

This item was submitted to [Loughborough's Research Repository](#) by the author.
Items in Figshare are protected by copyright, with all rights reserved, unless otherwise indicated.

Early-age finite element modelling of concrete industrial floors

PLEASE CITE THE PUBLISHED VERSION

PUBLISHER

© Professional Engineering Publishing Ltd

LICENCE

CC BY-NC-ND 4.0

REPOSITORY RECORD

Robins, Peter J., Jonathan W. Bishop, and Simon A. Austin. 2019. "Early-age Finite Element Modelling of Concrete Industrial Floors". figshare. <https://hdl.handle.net/2134/3905>.

This item was submitted to Loughborough's Institutional Repository (<https://dspace.lboro.ac.uk/>) by the author and is made available under the following Creative Commons Licence conditions.



For the full text of this licence, please go to:
<http://creativecommons.org/licenses/by-nc-nd/2.5/>

Early-age finite element modelling of industrial ground floors

Authors: P J Robins, J W Bishop and S A Austin

Department of Civil and Building Engineering, Loughborough University, Loughborough,
Leicestershire, LE11 3TU, UK

Abstract

A series of finite element models of ground floor slab behaviour has been developed and validated against in-situ data from site instrumentations. Good agreement was obtained for thermal flow models, and the negative impact of air movement over the top of the slab has been highlighted. Plane stress models of slabs exposed to uniform and differential thermal gradients representing hydration and shrinkage effects demonstrated that the stresses resulting from frictional restraint are significantly less than those caused by warping restraint, even though the latter are currently not considered in design.

Keywords: Finite Element; Mathematical modelling; Concrete floors; Industrial floors; Early-age.

Notation:

g Acceleration due to gravity (m/s^2)

C Capacitance

ρ_c Concrete density (taken as $24 \times 10^{-6} \text{ kg/mm}^3$)

r	Concrete maturity
α_c	Concrete thermal coefficient of expansion (taken as 10 μ strain/K)
k	Conductivity
r_0	Critical maturity at which concrete properties begin to develop
L_{cr}	Critical slab length (mm)
σ_w	Critical warping stress (N/mm ²)
ρ	Density (kg/m ³)
$\Delta\varepsilon_{sh}$	Differential movement between top and bottom slab faces
$E_c(r)$	Elastic modulus at concrete maturity r
ΔL	Element size
f	Load factor
k	Modulus of subgrade reaction (MN/m ³)
b	Parameter
ν	Poisson's Ratio
L	Slab length (mm)
h	Slab thickness (m)

c	Specific heat capacity (J/kg.K)
Δt	Step size
ΔT	Temperature difference between the top and bottom slab faces (K)
α	Thermal coefficient of expansion ($\times 10^{-6}/^{\circ}\text{C}$)
λ	Thermal conductivity (W/m.K)
σ	Warping stress (N/mm^2)

1 Introduction

The prime asset of many buildings is its concrete ground floor. If the floor becomes un-useable, for reasons of structural integrity or surface irregularity, the operation of the building can be seriously disrupted and the capital value of the entire structure put at risk. At present there is a lack of understanding of concrete floor slab construction materials and their behaviour, reflected in current design practice, and improvements are needed if effective and economic engineering solutions to the increasing demands being placed on floors by the end users are to be achieved.

A project has recently been undertaken at Loughborough University, with support from the Engineering and Physical Sciences Research Council and 12 industrial sponsors, to investigate the early-age behaviour of concrete industrial floors through a series of in-situ instrumentations (Figures 1 & 2).

This paper describes the use of the finite element (FE) method to model the early-age thermal and structural behaviour of concrete ground supported slabs; the latter simulating hydration and the impact of diurnal temperature variations, and the effects of moisture loss due to drying shrinkage. The modelling techniques presented here are not new; however, this work has validated an FE approach against in-situ data from six industrial floors.

2 Model Development

2.1 Thermal flow

The system can be simplified structurally to a slab resting on a frictional boundary (Pettersson, 1998), but this fails to account for the thermal interaction of the slab and the supporting layers during hydration. For this reason the slab, sub-base and subgrade system should be modelled as a whole.

Critical parameters for the thermal flow analysis are the convection coefficients for the air-slab boundary, the material thermal properties, the adiabatic temperature rise of the concrete, and the ambient temperature (Truman et. al., 1991). A literature review identified values for the material thermal properties and suitable boundary coefficients, which are summarised in Table 1. The adiabatic temperature rise and the concrete thermal properties were calculated using Hymostruc (Van Breugel, 1991). The calculated values for conductivity and capacitance fell within the ranges found in the literature, but could be made hydration-dependent in this way.

In-situ temperature data from six industrial floors slabs (Bishop *et. al.*, 2002), and in one case from the sub-base and the subgrade up to a depth of 800mm below the slab surface, have been used to validate the model.

The size of the model was determined according to theoretical predictions of temperature variations and the in-situ data. Tomlinson (1940a) showed that for a slab subjected to simple harmonic heating and cooling, as an approximation to the diurnal temperature changes, the daily temperature variation at a depth of 635mm is negligible.

At an early age, the air-slab boundary conditions remain similar to the simple harmonic approximation, though the hydrating cement in the slab also acts as a heat source. Truman *et. al.* (1991) modelled the hydration of two 2.75m high concrete lifts founded on soil layers 3m and 6m thick, showing that the change in the thickness of the soil layer altered the temperature at the bottom of the concrete lift by less than 1%.

The in-situ data collected during this project shows that in the subgrade, 800mm below the slab surface, the temperature increased by several degrees during the first couple of days after construction (Figure 3). Because the slab thickness, and hence the heat generated, is significantly less than that considered by Truman *et. al.* a subgrade thickness of 1.6m topped by a 0.2m thick sub-base was modelled. This is similar to the depth of 1.5m used by Rees *et. al.* (1995) when looking at the annual thermal changes under a building slab in the UK.

The relationship between element size and time-step duration can limit the model accuracy in a diffusion problem, with Truman *et. al.*(1991) identifying the following relationship between the density (ρ), capacitance (C), conductivity (k), element size (ΔL) and step size (Δt):

$$\Delta t > \left(\frac{\rho \cdot C}{6k} \right) \Delta L^2 \quad \text{equation 1}$$

The time steps were varied to reflect the rate-of-change of temperature in the model, with the initial 24 steps, whilst the concrete hydrated, having a duration of 1-hour. The step size was then increased to 2 hours for the next 24 steps as the concrete temperatures reduced to ambient levels, and following this a 4-hour time step was used for the remainder of the model runs.

The element sizes determined according to equation 1 are generally not limiting in the case of ground supported slabs. For example, with a 1-hour time step the concrete and sub-base elements could be up to 0.16m in size. However, in order to reproduce the thermal profile through the slab and sub-base, the elements have to be much smaller than this. In fact, six elements were used in the slab, with a further six elements through the thickness of the sub-base, giving a maximum nodal separation of 0.038m in the direction of heat transfer (Figure 4).

The maximum element size at the bottom of subgrade was determined using the 4-hour time step. Although this means it exceeds the theoretical maximum element size for the initial 1-hour time steps, the in-situ data show that there was no temperature change below a depth of 0.6m during this period, removing any dependence on the element size. Halving the nodal separations was found to have a negligible effect on the predicted thermal profile.

2.2 Structural models

Whilst the thermal boundary conditions can be clearly defined, this is not the case for the structural model. For this reason several models were developed to look at different aspects of the structural behaviour of a ground supported slab subjected to the thermal effects of hydration and varying ambient temperatures, and, in the longer term, the impact of hygral gradients caused by drying shrinkage from the slab surface.

The impact of the cement hydration has been incorporated by carrying out a linked flow-stress analysis. This imports the spatially varying, and time dependant, degree of hydration and temperature data from the thermal flow model described in section 2.1. In order to allow comparison with the site data collected as part of this project the approach proposed by de Borst and van den Boogaard (1994) was further developed by the addition of maturity dependent tensile strength. The influence of the shear stiffness of the slab/sub-base interface and the inclusion of drying shrinkage have also been assessed.

The constitutive material properties used in the finite element model cannot account for plastic shrinkage effects. However, Turton (1989) stated that plastic shrinkage cracking was not a problem for industrial ground floors in the UK. If the rate of moisture loss is not

excessive (Austin *et. al.*, 2002), the assumption that the concrete begins to set in a stress-free state is reasonable (ACI Committee 207, 1987).

The time-dependent elastic stiffness has been modelled using the maturity relationship of equation 2 (De Schutter & Tearwe, 1996). Where $E_c(r)$ is the elastic modulus at maturity r , r_0 is the critical maturity at which material properties begin to develop, and b is a parameter to fit the experimental data equal to 0.75.

$$\frac{E_c(r)}{E_c(r=1)} = \left(\frac{r - r_0}{1 - r_0} \right)^b \quad \text{equation 2}$$

A similar function was used to model the development of the tensile strength of the concrete, but with $b = 1$.

In a comparative study of modelling methods for concrete pavements, Kim *et. al.* (2000) showed that 2D plane stress models generally agreed well with 3D models, whilst 2D plane strain models over-predicted the stresses by around 15%. Furthermore, plane strain models are not well suited to the analysis of thermally induced stresses, as the restraint to out-of-plane movement leads to maximum stresses in this direction. For these reasons plane stress models have been used for these analyses.

Teller & Sutherland (1943) replaced non-uniform drying shrinkage with an equivalent temperature gradient (ETG) to simplify analysis. Initial models with constant material properties were analysed with uniform and differential (linearly varying) temperature changes to represent annual temperature variation and the long-term effects of drying shrinkage respectively. This allowed the slab behaviour to be assessed and comparisons with theory to

be made – for example, the degree of curling under a given drying shrinkage or equivalent thermal profile. Because the temperature fields were input and not calculated, these models were simplified to a slab resting on frictional interface elements including the gap criterion. This allowed slipping between the slab and the ground, whilst preventing tensile vertical restraint to upwards movement of the slab.

The models used in the flow-stress analyses for comparison with the site data simplified the instrumented section of slab to an 8m long panel. Site readings showed that there was no movement at the restrained movement joint so this was taken as a line of symmetry and horizontal movement was prevented at this section. The other end of the slab was left free to represent the free movement joint at this location. This oversimplifies the boundary conditions, as both ends of the panel were fully restrained until the free movement joint opened, and from this point on there was a finite, but indeterminate, restraint at this point. Additionally vertical movement was measured at the restrained movement joint, which indicates that rotation of the joint has occurred even if no joint opening was measured at the level of the reinforcement.

3 Analysis of the results

3.1 Thermal models

The thermal model was calibrated with the data from floor F, a 225mm thick jointed large area pour slab cast in March 2000, as temperature data were collected from the sub-base and the subgrade in addition to the slab (Bishop *et. al.*, 2002). This involved varying the material and air-slab boundary coefficients within the ranges found in the literature.

The slab temperature was found to be sensitive to all of the material properties of the subgrade and sub-base, with increases in conductance and reductions in capacitance reducing the maximum slab temperature and vice-versa. The rate at which the slab cooled and the maximum temperature achieved were also affected by the convection coefficient chosen for the air-slab boundary.

Lachemi and Aitcin (1995) demonstrated with a finite element model that reducing the initial concrete temperature delayed and reduced the peak temperature. Although the measured initial concrete temperature was used in the model, the degree of the delay which relates to the hydration behaviour of the cement, based solely on its chemical composition, has been underestimated. The fit of the model could probably have been improved had test data on the adiabatic hydration curve for the concrete been available to input into Hymostruc. However, the intention was to assess the accuracy of the model without resorting to laboratory determinations of all of the material parameters.

Good agreement was obtained between the measured and predicted temperatures at mid-depth in the slab, sub-base and subgrade (Figure 3), although a difference arises in the prediction of the timing of the maximum slab temperature; with the maximum temperature recorded on site occurring approximately 11 hours later than that predicted by the model. Similar differences between model predictions and experimental data were found by de Borst and van den Boogaard (1994). However, they concluded that discrepancies during the warming phase were insignificant, so long as agreement during the cooling phase, when cracking was likely, was good.

Model validation was achieved by comparison with the other instrumented sites. The material properties for the sub-base and subgrade were kept as previously determined, leaving the only variables as the physical dimensions of the slab and the concrete constituents. In each case the measured ambient temperatures were input to determine the rate of heat loss across the air-slab interface, and the concrete parameters were determined using Hymostruc according to the mix design and the chemical composition of the cement.

Overall, good agreement was obtained between the model predictions and the site data, although one discrepancy became clear. Three of the instrumented floors were constructed before the walls of the building had been completed, resulting in higher average daily air speeds over the floor surface than the other sites.

As an industrial floor is not exposed to direct sunlight and is distant from the roof of the building, thermal exchange across the air-slab boundary is primarily achieved through convection, which is sensitive to the air movement across the slab surface. Under still air conditions losses occur through natural convection; however, increases in air speed over the surface of the slab lead to forced convection conditions resulting in much higher rates of heat loss. The CIBSE guide (1999) gives the convection coefficient for losses upwards from a horizontal surface in still air as $4.3 \text{ W.m}^{-2}\text{K}^{-1}$, whilst Thelandersson *et. al.* (1998) proposed boundary coefficients of 7.7 and $13.5 \text{ W.m}^{-2}\text{K}^{-1}$ for air movement of 0.5 ms^{-1} and 2.0 ms^{-1} respectively. Figure 5 shows output from a finite element model demonstrating how the maximum temperature and the rate at which heat is lost to the environment are directly affected by this coefficient. The site data were from a floor constructed before the end wall of

the building and part of the nearest side wall of the building had been completed, resulting in measured daily average wind speeds in excess of 1.0 ms^{-1} .

Once the excess heat due to hydration is lost to the environment after about 72 hours, the thermal changes in the slab are driven solely by the diurnal ambient temperature change. Because the temperature differential between the slab surface and the air is small, the sensitivity to the air movement is much reduced, resulting in smaller variations between the different model predictions and good agreement with the measured temperature (Figure 6).

With the air-slab boundary coefficients corrected to represent the site conditions, the good agreement between the site data and the model predictions for all of the sites indicates that the material properties determined from the initial calibration can be applied with good accuracy to models of other locations.

This finding allows the impact of the change in environmental conditions to be assessed. Exposing the slab to moving air reduces the maximum temperature in the hydrating concrete, which is beneficial, as it reduces the thermal contraction potential of the slab. More worrying, is the more rapid drop in temperatures back to ambient levels, as this occurs at a time when the concrete is still weak in tension, and will increase the likelihood of cracking.

Once the initial heat of hydration has been lost the temperature variation in the slab, which is driven by the diurnal ambient temperature variation, becomes much smaller. Figure 7 shows how over a 24-hour period the slab surface temperature varies by about 0.8°C , whilst there is no variation at a depth of 0.4m. The temperatures at lower depths are generally influenced

only by seasonal temperature variations, though fluctuations lasting at least several days can have some impact.

3.2 Structural modelling

One of the critical behavioural trends of industrial floors is the tendency to warp or curl, which occurs because the moisture loss from a floor is essentially one-dimensional, normal to the slab surface, resulting in a shrinkage gradient, decreasing from the slab surface. Tomlinson (1940a & b) and Pettersson (1998) found that the stresses arising from these non-uniform strain changes were normally significantly larger than those resulting from uniform strain changes. Draft CUR design guidance (CUR 36) states that '*a shrinkage gradient must be taken into account under all circumstances*', though they are not considered in the current UK design guidance, TR34 (1994).

Warping of a slab is a maximum at the free ends, reducing with distance as the self-weight of the curled section of slab increases, until at some critical slab length l_{cr} the warping stresses and the self-weight are equal and opposite. Any further increase in slab length will leave a curled length of slab at each end with a flat central section (Figure 8).

Several authors have suggested methods for predicting the critical length of slab which will warp, and the resulting restraint stresses. Pettersson (1998) determined the critical length to be as shown in equation 3, where L_{cr} is the critical slab length (mm), ΔT is the temperature difference between the top and bottom slab faces (K), ρ_c is the concrete density (taken as $24 \times 10^{-6} \text{ kg/mm}^3$), g is the acceleration due to gravity (9.81m/s^2), α_c is the concrete thermal

coefficient of expansion (assumed to be 10 μ strain/K) and f is a load factor (taken as one in this case as only self-weight is considered).

$$L_{cr} = \sqrt{\frac{4\alpha_c \cdot E_c \cdot \Delta T \cdot h}{5 \cdot \rho_c \cdot g \cdot f}} \quad \text{equation 3}$$

Pettersson then showed how the maximum restraint stress in a slab exposed to linear thermal gradients varied as a function of the critical length.

$$\sigma = 1.2\sigma_w \cdot \left(\frac{L}{L_{cr}}\right)^2, \quad L \leq L_{cr} \quad \text{equation 4}$$

Losberg (1978) calculated the warping stresses according to equation 5, where $\delta\varepsilon_t$ is the difference in shrinkage or temperature strain between both surfaces of the slab.

$$\sigma_w = \frac{E_c}{1-\nu} \frac{\delta\varepsilon_t}{2} \quad \text{equation 5}$$

Once the critical length is exceeded the maximum stress is treated as a constant with a value of $1.2\sigma_w$.

The vertical movement resulting from the curling was found by Rollings (1997) to be

$$a_t = \frac{\Delta\varepsilon_{sh} \cdot L^2}{8 \cdot h} \quad \text{equation 6}$$

where $\Delta\varepsilon_{sh}$ is the differential movement between the top and bottom surfaces of the slab, and L and h are the length between joints and slab thickness respectively (m).

A simple structural FE model was developed for comparison with these suggested methods. This model analysed the behaviour of an 8m long, 160mm thick slab with an elastic stiffness of 25 GPa. Because of symmetry only half of the structure was input, with horizontal restraint applied to the centreline of the model. Plane stress elements with 8 nodes modelled the slab, whilst 6 node interface elements represented the subgrade response.

This model was first analysed with a negative temperature differential of 4°C – equivalent to a thermal gradient of 0.25 °C/cm – to simulate a drying shrinkage of 25 µstrain/cm. For comparison, Leonards & Harr (1958) found equivalent temperature gradients of between 0.66 and 1.33 °C/cm for internal slabs. This gives a theoretical critical length from equation 3 of 2.33m. The initial and deformed shape of the finite element model can be seen in Figure 9, showing the length of curled slab to be around 1.33m – a critical length of 2.66m, although some of this length has settled into the sub-base. The upward movement of the top corner node was 0.13mm, compared to a maximum vertical movement according to equation 6 of 0.17mm. However, Rollings assumed a slab, resting on a rigid support. If the settlement of the slab into the sub-base is accounted for, the finite element model predicts a total movement of 0.15mm.

The maximum warping restraint stresses according to equation 4 were $\pm 0.59 \text{ N/mm}^2$, whilst the finite element model predicts $\pm 0.61 \text{ N/mm}^2$, a difference of around 3%.

Further comparison has been carried out with site data. The stress distribution in a 260 mm thick floor resting on a sub-base with a modulus of sub-base reaction $k = 50 \text{ MN/m}^3$ (CBR = 7.5%) and exposed to an ETG of 1.1 °C/cm, as measured in one of the instrumented floors after 370 days, was calculated. The model used an age-adjusted elastic modulus of 9.8 GPa

was used to allow for creep effects, which assumes the curling began one week after the slab was constructed and continued developing over the monitoring period.

The stress distribution at two sections in the slab can be seen in Figure 10. Section A-A is 1m from the centre-line of the 8m long slab section, whilst Section B-B is 0.5m from the free end. The increase in restraint stress away from the free end caused by the slab's self-weight is clear to see.

The maximum tensile stress of 1.59 N/mm^2 is very similar to the value of 1.49 N/mm^2 calculated in Austin *et. al.* (2002) using the same age-adjusted modulus and the differences in strain gradients recorded from embedded strain gauges near to joints and midway between them. The frictional restraint reduces the maximum compressive stress in the slab bottom to 1.28 N/mm^2 (Figure 10). The predicted vertical movement produced by the strain gradient is 1.7 mm, which compares to measured mean vertical movement of 3.6 mm and 2.7 mm for the restrained movement joint and the free movement joint as determined by precision level surveys. This predicted movement is based purely on the elastic deformation of the sub-base and takes no account of the long-term settlement that will occur under the slab and increase the magnitude of the deformations. This can be demonstrated by reducing the subgrade modulus in the model – a value of 10 MN/m^3 (CBR = 1%) gives a vertical differential movement of 2.1 mm, whilst changing the warping restraint stresses in the slab by less than 1%.

The model was then analysed with uniform temperature drops applied to the slab to simulate the effects of seasonal temperature variation and the uniform portion of drying shrinkage. As the temperature drop was increased to 10°C the shear stress distribution along the interface

changed as shown in Figure 11. This shows how the shear stress increases linearly up to the point where slipping occurs; the reduction in shear stress as the distance from the line of symmetry increases is due to the redistribution of the ground pressure under the slab (Figure 12). The frictional restraint theory adopted in TR34 (1994) assumes a uniform limiting value of shear restraint along the interface; however, the sum of the restraint is the same. With an applied temperature drop of 10°C, the FE model predicts a tensile stress at mid-depth on the line of symmetry of the slab of 0.09 N/mm², which is the same as that predicted according to frictional restraint theory.

Kiamco (1997) suggested that as the restraint is applied to the bottom face of the slab the eccentricity should be accounted for in design. This results in a compressive stress at the slab surface and an increased tensile stress at the bottom of the slab, requiring more crack control reinforcement. FE results do not agree with this, showing a slight stress differential at the line of symmetry for an 8m long slab, with no compressive stress. Pettersson (1998) demonstrated that because of the redistribution of the ground pressure under the slab, the maximum tensile restraint in a slab longer than 10m can be treated as centrally applied.

The comparisons presented here demonstrate that very good agreement is obtained between the simple theoretical approximations and the more thorough finite element analyses for warping and longitudinal restraint. They also demonstrate that the stresses resulting from a differential strain change in the slab – such as caused by drying shrinkage – are greater than those caused by a uniform change of similar magnitude – such as caused by seasonal thermal effects. Additionally, whereas the stresses from the differential change are uniform for much

of the slab length, those due to the uniform change increase with the distance from the free end.

Structural models of the hydrating concrete were also compared with site data. The largest tensile stresses at early ages were found at mid-depth in the slab, where temperature drops had been greatest, whilst the differences in restraint with distance from the free end of the slab are also apparent (Figure 13).

The Diana finite element software (de Witte, 1999) includes drying shrinkage models from various design codes; these are not constitutive models, rather they facilitate the prediction of the mean drying shrinkage in a member. This makes them suitable for determining the long-term behaviour of a structure where the interaction of different elements is considered, but does not allow the deformation and resultant stress profile in an individual section to be modelled.

Models of the early-age behaviour of concrete slabs were run with shrinkage data from the B3 model (Bazant and Baweja, 1995a, b & c), which had been found to agree well with the site data, input in tabular form.

Because of the low hygral conductivity of concrete at early ages the strain measured by the embedment gauges at the bottom of the slab should not include much drying shrinkage. Figure 14 compares the measured and calculated strains for the top and bottom gauges at distances of 1m, 4m and 7m from the sawn free movement joint.

Clearly the movement from the upper gauge is under predicted, though good agreement is obtained with the readings from the bottom gauge. By comparison Figure 15 shows the results

of the models run with drying shrinkage, where the agreement with the upper gauges is now much better.

Though it was not possible to carry out a combined hygral and thermal diffusion analysis, the results from the models with and without shrinkage give good agreement with the site data for the top and bottom of the slab respectively. Although an oversimplification, this demonstrates that analyses may be performed using these two assumed moisture states to give reasonable predictions of the expected contraction from the top and bottom of the slab.

4 Conclusions

This paper describes the application of finite element models for predicting the thermal and hygral behaviour of concrete industrial floor slabs. It has been shown that using the material properties in the literature and data supplied by the cement manufacturers it was possible to model accurately the temperature changes both in the slab, and in the underlying layers during the first 28 days after construction. The impact of increased air movement has been demonstrated, leading to accelerated temperature changes and an increased risk of cracking.

Comparisons with simple structural models have demonstrated that a full FE analysis is not required to determine the effects of differential strain gradients on a slab due to non-uniform drying. Petersson (1998) and Rollings (1993) offer equations which give very good agreement with the finite element predictions of slab movements and restraint stresses.

The eccentricity of the frictional restraint in industrial floors has been shown to be negligible away from the end sections of a slab. Given that the frictional restraint increases with distance

from the free end, and generally only becomes significant with joint spacing $> 10\text{m}$, this eccentricity can safely be ignored in design.

The finite element models indicate that the maximum tensile stresses at early ages may occur at mid-depth in the slab. For this reason the crack control reinforcement in industrial ground floors may be more effective if placed at mid-depth, rather than in the slab bottom as has become common practice, particularly in large area pours. Not only would this reinforcement be in the most highly stressed part of the slab, making it efficient in controlling the spread of cracks but, because of the reduced cover to the top of the slab, this would also improve its effectiveness in controlling surface crack widths.

The finite element model of the long strip slab exposed to an ETG of $1.1\text{ }^{\circ}\text{C}/\text{cm}$ over a one year period predicted stresses which were very similar to those determined from the site data. The close agreement between the finite element models and the simplified expressions means that good agreement could also, therefore, be expected from the simplified approaches. This indicates that designers could now make some allowance for the intrinsic stresses in a slab resulting from warping restraint.

The stresses resulting from a uniform contraction, caused by seasonal temperature changes or the uniform portion of drying shrinkage, have been shown to be significantly less than those resulting from a differential contraction. This means that in the longer term, when warping and frictional restraint are accounted for, it is likely that the maximum tensile stresses in a slab will occur at the slab surface.

Acknowledgements

We acknowledge the financial support of the Engineering and Physical Sciences Research Council and thank the members of the project steering group, the Association of Concrete Industrial Flooring Contractors and our industrial partners BRC Ltd., Burks Green & Partners, The Concrete Society, Face Consultants Ltd., Fibercon UK Ltd., Fibermesh Europe, Stanford Industrial Concrete Flooring Ltd., Stuarts Industrial Flooring Ltd., The Sprigg Little Partnership, Tarmac Topmix Ltd. and the Quarry Products Association. The research team is also grateful to ANC, Asda, Birse, Booker, Gazeley Construction Ltd., Safeway, and Wincanton Logistics for granting access to their sites and for the assistance which we have received.

References

ACI Committee 207 (1987): *Mass concrete*, Technical Report ACI 207-1R-87. American Concrete Institute, Farmington Hills, Michigan. 44 pages.

Austin, S.A., Robins, P.J. & Bishop, J.W. (2002): Hygral effects in industrial ground floors, In preparation for: *Cement and Concrete Research*.

Ayotte, É., Massicote, B., Houde, J. and Gocevski, V. (1997): Modelling the thermal stresses at early ages in a concrete monolith, *ACI Materials Journal*. Vol. **94**, No. 6 (Nov-Dec), pp. 577-587.

Bazant, Z.P. and Baweja S. (1995a): Creep and shrinkage prediction model for analysis and design of concrete structures - model B3, *Materials and Structures*. Vol. **28**, No. 28, pp. 357-365.

Bazant, Z.P. and Baweja, S. (1995b): Justification and refinements of Model B3 for concrete creep and shrinkage. 1 Statistics and sensitivity, *Materials and Structures*. Vol. **28**, pp. 415-430.

Bazant, Z.P. and Baweja, S. (1995c): Justification and refinements of model B3 for concrete creep and shrinkage. 2 Updating and theoretical basis, *Materials and Structures*. Vol. **28**, pp. 488-495.

Bishop, J.W., Austin, S.A. and Robins, P.J. (2002): Industrial concrete ground floors: current knowledge and practice, In preparation for: *ICE Structures and Buildings*.

de Borst, R., van den Boogaard, A.H. (1994): Finite element modelling of deformation and cracking in early-age concrete, *ASCE Journal of Engineering Mechanics*, Vol. **120**, No. 12, pp. 2519-2534.

van Breugel, K. (1991): Simulation model for development of properties of early-age concrete. In: *Quality Control of Concrete Structures – Proceedings of the Second International RILEM/CEB Symposium, Ghent, June 1991*. RILEM Proceedings 14. (Eds. Taerwe, L. and Lambotte, H.) E & FN Spon, London, pp.139-151.

Byfors, J. (1980): *Plain concrete at early ages*. Swedish Council for Building Research. 464 pps.

CIBSE (1999): *CIBSE Guide, Volume A – Design Data*. 6th ed. The Chartered Institute of Building Services Engineers, London.

CUR 36 (Draft). Design, calculation and detailing of industrial floors in structural concrete. CUR Foundation, Ak Gouda. 69 pages.

Faraggi, V., Jofr, C. and Kraemer, C. (1987): Combined effect of traffic loads and thermal gradients on concrete pavement design, *Transportation Research Record*. No. **1136**, pp. 108-118.

Khan, A.A., Cook, W.D. and Mitchell, D. (1998): Thermal properties and transient thermal analysis of structural members during hydration, *ACI Materials Journal*. Vol. **95**, No. 3(May-June), pp. 293-303.

Kiamco, C. (1997): A Structural Look at Slabs on Grade, *Concrete International*. Vol. **19**, No. 7 (July), pp. 45-49.

Kim, S.-M., Won, M.C and McCullough, B.F. (2000): *Three dimensional nonlinear finite element analysis of continuously reinforced concrete pavements*. Centre for Transportation Research, The University of Texas at Austin, Austin, Texas. 75pps.

Lachemi, M. and Aïtcin, P.-C. (1997): Influence of ambient and fresh concrete temperatures on the maximum temperature and thermal gradient in a high performance concrete structure, *ACI Materials Journal*. Vol. **94**, No. 2 (March/April), pp. 102-110.

Leonards, GA & Harr, ME (1958): Analysis of concrete slabs on ground, *Journal of Soil Mechanics*, ASCE, No. SM3, pp. 35-58.

Losberg, A. (1978): Pavements and slabs on grade with structurally active reinforcement, *ACI Journal*. Vol. **75**, No. 66, pp. 647-657.

Mirambell, E. (1990): Temperature and stress distributions in plain concrete pavements under thermal and mechanical loads. In: 2nd International Workshop on the Theoretical Design of Concrete Pavements. 1st ed. Vol. 5. (Ed: PIARC Committee on Concrete Roads - Cembureau) Centre ROW, Ede, pp. 121-135.

Neville, A.M. (1995): *Properties of Concrete*. 4th ed. Longman Group Limited, Harlow. 844 pps.

Pettersson, D. (1998): *Stresses in Concrete Structures from Ground Restraint*. Ph.D. Thesis, Department of Structural Engineering, Lund Institute of Technology, Sweden. 113 pps.

Rees, S.W., Lloyd, R.M. and Thomas, H.R. (1995): A numerical simulation of measured transient heat transfer through a concrete ground floor slab and underlying substrate, *International Journal of Numerical Methods in Heat and Fluid Flow*. Vol. **5**, pp. 669-683.

Rollings, R.S. (1993): Curling failure of steel-fiber-reinforced concrete slabs, *Journal of Performance of Constructed Facilities*. Vol. **7**, No. 1 (February), pp. 3-19.

de Schutter, G. and Taerwe, L. (1996): Degree of hydration based description of mechanical properties of early age concrete, *Materials and Structures*. Vol. **29**, pp. 335-344.

Teller, L.W. and Sutherland, E.C. (1943): The Structural design of concrete pavements, Part 5, An experimental study of the Westergaard analysis of stress conditions in concrete pavements of uniform thickness, *Public Roads*, Vol. **23**, No. 8.

Thelandersson, S., Alemo, J. and Nagy, A. (1998): Cracking in concrete structures due to imposed strains with regard to design of reinforcement, *Materials and Structures*. Vol. **31**, No. 211. Aug-Sep, pp. 442-450.

Tomlinson, J. (1940a): Temperature variations and consequent stresses produced by daily and seasonal temperature cycles in concrete slabs, *Concrete & Constructional Engineering*, Vol.**35**, No.6 (June), pp. 298-307.

Tomlinson, J. (1940b): Temperature variations and consequent stresses produced by daily and seasonal temperature cycles in concrete slabs, *Concrete & Constructional Engineering*, Vol.**35**, No.7 (July), pp. 352-360.

Truman, K.Z., Petruska, D., Ferhi, A. and Fehl, B. (1991): Non-linear, incremental analysis of mass-concrete lock monolith, *Journal of Structural Engineering*, Vol. **117**, pp. 1834-1851.

TR34 (1994): Concrete Industrial Ground Floors - A guide to their Design and Construction. 2nd edition. The Concrete Society, Slough, UK. 145 pages

Turton, C.D. (1989): Discussion on paper 'Influence of mix proportions and construction operations on plastic shrinkage cracking in thin slabs' by Shaeles *et. al.* (1988). *ACI Materials Journal*. V.**86**, pp.531-532.

de Witte, F.C. (Ed.): Diana Finite Element Analysis, Online User's Manual – Release 7.2, TNO Building and Construction Research, 1999.