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MEASURING TEACHER AWARENESS OF CHILDREN'S UNDERSTANDING OF EQUIVALENCE

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Children's understanding of the equals sign has been widely studied and identified as an important issue for thinking flexibly about arithmetic and learning algebra. Experiences in primary mathematics lessons impact significantly on understanding, but relatively few studies have investigated primary teachers' awareness of children's understanding. One reason may be that while carefully validated instruments exist for measuring children's understanding, no such instrument is available for evaluating teacher awareness. We analyse the performance of a questionnaire administered to 197 primary teachers in New Zealand and the UK, and identify how individual items are likely to elicit different teacher responses.

INTRODUCTION

There has been an increased focus on the teaching and learning of algebraic reasoning in recent years (Kaput, 2008; Knuth, Stephens, McNeil & Alibali, 2006). An outcome of this focus is a growing consensus between researchers and educators that algebra can be introduced at a much younger age with a focus on the integration of teaching and learning arithmetic and algebra in classrooms (e.g., Department for Education and Employment, 1999; Ministry of Education, 2007). Essential to effective teaching and learning is understanding the equals sign as a representation of an equivalence relationship, and using this understanding to work flexibly with numbers and expressions.

Previous studies (e.g., Freiman & Lee, 2004; Knuth et al., 2006) have identified three types of student responses to problems involving equivalence. These responses reflect an *operational* view in which the equals sign is an indicator for a numerical result, a *sameness* view in which the equals sign indicates the same value is on each side, and a *relational* view in which arithmetic and algebraic relationships are exploited to reduce computational burden when establishing equivalence. For example, consider the problem $9 + 6 = _ + 5$. Students adhering to an operational view may put 15 in the blank space and those adhering to a sameness view may put 10. Those adhering to a relational view, which subsumes sameness, may put 10 by noticing that the solution must be 1 more than 9 and avoiding further computation.

The operational view has shown to be dominant and resistant to change in many countries (McNeil & Alibali, 2005), leading to inflexible thinking about equivalence, arithmetic and algebra (Knuth et al., 2006). Helping children overcome or avoid operational thinking requires careful planning and teaching of mathematics lessons (e.g. Li, Ding, Capraro & Capraro, 2008). However, while student misconceptions have been widely studied and findings replicated across different contexts there is less research available on primary teachers' knowledge of typical student misconceptions regarding the equals sign (exceptions are A. Stephens, 2006; Zhang and M. Stephens, 2013). It is therefore not well understood how aware primary teachers are that children view the equals sign in varied ways, or the impact that teacher awareness might have on student conceptions. One reason this important area is under-researched may be that whereas carefully validated instruments exist for assessing student knowledge (Rittle-Johnson, Matthews, Taylor & McEldoon, 2011), there is no standardised way of evaluating teacher awareness of student knowledge. In this paper we analyse the performance of items designed to evaluate teacher awareness of their students' conceptions of equivalence.

METHODOLOGY

Questionnaire design

As a starting point to developing an instrument we turned to studies by Zhang and M. Stephens (2013) and A. Stephens (2006). Zhang and M. Stephens (2013) analysed teachers' responses to students' solution strategies to two missing-number equations, similar to Item 3 in Table 1. The authors reported that the teachers responded in a variety of ways, suggesting the two items were appropriate for eliciting teacher knowledge. However, the results are likely to be partly dependent on the particular tasks presented to teachers, as has been reported to be the case for children (Rittle-Johnson et al., 2011). Similarly, A. Stephens (2006) investigated teacher trainees' responses to five equivalence items, one of which was a definitional item and four were equation items. A. Stephens reported that the number of trainee teachers providing relational responses varied across the five items. Two of the items (similar to Items 4 and 7 in Table 1) elicited relational responses 80% of participants, a further two items (similar to Item 1 below and Item 2 in Table 1) elicited relational responses from about two thirds of the participants, and one item (Item 8 in Table 1) elicited relational responses from about half of the participants. The study reported here seeks to build on this work by identifying the performance of different items, and the range of responses they elicit, for the case of practicing primary teachers.

Item	Prompt	Item	Prompt
2	$15 + 8 = _ + 10$	5	$8 + f = 8 + 6 + 4$
3	$24 - _ = 21 - 15$	6	$8 + 2 + h = 10 + 6$
4	$37 + 15 = 52$ is true. Is $37 + 15 + 8 = 52 + 8$ true or false?	7	$99 + 87 = 98 + 86 + p$
		8	The solution to $2n + 15 = 31$ is 8. What is the solution to $2n + 15 - 9 = 31 - 9$?

Table 1: The equation items administered to teachers.

We adapted the items used by Zhang and M. Stephens (2013) and A. Stephens (2006) to construct an instrument comprising eight items in total. This included one definitional item (Item 1) which asked participants to “list possible student responses to the question ‘what does the equals sign (=) mean?’” A further seven items involved equations (see Table 1) and were adapted from the examples discussed in the above paragraph. For these seven items, the participants were asked to identify “what answers would you expect students to give and what strategies might they have used to get those answers?”

A key difference between our approach and that used by Zhang and M. Stephens (2013) and A. Stephens (2006) is that we did not provide participants with sample student solutions. This was done to avoid prompting specific responses. Our instrument was therefore expected to produce relatively fewer relational responses overall than that used by A. Stephens. Moreover, we expected participants to provide fewer relational responses for Item 8 than for other the other items based on A. Stephens’s findings. We also expected participants would provide fewer relational responses for Item 1 than for the other items. Although A. Stephens found two thirds of trainee teachers provided relational responses to a similar definitional item, studies with children have most do not respond with relational definitions or examples of equivalence. For Items 2 to 7 the study was exploratory and no firm predictions about their performance were made.

Participants

Participants in the study consisted of 197 primary teachers. Forty-nine of the teachers were in the United Kingdom and all of these teachers completed the questionnaire online. One hundred and forty-eight of the teachers were in New Zealand of which ten completed the questionnaire online and the remaining 138 completed a paper version of the questionnaire.

Code	Solution and explanation	
Operational	23	Added the 2 numbers without considering the 10 on the RHS.
Sameness	13	Understand the concept of balancing the equation.
Relational	13	10 is 2 more than 8 so you need to take 2 away from 15.

Table 2: Examples of coding of responses for Item 2 ($15 + 8 = _ + 10$).

Coding procedure

Responses to all items were coded as operational, sameness and/or relational. Responses coded operational were those that referred to student misconceptions related to the equals sign. Responses coded sameness referred students understanding the need to maintain equivalence on either side of the equals sign. Responses coded relational referred to students drawing on arithmetic and algebraic relationships to maintain equivalence. Examples of codes for Item 2 are shown in Table 2.

The codes were not exclusive and a given response by a participant could reflect none, one, two or all three of the codes. Therefore for each item there were three binary codes producing a total of 24 responses per participant (eight questions \times three codes). The coding was undertaken independently by two researchers and the initial inter-rater agreement was 78.7%. Following the initial coding, meetings were held in which disagreements were resolved.

Analysis

To explore the performance of the eight items for measuring teachers' awareness of student understanding of the equals sign, we undertook two procedures. First, we assessed the internal consistency of the codes by calculating Cronbach's alpha for each (operational, sameness, relational). Typically, values of Cronbach's alpha $> .7$ are considered to reflect acceptable internal consistency.

Second, we subjected the codes across the eight questions to Rasch modelling (Bond & Fox, 2007) as is common for investigating the performance of an instrument. For the case of traditional test data, the Rasch model aligns item 'difficulties' and participant 'abilities' onto a single scale, as described below. For the case of the data presented here, which arises from researcher codes rather than traditional test scores, we refer instead to conception 'difficulties' for each item and participant 'awareness'. Rasch modelling was undertaken using the eRm package for R statistical software (Hatzinger & Mair 2007). To interpret the outcomes of the Rasch modelling procedure we used the outcomes to construct a Wright Map.

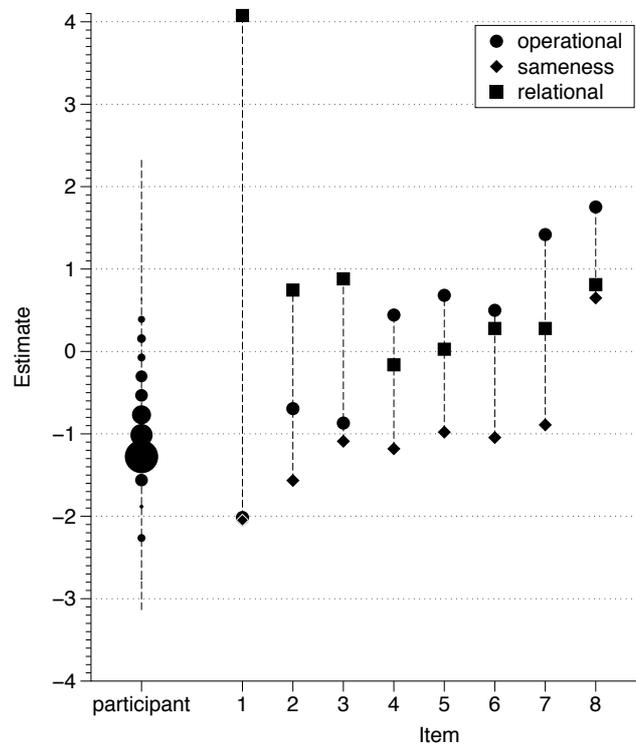


Figure 1: Wright Map of the outcomes of the Rasch modelling process. Participant ‘awareness’ estimates are shown on the left where the size of the dot indicates the number of participants at each level. ‘Difficulty’ estimates of the three conceptions are shown for Items 1 to 8.

RESULTS

Internal consistency. Cronbach’s alpha for the operational, sameness and relational coding was .79, .68 and .72 respectively. These figures suggest acceptable internal consistency and overall performance of the instrument, although the internal consistency of the sameness coding was marginal.

Rasch modelling. The outcome of the modelling procedure is the Wright Map shown in Figure 1. The Wright Map displays conception difficulties and participant awareness as z-scores for each item on a single scale, enabling direct comparison. The Rasch model frames outcomes in terms of the probability that a given participant will provide a given response to each item. This enables the analyst to make inferences between a participant’s estimated awareness and their likely responses to items (Andrich, 1988). Accordingly, our discussion of the results is framed in terms of the probabilities of teachers providing responses coded as operational, sameness and relational for each item. Where a conception difficulty is lower/higher on the Wright Map than a given participant’s awareness this indicates that the participant

had a more/less than 50% chance of being coded as aware of that conception for that item. For example, for Item 2 most participants were likely to provide a response coded as sameness, unlikely to provide a response coded as operational, and very unlikely to provide a response coded as relational.

All item responses were more likely to be coded as sameness than operational or relational. The definitional item and open number sentences responses were more likely to be coded as sameness than relational. Conversely, for the true/false arithmetic item and algebraic items responses were more likely to be coded as relational than sameness. Most participants (indicated by the largest dots on the participant scale) were likely to provide responses coded as sameness with a probability of about 50% for all the items except the definitional item (1) and one arithmetic item (2). Similarly, most participants were unlikely to provide responses coded as operational or relational for all the items, bar the definitional item for which most participants were likely to provide a response coded as operational.

Scrutiny of the Wright Map suggested two items are problematic for eliciting teacher awareness of children's understanding of equivalence. For the definitional item (1) almost all participants were very likely to be coded as sameness and operational, and all participants were very unlikely to be coded as relational (in practice only two out of 197 participants provided relational responses for this item). For the final item (8), almost all the participants were unlikely to be coded for any of the three conceptions. In fact 56.3% of participants provided no response that could be coded as any of the three conceptions for the final item.

DISCUSSION AND IMPLICATIONS

There have been limited studies which have addressed teacher understanding of students' mathematical thinking related to the equals sign with the exceptions of the studies by Zhang and Stephens (2013) and Stephens (2006). Our results suggest that while teacher awareness can be measured, there is a need to include a variety of items.

Overall we found that Items 2 to 7 performed well, suggesting these are the most appropriate of the items investigated to explore teacher awareness of children's understanding of equivalence. To investigate whether this may be the case, we re-estimated the internal consistency of the coded data with the two poorly performing items omitted. Cronbach's alpha was found to be higher for all three conceptions when Items 1 and 8 were omitted. For the operational coding it was .81 (up from .79), for sameness it was .73 (up from .68), and for relational it was .74 (up from .72), suggesting improved internal consistency.

Equation-based items may be preferable to definitional items when examining teacher awareness. Although this runs counter to the finding reported by A. Stephens (2006), an analogous definitional item was found to perform well for eliciting sameness and operational conceptions among children, although was less likely to elicit relational understanding than equation-based items (Rittle-Johnson et al., 2011). However, the item did not translate to eliciting teacher awareness in our study due to flooring effects for the relational conception, and ceiling effects for the operational and sameness conceptions.

Similarly, the results of our study indicated poor performance of Item 8, which assumes understanding that “solution” refers to the value represented by a letter. This was unsurprising as A. Stephens (2006) also found that few participants provided relational responses to Item 8. The poor response rate to this question may be due in part to primary teachers viewing this type of problem as unrelated to the level in which they teach. For example, one participant wrote simply “*The Year Sixes I work with would struggle with this.*”

For Items 2 to 7, which performed satisfactorily, the first two are more likely to elicit sameness than relational responses whereas the last four are more likely to elicit relational responses. This may be because Items 2 and 3 are ‘fill-the-blank’ items for which relational thinking reduces computational burden, but which nevertheless can readily be solved non-relationally. Conversely, Items 4 to 7 are not ‘fill-the-blank’ items and perhaps prompt a substitutive view (an arithmetic expression or letter must be replaced) which has been argued to be part of relational thinking (Jones et al., 2012).

CONCLUSION

We analysed the suitability of eight questionnaire items to investigate primary teacher awareness of children’s understanding of equivalence. Two of the items performed poorly and are not suitable for use, including a definitional item. The remaining six items involved missing number or substitution problems and performed satisfactorily but varied in the responses elicited. We conclude that teacher awareness can be validly and reliably measured and recommend a variety of equation-based items be used.

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