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What comes after nine?

Dave Hewitt

One, two, three, four, five, six, seven, eight, nine, ...

What is next? Well, as a mathematician I will say *ten* and expect others to say the same. However, as a teacher I do not think it is that simple. If my aim is to help children learn the number system, then is *ten* the most sensible choice?

What do I get for learning a name?

Consider the following:

besik, dua, telu, papat, lime, nenem, pitu, kutus, sia, ...

What number name is next?

Well, unless you know Balinese, you would have no idea. There is nothing from the first nine words which would give me a sense of what the tenth word would be. I will be kind to you and tell you the tenth word; it is *dasa*. So, now what is the eleventh word? Well, there is still no pattern that would help you know what the eleventh word would be. Even if I tell you the eleventh and twelfth words, *solas* and *roras*, you really are none the wiser about what comes next. So it is with English number names as well. If the sequence were to be continued up to the twelfth word there is really nothing which is helpful to know any other number names:

one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve,...

Little is gained from learning these extra names except the names themselves. By this I mean that there is no pattern within the number names that could give insight into the *structure* of how we say numbers. One extra word buys the name for just one extra number. So, the reward for learning 12 names is just 12 numbers.

In contrast, consider the Welsh number names from 1 to 12:

un, dau, tri, pedwar, pump, chwech, saith, wyth, naw, deg, un deg un, un deg dau,...

The names up to *naw*, give me no sense that *deg* might be next, just as the names up to *nine* in English give me no sense that *ten* would be next. However, the next few names, *deg, un deg un* and *un deg dau*, begin to show a structure in the words which also relates to the structure of number: *ten, one ten one, one ten two*. The latter two relating to the symbolic form: *one ten one* for 11, and *one ten two* for 12.

The Welsh number names show a pattern which allows a learner to gain a sense of that might come next: *un deg tri, un deg pedwar, un deg pump, un deg chwech, un*

deg saith, un deg wyth, un deg naw. Suddenly, the learning of the word *deg* brings with it the possibility of counting up to 19 without the need to memorise new names. Here, just 10 words buy me the names for 19 numbers; a much better return than for English.

The key issue here is that there is a significant difference for a learner to be in a position where they can *generate* new number names from the patterns they have noticed rather than just *memorising* names they have been told. This not only gives a sense of empowerment but also opens up the possibility of knowing many more names than those which they have already been told. For example, Welsh speakers might think that if 15 is said *one ten five*, then 35 might be said *three ten five* and so would be *tri deg pump*. The awareness of structure opens up the door to creativity where a learner can suddenly feel that they can generate a whole load of new number names with what they already know. It feels as if all these new number names come for free. There are no further names to memorise in order to know the names for a whole set of new numbers.

Gattegno (1988) gave the name of *ogden* for the price someone has to pay to memorise something. So, for the number names from 1 to 12 the cost is 12 ogdens for the English number names and just 10 ogdens for Welsh. Indeed the Welsh get a lot more for their 10 ogdens than just the numbers from 1 to 12. The new awareness of structure potentially opens the door up to 99 number names. However, things are not quite so simple, even for the Welsh. There are exceptions. I might expect 55 to be *pump deg pump* but I find out that it is *pum deg pump*. So, for this exception of *pum* rather than *pump* I will need to pay another ogden. The other exceptions are 60 being said as *chwe deg* rather than *chwech deg* and 20 being said as *dau ddeg* rather than *dau deg* (the double *dd* being pronounced more as a 'v' sound in English). So there are three exceptions bringing the total cost of just 13 Ogdens to learn the Welsh numbers from 1 to 99.

What is powerful educationally is that learners can gain a sense of the structure of the number names and a sense of power which that can bring. This sense of power can come from a sense that "one word gets me just one more number" to "one word buys me a whole host of numbers which I can work out myself without the need to be told". Of course, there may be exceptions, but these can be dealt with later. As a teacher, I argue that I would not want to diminish the sense of power over the number system which the awareness of structure can bring just because of a few exceptions. It is only when a learner feels confident with the overall structure that I would choose to introduce exceptions. Any earlier introduction of exceptions might prevent a learner developing that general overview of the structure.

Language: getting a lot for a little

I return to the English number names. How can I help my learners get a sense of the structure of number through the learning of number names? It is the awareness of structure which is key. The exceptions detract from that sense of structure and so I would want to stay with regularity first and bring in exceptions later.

The names *one* up to *nine* are required, since all our number names involve at least one of those names. In Welsh, the word for ten, *deg*, would be useful to know structurally, since it forms part of the structure for the numbers from 10-19. However, *ten*, *eleven* and *twelve* in English bring nothing which helps gain that sense of structure. Even the *-teens* is a one-off structure which is not to be repeated within the 20s, 30s, 40s, etc. So first I would go for the regular structure which does exist within the English number names. Figure 1 shows the Gattegno, or Tens, Chart. The orientation can be different with the decimals at the top rather than the bottom, according to preference. Also the number of rows can be reduced or expanded as required.

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

Figure 1: the Gattegno Tens Chart

This chart is structured to reflect the structure within the number names. Figure 2 shows this chart in terms of the names which are used.

1000	2000	3000	T H O U S A N D			7000	8000	9000
100	200	300	H U N D R E D			700	800	900
10	20	30	40	- 50	60	70	80	90
one	two	three	four	five	six	seven	eight	nine
0.1	0.2	0.3	T E N T H S			0.7	0.8	0.9
0.01	0.02	H U N D R E D T H S			0.07	0.08	0.09	
0.001	0.002	T H O U S A N D T H S			0.007	0.008	0.009	

Figure 2: the names for the rows and columns of the chart

These names are a mixture of *digit* names (*one, two, three, ..., nine*) and *value* names (*-ty, hundred, thousand, tenths, hundredths, thousandths*). Within the table, the digit names apply down columns and the value names apply across rows. Each number is made up of a combination of digit and value names (with silence being the value 'name' for the units row). So, Figure 3 shows how 900 is said as a combination of the row and column names within which it sits.

1000	2000	3000	T H O U S A N D			7000	8000	9000	
100	200	300	H U N D R E D			700	800	900	
10	20	30	40	- 50	60	70	80	90	
one	two	three	four	five	six	seven	eight	nine	
0.1	0.2	0.3	T E N T H S			0.6	0.7	0.8	0.9
0.01	0.02	0.03	H U N D R E D T H S			0.07	0.08	0.09	
0.001	0.002	0.003	T H O U S A N D T H S			0.007	0.008	0.009	

Figure 3: how 9000 is made from the digit and value names

The number name for 9475 is made up of coming down the chart saying each component part in turn (see Figure 4).

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

Figure 4: how 9475 is viewed in the chart

The *-ty* row, of course, has plenty of exceptions but if we ignore these for the moment, then there are only 15 names to learn (see Figure 2) which will buy the vast majority of the number names from 0.001 up to 9999.999, in steps of 0.001. These are generated from the structured use of those 15 names (along with the extra *and* which is said after the hundreds). That is a lot of numbers for such few names to remember.

What might follow the learning of the digit names *one* to *nine*? Well, the word *hundred* suddenly gives access to another 90 numbers (100, 101, 102,... 109; 200, 201, 202,... 209; up to: 900, 901, 902,... 909). It also opens up the structure of saying *digit name* followed by *value name*, which is at the heart of place value. It also opens up the structure of not only having a number such as *four hundred* (digit

name followed by value name) as well as a number such as *five*, but these can be combined to produce *four hundred and five*. This is the beginning of learning about numbers such as 407 being made from component parts: 400 and 7. The regularity of the hundreds and units allows a sense of structure to be developed, which learning the *-teens* at this stage would not.

Work on the structure of number can be done through work with any of the bold numbers in Figure 5, as these are all regular. I have included the decimals, although some may choose to avoid these initially (in which case those rows can be excluded from the chart).

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009

Figure 5: the regular parts of the Gattegno Tens Chart

Consideration might be given to maintaining regularity across the *-ty* row by saying *one-ty*, *two-ty*, *three-ty* and *five-ty* for 10, 20, 30 and 50 in the early stages of learning numbers. This would be down to pedagogic considerations to keep a strong sense of regularity in order for learners to become confident with the structure of how numbers are said. It also would allow counting to happen whilst still maintaining a strong sense of structure within the number names. In the past I have used this regularisation of the names with secondary school students who had low confidence with number generally (Hewitt and Brown, 1998). Even though we all knew that 15 was said *fifteen*, they were prepared to go along with saying it as *one-ty five* since it helped make sense of the structure of numbers in a way which *fifteen* did not. Once the confidence with the structure of numbers improved, this new confidence was carried through whilst reverting back to saying irregular names such as *fifteen*.

Decimals

Decimal numbers can be seen either as a whole new level of complexity and put off until sometime in the future. However, some meaningful engagement with decimals can be gained at a very accessible level. The decimal rows are just new rows in the same way as the *-ty*, *hundred* and *thousand* rows were new at one point in time. Just three new names buy access to these three new rows. The mechanism of counting in tenths is the same mechanism for counting in units except one row down

and questions such as how many counts in tenths are needed to get to *one* can begin to give a sense for what 0.1 might represent.

The names for the decimal number rows (see Figure 2) give the value of the digit (e.g. *hundredths*) and so there are advantages to using this language rather than the language of *nought point nought one*, for example. A number such as 687.942 could be said as *six hundred and eighty seven point nine four two*. However, the use of that language does not support the learning of the place value of each digit in the decimal part. There would still have to work done to learn what the value is of the digit 4 in that number. By choosing to use the value names and saying the number as *six hundred and eighty seven, nine tenths, four hundredths and two thousandths* allows the place value of each digit to be part of the language of the number name. As a consequence place value is dealt with as part of the language and does not have to be taught as a separate topic.

Ordinality

The approach I have discussed here is an ordinal approach to number as opposed to a cardinal approach. The cardinal aspect of number has a focus on counting *things* and the task of associating a number name with a collection of objects, such as being able to say that there are six cups on the table. This is an important aspect of teaching number and one which I am not discussing here. However, in order to be successful in counting objects, someone has to know the number 'rhyme' of *one, two, three, ...* in the first place. The emphasis placed within my writing above is one where someone gets to know about the structure of number, how numbers are said and written, how they are made from their component parts, and also how they are related to one another. This latter aspect is what is essentially ordinal about this approach and can come from counting activities which can be carried out with the chart (e.g. counting up in ones, counting up fives, counting down from a certain number, counting in 10s or 100s, saying one more/less than a given number) where the structure of the chart can also offers visual support.

What comes after nine?

So, let me return to my question: *what comes after nine?* Mathematically I said that I would answer *ten* but pedagogically I feel that it would NOT be *ten*. For the reasons I have outlined above there are a number of possibilities which will help learners gain an awareness of the structure of number, none of which are *ten*. It could be the name *hundred* or *thousand* or indeed *-ty* (staying with the regular *-ty* names). These offer opportunities for learners to become empowered through their awareness of structure which can give them a sense of being able to generate names themselves.

There is also an additional reason why it is best to stay with the rhyme *one* to *nine* and not include *ten* too soon. It is the names *one* to *nine* which form the basis of our

number system as indicated in Figure 2, not *ten*. If learners get used to the number rhyme from *one* to *ten* (as opposed to *one* to *nine*) then this will only increase the likelihood of them continuing rhymes such as *sixty-one*, *sixty-two*,... to include *sixty-ten* as they are so used to that rhyme. The word *ten* offers nothing more than how to say the number 10, whereas a name such as *hundred* or *thousand* or *-ty* can open a door to a whole new world.

What comes after *nine*? Not *ten*.

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