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## **Is there a relationship between real exchange rate movements and the output cycle?**

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
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**Department of Economics**

***Business Cycle Volatility and Economic Growth***  
***Research Paper No. 01/2***

***IS THERE A RELATIONSHIP  
BETWEEN REAL EXCHANGE RATE  
MOVEMENTS AND THE OUTPUT  
CYCLE?***

***Terence C. Mills and Eric Pentecost***

*June 2001*

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## **1 Introduction**

The Neo-classical theory of exchange rate determination, with a stock view of capital movements, has the equilibrium exchange rate dependent on purchasing power parity (PPP); that is, the bilateral nominal exchange rate is determined by the ratio of domestic to foreign price levels. Thus the real exchange rate is predicted to be constant. This prediction, however, is largely rejected by the data (see, for example Rogoff, 1996). Less restrictive models of the equilibrium exchange rate, such as the traditional Mundell-Fleming model (Mundell, 1963) or generalised portfolio balance (Branson and Buiter, 1983), assume that output is not fixed at the level of full employment, and postulate that the current account balance determines the equilibrium exchange rate. In other words, the real exchange rate, rather than assumed to be constant, is related to the relative output levels. The empirical implication of this hypothesis is that the real exchange rate should be co-integrated with the relative levels of domestic and foreign output. This hypothesis is, however, rejected by the data in this paper, just as the PPP relationship has been empirically rejected. This result notwithstanding, the real exchange rate does show a long, cyclical pattern, not dissimilar to a business cycle pattern. Cyclical output patterns could translate into real exchange rate cycles, since the international transmission of the business cycle traditionally takes place through the trade balance (see, for example, Williamson and Milner, 1991). This cycle, however, is not apparent from either monthly or quarterly data because, as noted by Baxter (1994) in a different context, the frequency is too high. At lower frequencies, however, a cyclical relationship may emerge between output and the real exchange rate. If confirmed, this analysis also explains why PPP is rejected in the short to medium term, since it implies that the real exchange rate, rather than remain constant as suggested by PPP, in fact fluctuates over the business cycle.

This paper examines this relationship for the U.S. and U.K. over the floating exchange rate period from 1973 to 1999. Section 2 posits a simple model of the relationship between U.S. and U.K. output and the real dollar-sterling exchange rate. Section 3 considers the detrending of time series and models the cyclical components of the series to reveal the relationship between the real exchange rate and the two output cycles. Section 4 offers a brief conclusion.

## **2 The Basic Equilibrium Model**

With a stock-view of international capital flows and a freely floating nominal exchange rate regime, in equilibrium the current account of the balance of payments

must be in balance. This external balance condition, expressed in terms of the home country, is:

$$X(Q, Y) = M(Q, Y) \tag{1}$$

$$X_Q > 0, \quad X_{Y^*} > 0, \quad M_Q > 0, \quad M_Y > 0$$

where home exports,  $X$ , are identical to foreign imports and home imports,  $M$ , are identical to foreign exports.  $Q$  is the real exchange rate, measured as the foreign currency price of domestic currency,  $Y$  is domestic output,  $Y^*$  is foreign output, and the respective partial derivatives are given below (1)<sup>1</sup>. According to (1), domestic exports depend inversely on the real exchange rate, such that as  $Q$  rises domestic exports become more expensive in foreign markets and the demand for them falls. Conversely, a rise in  $Q$  is expected to lead to an increase in domestic imports. An upturn in the foreign economy will lead to  $Y^*$  rising above trend, raising the demand for home exports. Similarly, a rise in  $Y$  will raise the domestic demand for foreign imports

Equation (1) is more general than the usual PPP equilibrium since we do not assume that domestic and foreign goods are perfect substitutes such that  $|X_Q| = |M_Q|$ . Equation (1) also permits price adjustment to be slow, and hence output prices to be sticky, which is again an appealing feature consistent with New Keynesian type research strategies. If prices are sticky in the short run, the real exchange rate may change, altering relative prices and hence the demand for national output. Suppose, for example, there is an increase in foreign demand for home output. If the exchange rate does not appreciate to offset the effect of the higher demand then output will rise. Thus, in the short run, over which prices are sticky, there may still be an international transmission of the business cycle despite the floating exchange rate policy. The need to maintain this equilibrium implies that there is a relationship between the business cycle and the real exchange rate.

To examine this potential relationship (1) can be assumed to be log-linear, where in equation (2) lower case letters denote the logs of their upper case counterparts:

$$q = \frac{m_y}{x_q m_q} y = \frac{x_{y^*}}{x_q m_q} y^* \tag{2}$$

This says that the real exchange rate is directly related to foreign income and inversely related to domestic income. There are two restrictions that can be tested on this simple quasi-reduced form model of external balance. First, if PPP is to hold then  $x_q = m_q$  and (2) would collapse to  $q = 0$ , implying a constant real exchange rate. Second, if there is symmetry between the two economies such that  $x_{y^*} = m_y$ , then (2) reduces to  $q = [m_y / (x_q = m_q)](y^* = y)$ . Alternatively, the relative sizes of the two economies could lead to asymmetric income effects on the real exchange rate. Thus, if the foreign economy is larger than the domestic then a foreign boom could be associated with an appreciation of the real foreign currency price of domestic currency.

Since all variables in (2) are strictly endogenous, this model is best tested within a three-equation VAR framework using the cyclical components of domestic and foreign output and the real exchange rate. This will also allow disequilibrium dynamics into the structure, which may be empirically important given the apparent empirical evidence in support of J-curves.

### 3 Empirical Implementation of the Model

In this section we construct the cyclical components of the variables in (2) and hence analyse the cyclical relationship between U.K. and U.S. output and the real dollar-sterling exchange rate. The data period is the floating rate regime from 1973 to 1999, with quarterly observations being used. The output series are logarithms of real GDP for both countries and are denoted  $y_t^{UK}$  and  $y_t^{US}$ . The (logarithm of the) real exchange rate is formally defined as:

$$q_t = s_t = p_t^{UK} = p_t^{US} \quad (3)$$

where  $s_t$  is the logarithm of the nominal exchange rate and  $p_t^{UK}$  and  $p_t^{US}$  are the logarithms of the respective consumer price indices. The output series are shown in Figure 1, and the real exchange rate is shown in Figure 2. These series display familiar properties. The two output series both contain prominent trends and short-run movements about these trends, although for the U.S. such ‘cyclical’ fluctuations have been much attenuated in the last decade. The real exchange rate, on the other hand, contains no trend but is characterised by long swings, particularly up to 1990. All three series thus have the characteristics of integrated processes containing a

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<sup>1</sup> We assume in this section that the Marshall-Lerner condition holds such that  $\partial Q / \partial X > \partial Q / \partial M > 1 > 0$ , although this is strictly an empirical question and does not in any way affect our results.

single unit root, although the real exchange rate does not contain a drift. Standard Dickey-Fuller tests confirm this supposition.

As Baxter (1994) and Baxter and King (1999) point out, relationships between business cycle components are often difficult to detect when conventional trend removal techniques, such as linear de-trending or first differencing, are employed. Stochastic trends associated with the presence of a unit root cannot be removed by linear de-trending, and this is a major drawback since permanent (trend) components will remain in the cyclical component obtained as the residual from the fitted trend (short-run noise will also be included in this residual). Although first differencing will remove unit root components, this filter has several drawbacks. It alters timing relationships between variables by inducing a substantial phase shift and it involves a dramatic re-weighting of frequencies – high frequency (noise) components are emphasised at the expense of down-weighting lower frequencies: in particular, much of the ‘cyclic’ variation is removed.

A popular choice of business-cycle filter is that proposed by Hodrick and Prescott (1997). This filter has some desirable properties. It will remove unit root trend components, it has no phase shift and, for an appropriate choice of its smoothing parameter, it closely approximates the optimal filter that isolates only components having business-cycle frequencies, defined to be those having periods between 6 and 32 quarters.

An arguably better choice is the band-pass filter proposed by Baxter and King (1999), which avoids the distortions associated with the Hodrick-Prescott filter that are caused by rapidly changing weights at the ends of the sample, which result in substantial distortions of these cyclical observations. Pedersen (2001), for example, recommends the use of this filter in situations such as those found here. For the quarterly time series  $x_t$ , the band-pass filter that passes components having periodicities between 6 and 32 quarters is defined as

$$x_t^* = \sum_{k=6}^{32} a_k x_{t-k}, \quad a_k = a_{32-k}, \quad \sum_{k=6}^{32} a_k = 0$$

$$a_0 = 0.2777, \quad a_1 = 0.2204, \quad a_2 = 0.0838, \quad a_3 = 0.0521, \quad a_4 = 0.1184, \\ a_5 = 0.1012, \quad a_6 = 0.0422, \quad a_7 = 0.0016, \quad a_8 = 0.0015, \\ a_9 = 0.0279, \quad a_{10} = 0.0501, \quad a_{11} = 0.0423, \quad a_{12} = 0.0119$$

This filter is employed here to extract the cyclical components from the two output series and real exchange rates, all of which contain single unit roots, although the

latter does not contain a drift: as we have pointed out above, it displays long swings rather than any trend.

Our first empirical modelling exercise, however, is to investigate the relationships between the observed series. The presence of unit roots in the three series opens the possibility that the series are cointegrated, as implied by the equilibrium exchange rate models mentioned in the introduction, in which case they can be modelled using a VECM framework. If they are not cointegrated, then a VAR in first differences is appropriate. In fact, there appears to be no compelling evidence of cointegration. For example, a likelihood ratio test of the null of no cointegrating vectors against the alternative of there being one, with a trend in the data but not in the cointegrating vector, yields a test statistic of 25.95 compared to a 5% critical value of 29.68.

We thus considered a VAR(2) in the differences,  $\Delta y_t^{US}$ ,  $\Delta y_t^{UK}$  and  $\Delta q_t$ , a second order being chosen by a sequential hypothesis testing procedure. This setting is consistent with the lag order used in the test for cointegration reported above and is general enough to ensure that the residuals from the estimated VAR passed diagnostic tests for residual autocorrelation, etc.. An implication of this framework is that we are de-trending via first differencing to use output growth rates and real exchange rate appreciation. The impulse responses for this ordering of the variables are shown in Figure 3. (The correlations between the three residual series are 0.15, -0.12 and -0.07, respectively, so that the impulse responses are virtually unaffected by alternative orthogonalisations.) When compared to their two standard error bounds, only the response of  $\Delta y_t^{UK}$  to an innovation in  $\Delta y_t^{US}$  looks to be at all significant, which is not surprising, but there does not appear to be any relationship between output growths and real exchange rate appreciation.



If the relationship between output and the real exchange rate is a cyclical one, then using first differences will remove most of these components, so that it is hardly surprising that little can be gleaned from Figure 3. Figure 4 presents pair-wise plots of the band-pass filtered cyclical components  $y_t^{*US}$ ,  $y_t^{*UK}$  and  $q_t^*$ , from which it can be seen that lead-lag relationships certainly appear to be present. Figure 5 presents the impulse responses from fitting a VAR(4) and using the above ordering. This lag order was again selected to ensure that all residual diagnostic checks were satisfied.

It is clear that there are important cyclical relationships present. Innovations to  $y_t^{*UK}$  and  $y_t^{*US}$  influence  $q_t^*$  with approximately seven and 13 quarter delays before the maximum response is felt. Innovations to  $q_t^*$ , on the other hand, have a much more muted effect on  $y_t^{*UK}$  and  $y_t^{*US}$ .  $y_t^{*UK}$  responds to an innovation in  $y_t^{*US}$  with a delay of around seven quarters, with little evidence of feedback. This is consistent with the UK being a small country relative to the US.

The pattern of causality would thus appear to run from  $y_t^{*US}$  to  $y_t^{*UK}$ , with a delay of seven quarters, and thence to  $q_t^*$ , with a delay of another seven quarters. There is then a weak feedback effect from  $q_t^*$  to  $y_t^{*UK}$  that peaks after a further 12 quarters, and an even more muted effect on  $y_t^{*US}$  after another six quarters. This pattern suggests that over a three-year time horizon the exchange rate is driven by the relative output cycle, which is consistent with exchange rate theory and that the subsequent feedback from the exchange rate to UK output is relatively weak. This weak feedback effect suggests that any overvaluation of sterling takes approximately three years to affect UK output, but even then the effect is relatively small.

#### 4 Conclusions

This paper has demonstrated that there is no long-run empirical relationship between the real US dollar-sterling exchange rate and relative outputs, as may be expected from a neo-Keynesian approach to the equilibrium exchange rate. On the other hand, it demonstrates a cyclical relationship between the real exchange rate and relative outputs, where the direction of causality is from outputs to the exchange rate, with only weak feedback effects. Thus the medium run relationship is between the cyclical components of real outputs and the real exchange rate series and not the trends.

This empirical result may also be taken as supportive of the neo-Keynesian, sticky-price model of the equilibrium exchange rate, if the benchmark is taken to be the business cycle and not the trend growth in outputs, while also explaining the rejection of the neo-classical PPP hypothesis.

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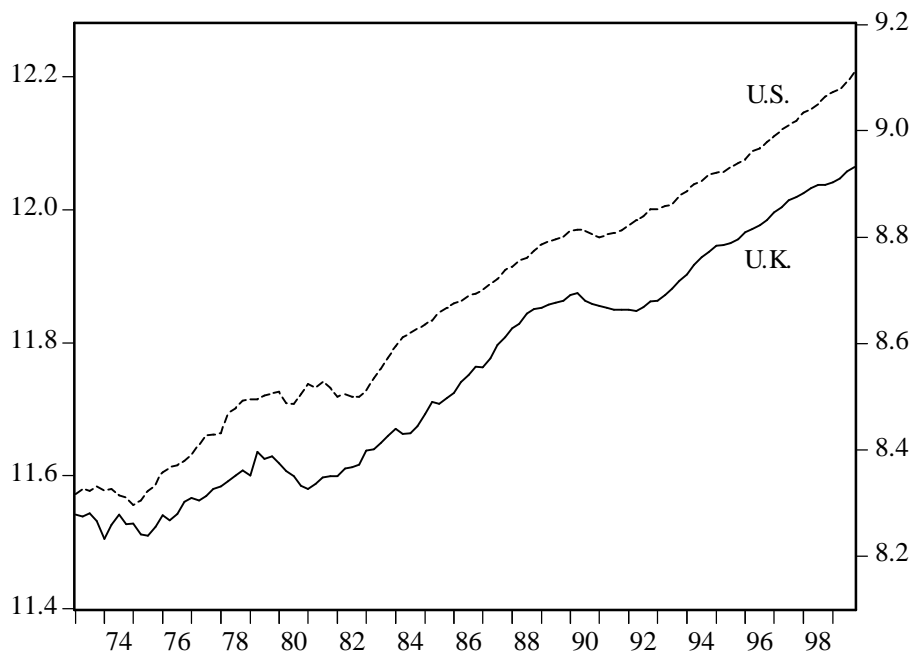


Figure 1. U.K. and U.S. output



Figure 2. Real exchange rate

Response to One S.D. Innovations  $\pm 2$  S.E.

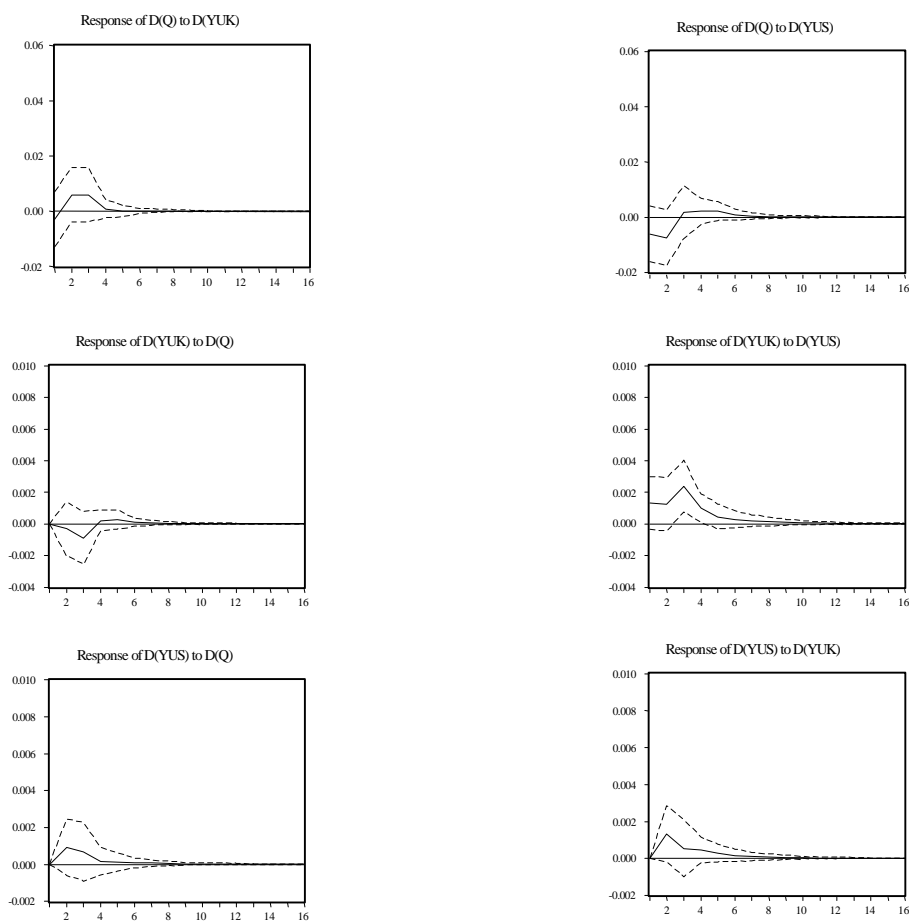


Figure 3. Impulse responses from the VAR(2) of  $y_t^{US}$  (denoted D(YUS)),  $y_t^{UK}$  (D(YUK)) and  $q_t$  (D(Q)), surrounded by two standard error bounds.

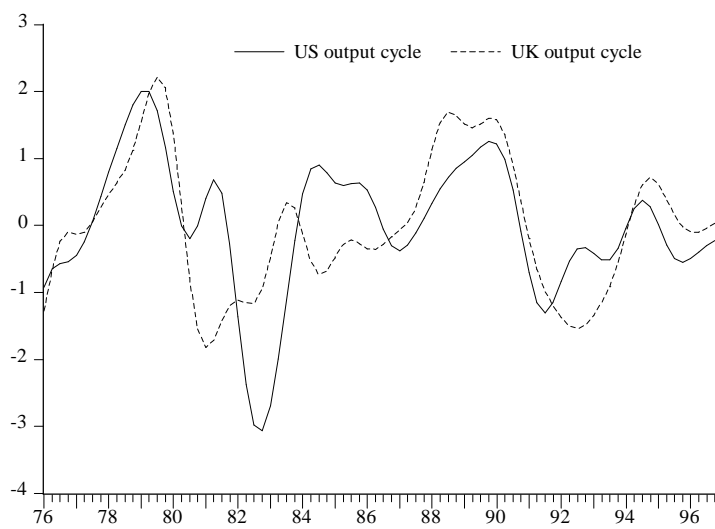
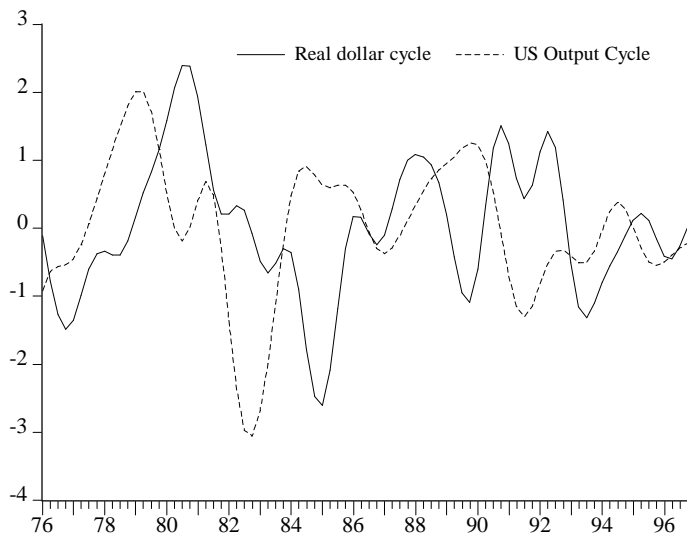
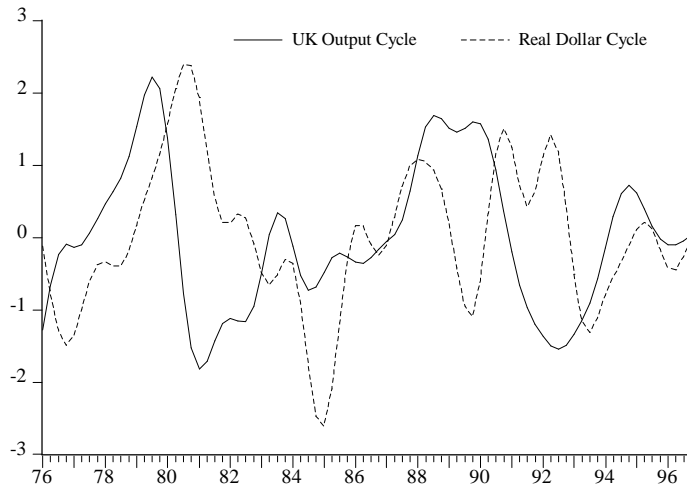


Figure 4. Cyclical components,  $y_t^{*US}$ ,  $y_t^{*UK}$  and  $q_t^*$ .

Response to One S.D. Innovations  $\pm 2$  S.E.

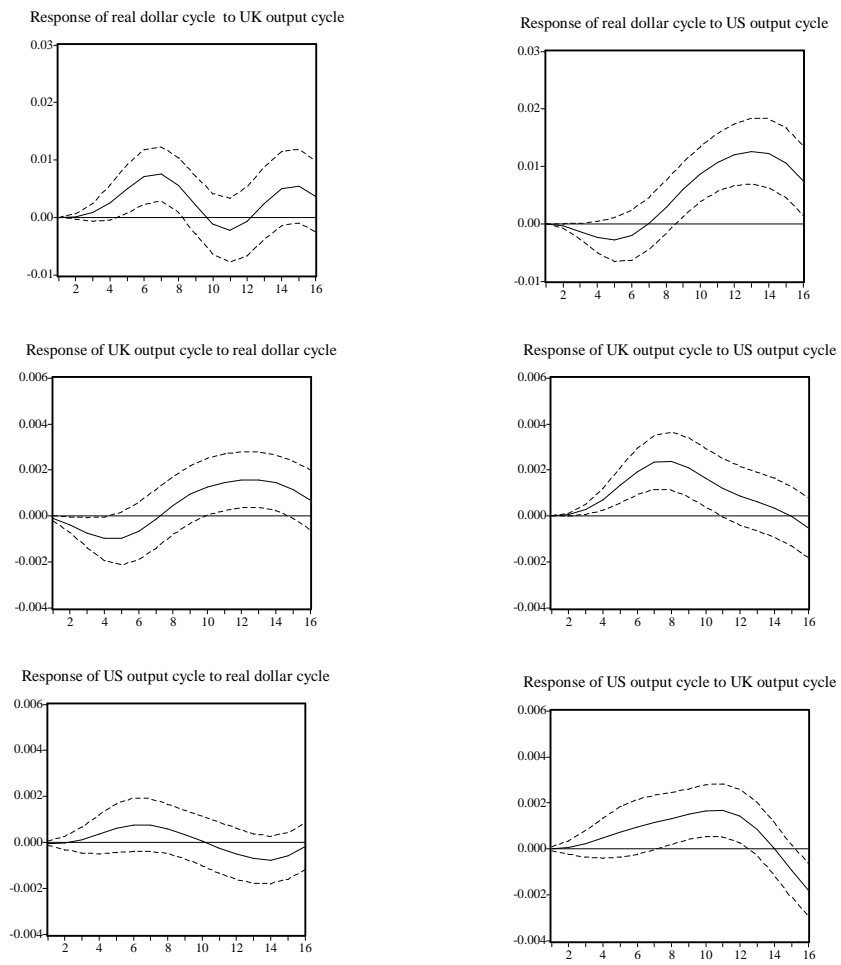


Figure 5. Impulse responses from the VAR(4) of  $y_t^{*US}$ ,  $y_t^{*UK}$  and  $q_t^*$ , surrounded by two standard error bounds.