared to be very much lower than the values obtained with a conventional rheometer, using an oscillating rod with operating frequencies from 500 kHz to 700 kHz and an estimated maximum shear rate of 5000 s^{-1} . This anomalous behaviour was explained by molecular relaxation phenomena, which has a significant effect on the viscosity of polymer-based fluids at the probe frequency due to their long chain molecules. The data were successfully fitted to a relaxation model [10].

It would thus appear that the ultrasonic oscillating rod viscosity probe successfully measures the viscosity at its operating frequency, which may be different from the viscosity at a lower frequency. Since for non-Newtonian fluids, viscosity is strongly dependent on shear rate, it is important to characterise the shear rate produced by the probe. This paper comprises an analytical solution for the estimation of apparent shear rate in a cylindrical rod under torsional oscillation surrounded by a viscous medium and an experimental programme to determine indirectly the share rate. Surface displacement and velocity of an oscillating rod viscosity probe were measured and the maximum shear rate was estimated from these data. As far as the current authors are aware, these important considerations have not been discussed in the literature to date.

2. Theoretical basis

Propagation of torsional stress waves along the rod results in energy leakage into the surrounding fluid in the form of bulk shear waves. The fluid adjacent to the rod can be considered incompressible since the compressional waves play no part and the fluid movement can be considered laminar with constant velocity on any cylindrical locus concentric to the rod [11].

Defining cylindrical polar coordinate system, (r, θ, z) , for the rod with z axis along the rod axis (Fig. 1), the velocity field for shear waves, $= (v_r, v_\theta, v_z)$, produced in the fluid by the probe can be expressed using a vector potential Φ such that:

$$\mathbf{V} = \nabla \times \Phi \quad (1)$$

where Φ satisfies the Helmholtz equation:

$$\nabla^2 \Phi + k^2 \Phi = 0 \quad (2)$$

with k wavenumber:

$$k = (1 + i) \left(\frac{\omega}{2\nu} \right)^{\frac{1}{2}} \quad (3)$$

where ν is the kinematic viscosity of the fluid ($\nu = \frac{\eta}{\rho}$) and ω is the angular frequency of the bulk shear waves.

Fig. 1 shows cylindrical polar coordinates at a cross-sectional plane of the probe with axial symmetry and the illustrated configuration. In this configuration the vector potential has only z component, hence, $\Phi = (0, 0, \varphi)$ and the appropriate solution for this geometry, with harmonic dependence, for a single frequency, ω , is in the form of Hankel functions [12]. Since ka is large, the solution can be taken in the limit of $kr \gg 1$ with an amplitude of φ_0 , so that:

$$\varphi(r, t) = \varphi_0 \frac{e^{i(kr - \omega t)}}{\sqrt{kr}} \quad (4)$$

From Eqs. (1) and (4), the velocity in the fluid is:

$$v_\theta(r, t) = -ik\varphi_0 \frac{e^{i(kr - \omega t)}}{\sqrt{kr}} \quad (5)$$

For the simple shear case, the maximum shear rate is:

$$|\dot{\gamma}_{max}| = \left(\frac{\partial v_\theta}{\partial r} \right)_{max} = \left(\frac{\omega}{\nu} \right)^{\frac{1}{2}} |v_{\theta_{max}}| \quad (6)$$

where $|v_{\theta_{max}}|$ is the velocity amplitude at $r = a$. Thus, by measuring $|v_{\theta_{max}}|$ one can estimate the amplitude of the shear rate in the fluid at the surface of an oscillating cylindrical rod.

3. Experiments

Measurement of surface displacement and corresponding surface velocity of the oscillating rod is required for the estimation of rate of shear induced by the rod oscillation. A series of experi-

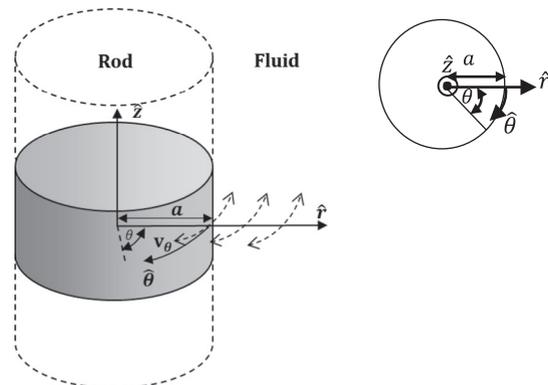


Fig. 1. Cylindrical polar coordinate system for the probe and velocity at the rod – fluid interface.

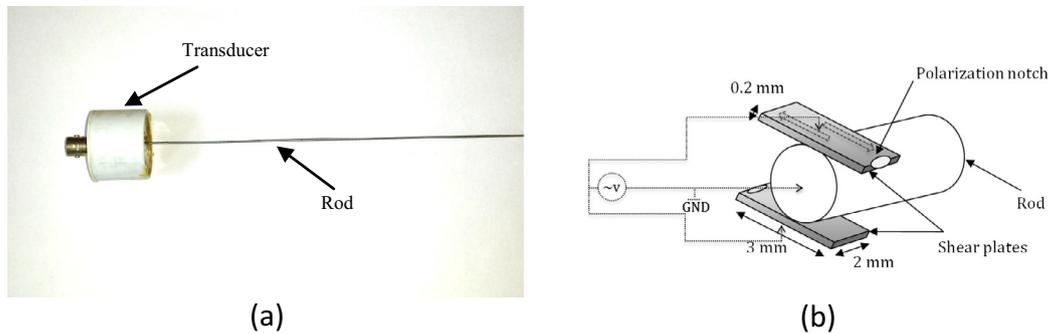


Fig. 2. (a) Oscillating rod viscosity probe with 1 mm carbon steel rod attached to a piezoelectric based transducer. (b) Internal arrangement of the transducer: two shear plates in contact with the rod.

ments were conducted using an ultrasonic oscillating rod viscosity probe and a laser interferometer.

The viscosity probe used in the experiments was made of circular cross-section carbon steel (piano wire) with 1 mm diameter (Fig. 2a). PIC 255 piezoelectric ceramic plates (PI Ceramic GmbH, Lederhose, Germany) which operate in thickness shear mode were used to generate torsional oscillations. In order to excite torsional oscillations in the rod, the method of Vogt [13], was utilised. Two rectangular shear plate transducers (3 mm × 2 mm × 0.2 mm) were coupled on opposite sides at one end of the axially-elongated rod with direction of polarisation opposed each to other (Fig. 2b); when a voltage is applied to the plates in a direction perpendicular to the direction of the polarisation, shear stresses couple in a plane perpendicular to the axis of the rod which generates a torsional wave in the rod.

The shear plates are connected to the driving voltage source using wires soldered to their outer faces. They are bonded to the rod using a very thin layer of low viscosity conductive epoxy (CW2400, Circuitworks, Kennesaw, USA). The ground electrode is connected to the rod using the same type of conductive epoxy. The electrical connections for the transducer are also shown in Fig. 2b. Hanning windowed 10-cycle tone bursts were used for excitation of the torsional oscillations in the rod. Further details of the design and operation of the probes are given in Rabani [14].

An in-plane laser Doppler interferometer (two oblique incidence OFV 505 sensor heads, with OFV 5000 controllers, Polytec GmbH, Waldbronn, Germany) was used to measure the in-plane velocity (θ direction) and displacement at the surface of the rod.

In laser vibrometry the induced Doppler effect imposed on the reflected laser beam that results from movements of the test object enables the calculation of velocity and displacement at the surface of the object [15,16].

The application of a laser interferometer to the measurement of in-plane vibration in a cylindrical rod is illustrated in Fig. 3. The rod was placed horizontally on a table in air. Two laser beams were aligned to the surface of the rod in the plane normal to the rod axis. The intersection of the laser beams was set to the midpoint of the rod length and all the measurements were made at a single point on the surface of the rod.

Setting the angle α to be 30° , the in-plane velocity, v_θ , can be calculated by subtracting the measured velocity components of the beams, v_R and v_L , from each other. Accordingly, for the case of fundamental torsional stress waves propagating along the rod the in-plane displacement, u_θ can be obtained by dividing the in-plane velocity, v_θ , by the angular frequency, ω , of the propagating shear waves.

A digital oscilloscope (DSO, LeCroy 9400A, New York, USA) was used to digitise the received analogue signal. The oscilloscope used a GPIB interface (IEEE 488) to transfer the digitised data to a personal computer for further processing.

It was assumed that the propagating waves have constant properties along the length of the rod. A very thin reflective tape was stuck to the rod to enhance the reflection of the laser beams from the surface of the rod. Some errors are expected in the measurement results due to the possible shift between the laser decoders as the rod was very thin and therefore difficult to place exactly at the intersection of the two laser beams. Measurements were made at eight different frequencies ranging from 525 kHz to 700 kHz in steps of 25 kHz and three different excitation voltages, 50, 60 and 75 V; 20 averages were made on the received signals to suppress noise.

4. Results

Surface displacement and velocity of the torsional waves excited in the probe rod for three different excitation voltages were calculated and are shown along with the linear fit through the origin for a range of frequencies in Fig. 4a and b. The surface displacement of the probe rod is of the order of nanometres and the surface velocity is of the order of millimetres per second. Surface displacement and ultimately the torsional velocity increase with an increase in the excitation voltage irrespective of the frequency of the oscillations; this agrees with the theory (see Eq. (5)). There is maximum $\pm 5\%$ variation associated with the measured data from the fitted lines. The possible sources of these variations are (a) the presence of elements of other wave modes such as flexural and longitudinal in the excited waves due to the transducers

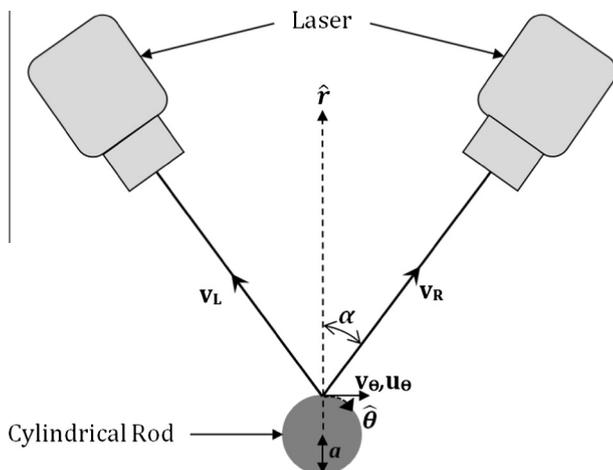


Fig. 3. Schematic of the measurement of in-plane vibration (velocity and displacement) for cylindrical rod under ultrasonic torsional stress waves.

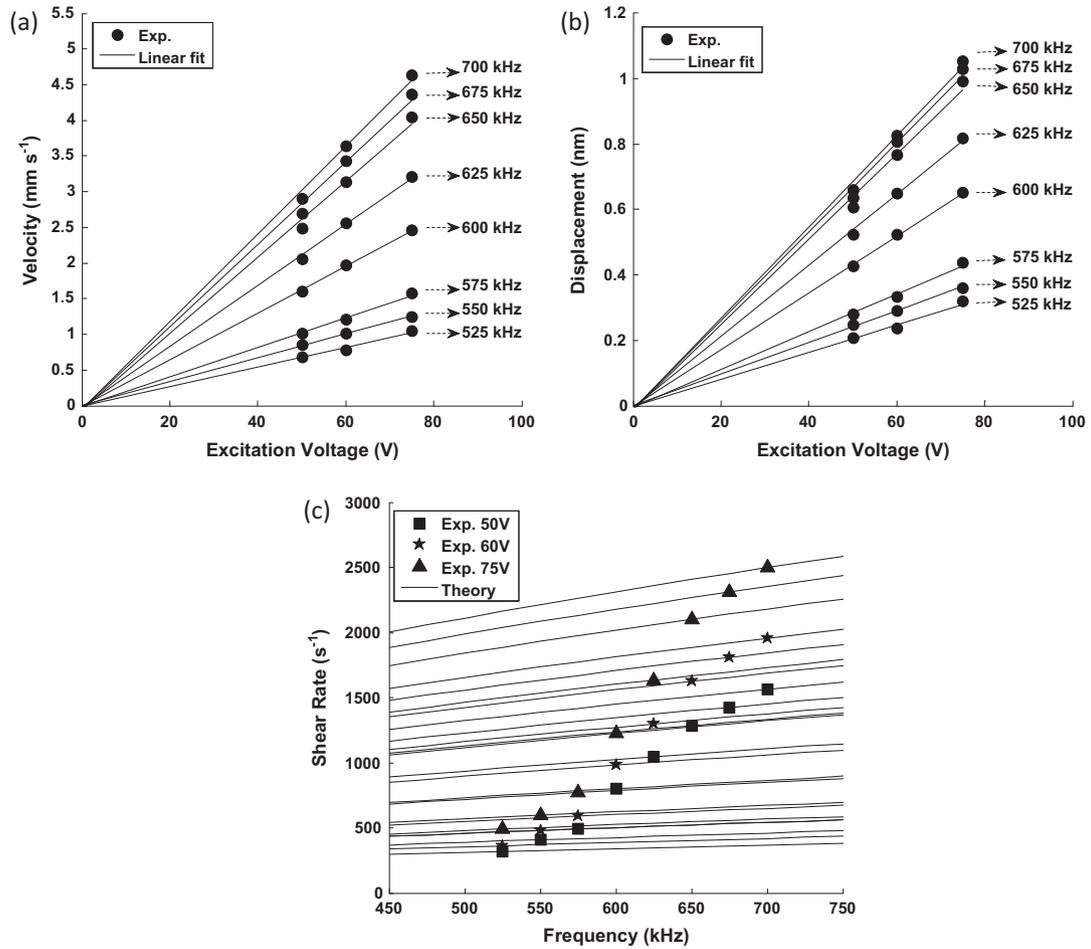


Fig. 4. (a) Surface velocity and (b) surface displacement of ultrasonic oscillating rod viscosity probe with 1 mm carbon steel rod waveguide in air. (c) Estimated maximum shear rate produced by the probe in a hypothetical fluid of viscosity of 18.6 $\mu\text{Pa s}$ and density of 1.225 kg m^{-3} .

design and assembly; (b) a slight shift between the laser beams due to the experimental setup constraints.

The maximum shear rates are calculated using Eq. (6) from the measured in-plane rod surface velocities shown in Fig. 4a, and are shown in Fig. 4c. In the calculations of maximum shear rate a hypothetical fluid with viscosity of 18.6 $\mu\text{Pa s}$ and density of 1.225 kg m^{-3} equal to those for air were considered.

Therefore, the maximum shear rates produced by the probe in air are of the order of 1000 s^{-1} that can be regarded as the amplitude of the oscillating shear rate for the probe at its surface at the measured excitation voltage and frequency. The actual shear rate in the fluid in the vicinity of the probe will in reality be lower due to the viscous drag forces and corresponding reduction of the surface velocity of the rod.

The results show that the shear rates ($\leq 2500 \text{ s}^{-1}$) generated by the 1 mm diameter carbon steel ultrasonic oscillating rod viscosity probe are within the range of the shear rates (1–20,000 s^{-1} [17]) that conventional rotational viscometers can generate. However, the frequency of the torsional probe (525–725 kHz) is much higher than that of most rotational viscometers. The frequency–diameter product of the probe should be selected in a way that the required torsional mode travels at a speed that is significantly different from that of the unwanted modes to allow time gating of the torsional signal for further processing. This requires an operating frequency well below 1 MHz for the 1-mm-diameter carbon steel probe. If the diameter was increased, this frequency limit and consequently the shear rate would be decreased. Therefore, it is possible to design probes that can collectively measure viscosities at shear rates rang-

ing from 10- s^{-1} up to 1000- s^{-1} ; this range adequately covers a large span of shear rates of technical processes (10⁻³–10⁷ s^{-1} [18]). For each individual probe the magnitude of the generated shear rate can be controlled by adjusting the excitation voltage and frequency. This provides the possibility to make measurements at several shear rates and extrapolate the data to the projected values where knowledge of viscosity at the projected shear rates is essential.

5. Conclusions

This paper has described the theory for shear rate estimation for ultrasonic oscillating rod viscosity probes. A method based on laser interferometry for measurement of surface displacement, velocity and corresponding maximum shear rate of the oscillating probe has been developed. The surface displacement and the surface velocity of the probe rod in air were measured and corresponding shear rates were estimated for a 1 mm diameter probe. Shear rates were found to be of the order of 1000 s^{-1} in air at the rod surface. The shear rate varies with applied voltage and increases at higher frequencies. Consequently, the maximum limit for the shear rates that would be induced into a fluid in contact with the probe surface at measured voltages and frequencies has been established; whereas, previously there was no actual knowledge of generated shear rates at the time of viscosity measurements using such probes. The probe shear rate is expected to be reduced when the probe is immersed in a viscous liquid. Therefore, further evaluation

of the rod surface displacement and velocity in a liquid are required in order to better estimate the shear rate in that liquid. In this case, the reduction in the amplitude of the shear rate can potentially be measured either by measuring the surface displacement and velocity of a free section of the probe out of the test fluid using interferometry or by calculation of the reduction in surface displacement and velocity from the attenuation of the travelling torsional waves along the probe.

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