

---

This item was submitted to [Loughborough's Research Repository](#) by the author.  
Items in Figshare are protected by copyright, with all rights reserved, unless otherwise indicated.

## R&D policy and privatization in a mixed oligopoly

PLEASE CITE THE PUBLISHED VERSION

PUBLISHER

© Loughborough University

VERSION

VoR (Version of Record)

LICENCE

CC BY-NC-ND 4.0

REPOSITORY RECORD

Gil-Molto, Maria Jose, Joanna Poyago-Theotoky, and Vasileios Zikos. 2019. "R&D Policy and Privatization in a Mixed Oligopoly". figshare. <https://hdl.handle.net/2134/4159>.

This item was submitted to Loughborough's Institutional Repository (<https://dspace.lboro.ac.uk/>) by the author and is made available under the following Creative Commons Licence conditions.



CC creative commons  
COMMONS DEED

**Attribution-NonCommercial-NoDerivs 2.5**

**You are free:**

- to copy, distribute, display, and perform the work

**Under the following conditions:**

 **Attribution.** You must attribute the work in the manner specified by the author or licensor.

 **Noncommercial.** You may not use this work for commercial purposes.

 **No Derivative Works.** You may not alter, transform, or build upon this work.

- For any reuse or distribution, you must make clear to others the license terms of this work.
- Any of these conditions can be waived if you get permission from the copyright holder.

**Your fair use and other rights are in no way affected by the above.**

This is a human-readable summary of the [Legal Code \(the full license\)](#).

[Disclaimer](#) 

For the full text of this licence, please go to:  
<http://creativecommons.org/licenses/by-nc-nd/2.5/>

ISSN 1750-4171

**DEPARTMENT OF ECONOMICS**

**DISCUSSION PAPER SERIES**

**R&D Policy and Privatization in a Mixed  
Oligopoly**

**Maria Jose Gil Molto, Joanna Poyago-Theotoky  
and Vasileios Zikos**

**WP 2006 - 22**

Dept Economics  
Loughborough University  
Loughborough  
LE11 3TU United Kingdom  
Tel: + 44 (0) 1509 222701  
Fax: + 44 (0) 1509 223910  
<http://www.lboro.ac.uk/departments/ec>



# R&D policy and privatization in a mixed oligopoly\*

Maria José Gil Moltó, Joanna Poyago–Theotoky and  
Vasileios Zikos<sup>†</sup>

*Loughborough University, Department of Economics*

## Abstract

We introduce R&D activity and R&D subsidies in the context of a mixed oligopoly and evaluate the effects of privatization on welfare. We show that when R&D subsidies are employed, privatization is welfare and R&D promoting provided that the number of competitors is sufficiently large.

*Keywords:* mixed oligopoly, process innovation, R&D subsidy, privatization.

*JEL Classification:* L31, L32, O38, L13, L50.

## 1 Introduction

There is a large body of literature on mixed oligopolies analyzing the effects on welfare of privatization. Interestingly, White (1996) and Poyago–Theotoky (2001) showed that when policy makers use output subsidies as a policy instrument, the issue of privatization is not welfare related. Further, Fjell and Heywood (2004) proved that privatization will bear negative consequences on welfare if the public firm remains as a leader in the post-privatization regime.

The analysis in these papers has been confined to output production and consequently, subsidies to output. However, the study of the R&D activity

---

\*We would like to thank the audience at the 33rd EARIE Conference (Amsterdam, August 25-27, 2006) as well as seminar participants at Alicante and Leicester for helpful comments. The last author gratefully acknowledges financial support by Loughborough University under its grant scheme.

<sup>†</sup>Corresponding author: Department of Economics, Loughborough University, Loughborough LE11 3TU, United Kingdom. Email: V.Zikos@lboro.ac.uk, tel.: + 44 (0) 1509 222708, fax: + 44 (0) 1509 26317.

and R&D subsidies in the context of mixed oligopolies has not yet been considered, despite the significant empirical evidence citing the importance of public funding towards R&D (Katz, 2001) and the substantial presence of public firms in innovative industries (examples are the health-care sector, Aanestad, 2003, and bioagriculture, Oehmke, 2001).

In this paper, we study the use of R&D subsidies in the context of a mixed oligopoly and evaluate the effects of a privatization. We show that, apart from addressing the market failures arising from the R&D activity, the use of R&D subsidies corrects (to some extent) the inefficient distribution of production costs which arises in mixed industries. In that sense, an R&D subsidy may (at least partially) serve the same purpose as an output subsidy.

Our results indicate that the optimal subsidy to R&D output is non-monotone in the number of private firms both in the private and the mixed markets and it is always lower for the former than for the latter. Further, when R&D subsidies are employed, privatization may increase total R&D and welfare provided that the number of private firms is sufficiently large. The latter contrasts with the results obtained in previous contributions, where output subsidies are employed.

## 2 The model

Consider an industry consisting of  $n$  identical private firms and a public firm producing a homogeneous good. The inverse demand function is linear and given by  $p(Q) = a - Q$ ;  $Q$  is aggregate output,  $Q = q_0 + \sum_{i=1}^n q_i$ ,  $q_0$  denotes the output of the public firm and  $q_i$ ,  $i = 1, \dots, n$ , is the output of the  $i$ -th private firm. We postulate that all firms engage in cost-reducing (process) R&D and there are no spillovers.<sup>1</sup> Thus, the production cost of each firm is represented by the quadratic function  $C_j(x_j, q_j) = (c - x_j)q_j + q_j^2$ ,  $j \in \{0, 1, \dots, n\}$ , where  $x_j$  is the cost reduction of the  $j$ -th firm and  $a > c > 0$ .<sup>2</sup> We also make the standard assumption that R&D spending is subject to diminishing returns to R&D expenditure,  $\Gamma_j(x_j) = x_j^2$ ,  $j \in \{0, 1, \dots, n\}$ .

---

<sup>1</sup>In other words, the patent system is fully effective.

<sup>2</sup>The presence of the quadratic term is standard in the mixed oligopoly literature and rules out the possibility of a public monopoly by introducing diminishing returns in production.

A firm's profit function is given by

$$\pi_j = q_j \left( a - \sum_{j=0}^n q_j \right) - (c - x_j) q_j - q_j^2 - x_j^2 + s x_j, \quad j \in \{0, 1, \dots, n\}, \quad (1)$$

where  $s$  denotes the (per unit) subsidy to R&D output. Social welfare, defined as the sum of consumer surplus,  $CS = (1/2)Q^2$  and producer surplus net of R&D subsidies is given by

$$SW = CS + \sum_{j=0}^n \pi_j - s \sum_{j=0}^n x_j. \quad (2)$$

The timing of the game is as follows: In stage one, the government commits to a subsidy on R&D output so as to maximize welfare. In stage two, firms make their R&D decisions and in the last stage, a standard Cournot game is played. We solve the entire game by backward induction to obtain the Subgame Perfect Nash Equilibria (SPNE henceforth) for both a mixed and a private oligopoly and compare the results across the two market arrangements.

### 3 Mixed oligopoly

Solving the last stage of the game, the respective equilibrium quantities of the public and the private firms are

$$q_0^m(x_0, x_i) = \frac{3(a - c) + (3 + n)x_0 - \sum_{i=1}^n x_i}{2n + 9} \quad (3)$$

$$q_i^m(x_0, x_i) = \frac{2(a - c) - x_0 + (3/n) \sum_{i=1}^n x_i}{2n + 9} \quad (4)$$

In the second stage, the associated equilibrium R&D output levels are<sup>3</sup>

$$x_0^m(s) = \frac{(a - c)[-3 + n(47 + 6n)] - 2n^2(6 + n)s}{-15 + n[235 + 102n + 12n^2]}, \quad (5)$$

$$x_i^m(s) = \frac{3(a - c)(3 + n)(1 + 2n) + n[135 + n(56 + 6n)]s}{-15 + n[235 + 102n + 12n^2]}, \quad i \in \{1, \dots, n\} \quad (6)$$

As it can be seen from  $x_0^m$  and  $x_i^m$ , the subsidy exerts a positive effect on

---

<sup>3</sup>The second order condition for the public firm requires  $135 + 56n + 6n^2 > 0$  and for each private firm,  $9 - 21n + 75n^2 + 36n^3 + 4n^4 > 0$ . Indeed, both conditions are fulfilled.

the R&D output of a private firm, whereas the reverse holds for the public firm. This implies that, similarly to an output subsidy (see White 1996; Poyago–Theotoky 2001), a subsidy to R&D has a cost redistribution effect. Hence, we can state that R&D subsidies may serve (at least partially) the same purpose as output subsidies.

Substituting (3) – (6) into (2) and performing the maximization with respect to  $s$ , we obtain the optimal subsidy<sup>4</sup>

$$s^m = \frac{(a - c)(5n - 3)}{n(5 + n)(7 + 2n)}. \quad (7)$$

The SPNE solutions of the entire game are the following

$$\begin{aligned} q_i^m &= \frac{(a - c)(9 + 2n)}{(5 + n)(7 + 2n)}; q_0^m = \frac{(a - c)(14 + 3n)}{(5 + n)(7 + 2n)} \\ x_i^m &= \frac{(a - c)(6 + n)}{(5 + n)(7 + 2n)}; x_0^m = \frac{(a - c)(7 + n)}{(5 + n)(7 + 2n)} \\ \pi_i^m &= \frac{(a - c)^2[-18 + n(153 + 65n + 7n^2)]}{n(5 + n)^2(7 + 2n)^2} \\ \pi_0^m &= \frac{(a - c)^2[-21 + n(179 + 75n + 8n^2)]}{n(5 + n)^2(7 + 2n)^2} \\ CS^m &= \frac{2(a - c)^2[7 + n(6 + n)]^2}{(5 + n)^2(7 + 2n)^2}; \end{aligned} \quad (8)$$

$$SW^m = \frac{(a - c)^2[7 + n(7 + n)]}{(5 + n)(7 + 2n)}. \quad (9)$$

It is important to note that although the optimal R&D subsidy improves the distribution of total costs, it does not restore cost efficiency. Thus, it attains a second best in the sense that complete equalization of production costs would require an additional instrument—a subsidy to output quantity—at the government’s disposal.

## 4 Private oligopoly

The industry now consists of  $(n + 1)$  profit-maximizing (private) firms. The SPNE outcomes of the game are<sup>5</sup>

<sup>4</sup>With requirement for second order condition  $6n^3(5 + n)(7 + 2n)(135 + 56n + 6n^2) > 0$ .

<sup>5</sup>The associated second order conditions are all satisfied and available upon request.

$$\begin{aligned}
s^p &= \frac{4(a-c)(3n-2)}{3n(3+n)(9+2n)} \\
q_i^p &= q_0^p = \frac{2(a-c)(4+n)}{(3+n)(9+2n)} \\
x_i^p &= x_0^p = \frac{(a-c)(5+n)}{(3+n)(9+2n)} \\
\pi_i^p &= \pi_0^p = \frac{(a-c)^2[-40+n(361+174n+21n^2)]}{3n(3+n)^2(9+2n)^2} \\
CS^m &= \frac{2(a-c)^2(1+n)^2(4+n)^2}{(3+n)^2(9+2n)^2} \\
SW^p &= \frac{(a-c)^2(1+n)(5+n)}{(3+n)(9+2n)}. \tag{10}
\end{aligned}$$

## 5 Results and Discussion

Comparing the results obtained in the mixed oligopoly and the private oligopoly cases, we can state the following results:<sup>6</sup>

**Proposition 1** *The optimal subsidy to R&D output in the mixed oligopoly is always greater than the subsidy in the private oligopoly,  $s^m(n) > s^p(n)$ .*

The intuition behind proposition 1 follows: In the case of a private oligopoly, two sources of market failure exist: (i) the imperfect competition, which will lead to underproduction (and hence, allocative inefficiency), and (ii) the R&D undervaluation effect (as defined by Ulph, 1999<sup>7</sup>), which will lead to under-investment in R&D by private firms. In the case of a mixed oligopoly a further source of market failure exists, the different nature (public or private) of the firms in the market. As a result, the production costs are inefficiently distributed. Hence, it is optimal for the decision-maker to subsidize more heavily a mixed market.

**Proposition 2** *The optimal subsidy to R&D output,  $s^i(n)$ ,  $i = m, p$ , is always positive and increasing in the number of private firms  $n$ , as  $n$  goes from 1 to 2, but decreasing in  $n$  if  $n > 2$ .*

<sup>6</sup>All proofs are relegated to the appendix.

<sup>7</sup>Private firms do not take into account the increases in Consumers Welfare as a consequence of the investment on R&D (as Consumers Welfare does not belong to their objective function). This will result in underinvestment in R&D.

Proposition 2 shows that, in contrast to the monotonic nature of an output subsidy identified in previous contributions, the optimal R&D subsidy is initially increasing and after a threshold value of the number of firms ( $n = 2$ ) decreasing. This is a result of the interaction of the market failures identified above. The effect of imperfect competition becomes less important as the number of firms increases whereas the undervaluation effect will have an inverted U-shape with respect to the number of firms, as identified by Suzumura (1992). The combination of these two effects (plus the inefficiency in the distribution of the production costs in the mixed case) results in an inverted U-shape of the optimal subsidy.

The next proposition compares total R&D output, output quantity and profits between the mixed and the private oligopolies:

**Proposition 3** *(i) Total R&D output in the private oligopoly is higher than in the mixed oligopoly if  $n > 4$ ;  $(x_0^m + nx_i^m) < (n + 1)x_i^p$ . (ii) Total output quantity in the mixed oligopoly always exceeds total output quantity in the private oligopoly;  $(q_0^m + nq_i^m) > (n + 1)q_i^p$ . (iii) Total profit in the private oligopoly always exceeds total profit in the mixed one;  $(\pi_0^m + n\pi_i^m) < (n + 1)\pi_i^p$ .*

With regard to proposition 3, it is relevant to note that the public firm will tend to reduce its R&D investment more than a private firm as  $n$  increases, i.e.,  $|\frac{\partial x_0^m}{\partial n}| > |\frac{\partial x_i^m}{\partial n}|$ , leading therefore to higher levels of total R&D output in the private oligopoly than in the mixed one for sufficiently large  $n$  ( $n > 4$ ). It turns out, however, that the public firm's behavior will not impact total output quantity in the same way and output will be always higher in the mixed oligopoly than in the private one. Regarding equilibrium profits, the underproduction problem will be more serious in the private oligopoly than in the mixed one as a result of the lower intensity of competition, leading thus to higher oligopoly rents and allocative inefficiency.

The next proposition contains a welfare assessment of privatization policies and is largely a consequence of Proposition 3:

**Proposition 4** *When government policy takes the form of an optimal subsidy to R&D output, then privatization enhances total welfare if  $n > 4$ ,  $SW^m(n) < SW^p(n)$ .*

The intuition for the above proposition follows: First, we can state that typically privatization improves productive efficiency. The reason is that in the move from the mixed to the private oligopoly optimum, the inefficiency in the distribution of production costs vanishes. However, privatization worsens allocative efficiency as it promotes higher oligopoly rents. It turns out that the gains in terms of productive efficiency will outweigh the losses in terms of allocative efficiency only if  $n$  is sufficiently large.

## 6 Conclusion

This paper aims at filling a gap in the literature on mixed oligopolies and privatization by introducing R&D activity and R&D subsidies. Our results indicate that the optimal subsidy to R&D output is non-monotone in the number of private firms both in the private and the mixed markets and that the mixed industry should be more heavily subsidized than the private one. Similarly to an output subsidy, a subsidy to R&D can address the inefficient distribution of costs. However, in contrast to the welfare results of privatization when output subsidies are provided, privatization is welfare enhancing if the number of firms in the industry is sufficiently high. Further, under the same condition, privatization yields increases in the total R&D levels. In industries with a small number of firms, privatization would result in a loss of surplus and decreases in the R&D activity.

## References

- [1] Aanestad, M., Mork, E. B., Grisot, M., Hanseth, O., Syvertsen, M. C., 2003. Knowledge as a Barrier to Learning: A Case Study from Medical R&D, paper presented at the 4th European Conference on Organisational Knowledge, Learning and Capabilities, April.
- [2] Fjell, K. and Heywood, J., 2004. Mixed oligopoly, subsidization and the order of firms' moves: the relevance of privatization, *Economics Letters* 83, pp. 411-416.
- [3] Katz, J., 2001. Structural reforms and technological behavior. The sources and nature of technological change in Latin America in the 1990s, *Research Policy* 30, pp. 1-19.

- [4] Oehmke, F. J., 2001. Biotechnology R&D races, industry structure, and public and private sector research orientation”, AgBioForum 4, pp. 105-114.
- [5] Poyago-Theotoky, J., 2001. Mixed oligopoly, subsidization and the order of firms’ moves: an irrelevance result, Economics Bulletin, 12, pp. 1-5.
- [6] Suzumura, K., 1992. Cooperative and noncooperative R&D in an oligopoly with strategic commitments, American Economic Review 82, pp. 1307-1320.
- [7] White, M.D., 1996. Mixed oligopoly, privatization and subsidization, Economics Letters 53, pp. 189-195.

## 7 Appendix

*Proof of Proposition 1:* It is immediate to show that  $s^m(n) > 0$  as  $a - c > 0$  and  $-3 + 5n > 0 \forall n \in Z_+$ . Next,  $s^m(1) < s^m(2)$  because  $\frac{a-c}{27} < \frac{a-c}{22}$ . To show that  $s^m$  is decreasing in  $n$  iff  $n > 2$ , ignore that  $n$  is an integer; it must be shown that  $\frac{ds^m}{dn} < 0$  iff  $n > 2$ . This derivative has the same sign as the expression  $105 + 102n - 67n^2 - 20n^3$ . The latter is negative iff  $n > 2$ . (The proof for  $s^p(n)$  is similar and hence is omitted). QED

*Proof of Proposition 2:* It will be shown that  $s^m(n) - s^p(n) > 0 \forall n \in Z_+$ . To this end, ignore that  $n$  is an integer. Then it suffices to show that  $37 - 14n + 19n^2 + 6n^3 > 0 \forall n$ , which in turn is always true. QED

*Proof of Proposition 3:* Let  $\Gamma = (3+n)(9+2n) > 0$  and  $\Delta = (5+n)(7+2n) > 0$ . We have the following (ignoring that  $n \in Z_+$ ):

(i)  $(x_0^m + nx_i^m) - (n+1)x_i^p = \frac{(a-c)(14-n-n^2)}{\Gamma\Delta} < 0$  iff  $14 - n - n^2 < 0$ . The inequality holds iff  $n > 4$ .

(ii)  $(q_0^m + nq_i^m) - (n+1)q_i^p = \frac{2(a-c)(49+24n+3n^2)}{\Gamma\Delta} > 0 \forall n$ .

(iii)  $(\pi_0^m + n\pi_i^m) - (n+1)\pi_i^p = -\frac{(a-c)^2\Psi}{3n\Gamma^2\Delta^2} < 0$  iff  $\Psi > 0$ , where

$\Psi = -3073 + 44548n + 151318n^2 + 139764n^3 + 59759n^4 + 13400n^5 + 1540n^6 + 72n^7$ . It is not difficult, although tedious, to show that  $\Psi > 0 \forall n$ , which completes the proof. QED

*Proof of Proposition 4:* In proving that  $SW^m(n) < SW^p(n)$  iff  $n > 4$ , it suffices to show that  $-14 + n + n^2 > 0$  iff  $n > 4$ . The result follows immediately. QED