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# Asymmetric Entry Equilibrium in a Symmetric Trading Oligopoly Model

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## Abstract

We examine the R&D and export decisions of two *ex ante* symmetric firms in symmetric countries, with both unit trade costs and fixed entry costs to the export market. When both trade costs are low, there will be a symmetric, cross-hauling duopoly, but if fixed costs are fairly high, unit trade costs are low and R&D is relatively cheap, there will be an asymmetric entry equilibrium, in which the exporting firm carries out higher R&D, has lower costs and larger profits. With higher R&D costs and/or higher unit trade costs, there will also be a zone where crosshauling duopoly and non-trading are simultaneously Nash equilibria.

JEL Codes: F12, L13

Keywords: Trade, Oligopoly, market entry, asymmetry

## 1 Introduction

While there is a considerable literature on oligopolistic and monopolistically competitive market structures in trade, asymmetry is rarely examined except where it reflects underlying heterogeneity. Following Melitz (2003), economists have come to link entry decisions to firm heterogeneity and fixed entry costs. Firms which engage in trade are seen as larger and more efficient (Bernard and Jensen, 1995; Baldwin and Gu, 2003) than their rivals, partly as a result of firm selection.

Caution is needed. Where does the heterogeneity stem from - is it truly *ex ante*, or consequential upon market entry, particularly if there are firm-level economies? Evidence of greater size or efficiency prior to starting exporting (Lileeva and Trefler, 2010) is not, of itself, sufficient to rule out that firms were *ex ante* homogeneous, given that what matters for subsequent behaviour is exporting intent, rather than actual entry.

In this short note we show first that the export decision, particularly in industries intensive in research and development (R&D), is a game of 'chicken', where, in some cost zones, we should expect an asymmetric equilibrium. There are similarities with Mills and Smith (1996), where some firms in a market will choose large-scale, while others choose small-scale technology, in cases where

the technology set is 'insufficiently convex'. Götz (2005) extended this to a free entry Cournot case with discrete technology, also finding zones of multiple equilibria - a feature of our model, too.

We extend Brander's (1995) 'reciprocal markets' Cournot model, with two symmetric firms in symmetric countries. Our first modification is that firms need not enter the foreign market. Secondly, they engage in noncollaborative cost-saving R&D. This introduces firm-level economies of scale, which reduce marginal costs.<sup>1</sup>

## 2 The basic duopoly model

Consumer preferences in both countries are identical. We choose units such that inverse demand in country  $i$ ,

$$P_i = 1 - q_{ii} - q_{ji}. \quad (1)$$

The game is: i) firms set R&D effort,  $x_i$ , which imposes a quadratic fixed cost  $\frac{\gamma}{2}x_i^2$ , but lowers marginal cost.<sup>2</sup> ii) Firms decide whether to export, subject to a unit trade cost,  $\tau$ , and, following Melitz (2003) a fixed entry cost,  $F$ . Generally, it does not affect equilibrium whether or not firms decide on R&D before the entry decision - we assume firms set R&D first for reasons of realism. iii) Firms set output noncooperatively in both markets. As R&D has already been set, firms take marginal costs as exogenous when setting output. Importantly, we start by assuming that both firms enter each other's market. We derive the SPNE in terms of R&D, output and profits for a series of possible regimes, allowing for entry/exit.

## 3 Equilibrium allowing for entry/exit

A firm can avoid the fixed cost,  $F$ , by not exporting. Sales and R&D will be reduced. Below are the R&D and associated profit matrices.

R&D matrix		Firm 2	
		Exporting	Non-Exporting
Firm 1	Exporting	$x^{*c}; x^{*c}$	$\hat{x}^A; x'^A$
	Non-Exporting	$x'^A; \hat{x}^A$	$x^N; x^N$

Profit matrix		Firm 2	
		Exporting	Non-Exporting
Firm 1	Exporting	$\Pi^{*c}; \Pi^{*c}$	$\hat{\Pi}^A; \Pi'^A$
	Non-Exporting	$\Pi'^A; \hat{\Pi}^A$	$\Pi^N; \Pi^N$

<sup>1</sup>Evidence that firms' productivity performance benefits from investment has been shown recently by Lileeva and Treffer, 2010.

<sup>2</sup>c.f. D'Aspremont and Jacquemin (AJ, 1988)

There are four possible levels of equilibrium profits, for a crosshauling duopoly, for two nontrading monopolies and for the exporting and nonexporting firms in the asymmetric cases. These are listed in the Appendix.

Before deriving the full conditions for each possible pure strategy SPNE, we make a couple of simplifications. First, we choose a 'middling' value of base cost ( $c = 0.5$ ). Secondly, we do not want unit marginal costs to be reduced below zero in any of our cases: this can be ruled out by making R&D 'sufficiently expensive' ( $\gamma \geq 2.7$ ).<sup>3</sup>

It is also useful to define two critical values of  $F$ .

**Definition:** We define  $F = F'$  as the value which equates  $\Pi_i^c = \Pi'^A$ . In addition,  $F = \hat{F}$  equates  $\Pi^N = \hat{\Pi}^A$ .

Both  $F'$  and  $\hat{F}$  are quadratic functions of  $\tau$  (full formulae, which are messy, are in the Appendix).

Using these we can derive:

**Proposition 1:** The SPNE, dependent upon the relative values of  $F$ ,  $F'$  and  $\hat{F}$ , implies:

- a) A symmetric, cross-hauling duopoly, if the fixed entry cost,  $F < F'$ .
- b) No trade if the fixed entry cost,  $F > \hat{F}$ .
- c) An asymmetric entry equilibrium, where one firm will export and the other will not, where  $F' < F < \hat{F}$ .

**Proof:** In the case of a),  $F < F' \implies \Pi^{*c} > \Pi'^A$ , so assuming one firm is exporting, the other also prefers to export. For b),  $F > \hat{F} \implies \Pi^N > \hat{\Pi}^A$ , so if one firm is not exporting, the other will also prefer not to export. For c) the two possible pure-pure-strategy, symmetric SPNEs are ruled out. Note, however, that a mixed strategy equilibrium (which is symmetric ex ante) will still exist. **QED**

Note that, in the asymmetric case, which firm exports is essentially a random decision (maybe reflecting some small-scale event, such as an approach from a potential export customer).

If is also conceivable that  $F' > F > \hat{F}$ , in which case, both C and N will be SPNE.

These equilibria are best examined numerically, plotting ranges of  $F$  and  $\tau$  for which the various outcomes are possible. This is shown in *Figure 1*, below, for the case where  $c = 0.5$  and  $\gamma = 2.75$ .

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<sup>3</sup>The case with the highest R&D, and hence lowest unit marginal costs, is with the exporting firm in an asymmetric equilibrium. Numerical analysis shows that  $\gamma \geq 2.7$  ensures  $\hat{x}^A \leq 0.5$ .

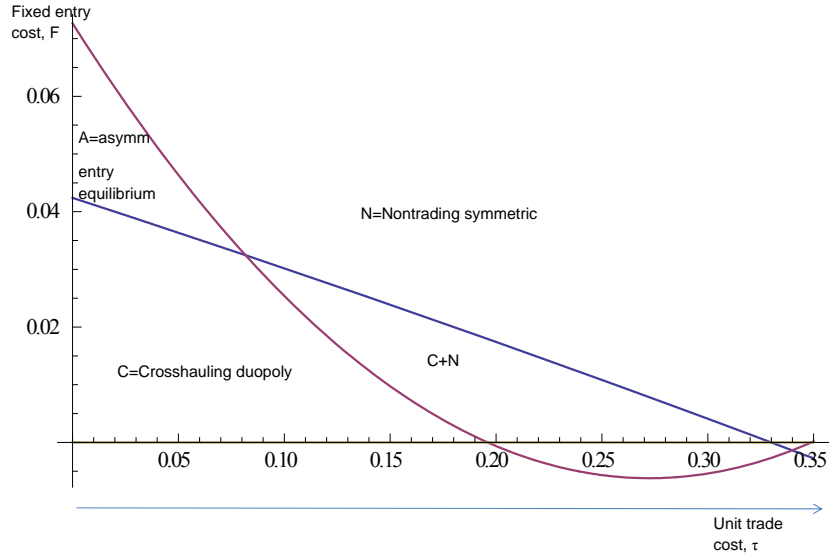


Figure 1: ranges of  $\tau$  and  $F$  which yield the various SPNEs of the entry game. R&D cost,  $\gamma$ , is assumed to be 2.75.

There are four ranges present: for low  $\tau$  and  $F$ , C prevails. For low  $\tau$  but higher  $F$ , there is asymmetric entry (A). For high  $\tau$  and  $F$  neither enters the other's market (N). There is also a zone of overlap between  $F'$  and  $\hat{F}$ , where either C or N can be an equilibrium (multiple equilibrium).

Note that when  $\gamma = 3$  (Appendix figure 1), implying costlier R&D, the possibility of asymmetric entry almost disappears (but the zone of multiple equilibria widens).

## 4 Conditions where there is no single, symmetric equilibrium

### 4.1 Asymmetric entry equilibrium

The possibility that increasing returns in R&D may enhance the difference between exporting and non-exporting firms has been known, at least since Bernard et al. (2003). Ledezma (2010) shows that such equilibria can exist in a multi-firm monopolistically competitive world, when firms are initially homogeneous. This paper shows that a similar situation can occur in a Cournot duopoly, although dependent upon entry and trade costs.

In particular, comparing the firms case A shows that they retain many of the characteristics usually attributed to underlying firm heterogeneity.

**Proposition 2:** *The exporting firm has higher R&D, lower unit costs and higher profits than the non-exporter.*

**Proof:**

If 2 is the exporter, then

$$x_1' = \frac{8(1-c)(3\gamma-4)}{-32+72\gamma-27\gamma^2}, \hat{x}_2 = \frac{4(1-c)(3\gamma-8)}{-32+72\gamma-27\gamma^2}. \quad (2)$$

Assuming  $\gamma > 2$ , then the denominator is positive and, assuming  $\hat{x}_2 > 0$ ,  $\hat{x}_2 > x_1'$ . Higher R&D implies lower unit costs. Also, if firm 2 enters 1's market, then 1's profits must be less than  $\Pi^{*c}$ , while 2 will only export if  $\hat{\Pi}^A > \Pi^{*c}$ . QED

Viewing this result in context, the literature recognises (Lileeva and Treffer, 2010) that, while firm heterogeneity may lead to differential entry decisions, within-firm productivity also benefits from the entry decision (as feedback). What is perhaps not enough appreciated is the role of industry structure (some markets are much 'thinner' than others, having barriers to initial entry) and, secondly, the initial causes of observed firm heterogeneity (does it, in fact, result from the cumulative result of a long-run series of games each of which is, *ex ante*, symmetric?).

## 4.2 Multiple symmetric equilibria

There is also a zone,  $\hat{F} < F < F'$ , where both C and N are Nash equilibria. In these circumstances, A cannot be an equilibrium.

**Proposition 3:** *From the point-of-view of the firms (though not consumers), the crosshauling duopoly equilibrium represents a Prisoner's Dilemma.*

**Proof:** Starting from a crosshauling duopoly, assume firm 1 exits the export market. This removes competition from firm 2's home market, so firm 2's profits rise. Hence  $\hat{\Pi}^A > \Pi^{*C}$ . Despite this, for N to be a Nash equilibrium, it must still be in firm 2's interests to exit its export market as well. Hence  $\Pi^N > \hat{\Pi}^A > \Pi^{*C}$ . It follows that the nontrading equilibrium is a Pareto improvement for the firms, compared to crosshauling duopoly, but if the firms reach the crosshauling equilibrium, they will be stuck there. QED

## 5 Conclusions

We should advise caution to researchers concluding that asymmetric ex post behaviour necessarily reflects ex ante heterogeneity of costs or efficiency. Models of market entry can easily produce asymmetric results, where trade costs - particularly fixed entry costs - mean that not all firms can be sustained as exporters. In those circumstances, difference of intent may be as important as

differences in initial efficiency. Previous studies of single markets suggests that asymmetry may carry over to cases with multiple firms.

A second caution is that multiple symmetric equilibria are feasible in this game, where the welfare results may vary considerably between equilibria. We are not aware of how easily this carries over to the multiple firm case.

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## 6 Appendix

1. Equilibrium values of profit and R&D for the various entry cases are:

$$\Pi_i^c = \frac{(1 + 2x^{*c} + 2\tau)^2}{36} + \frac{(1 + 2x^{*c} - 4\tau)^2}{36} - \frac{\gamma}{2}x^{*c2} - F; \text{ where } x^{*c} = \frac{2(1 - 2\tau)}{9\gamma - 8}. \quad (3)$$

$$\Pi^N = \frac{(1 + 2x^N)^2}{16} - \frac{\gamma}{2}x^{N2}; \text{ where } x^N = \frac{1}{4\gamma - 2}. \quad (4)$$

$$\hat{\Pi}^A = \frac{(1 + 4\hat{x}^A - 2x'^A + 2\tau)^2}{36} + \frac{(1 + 4\hat{x}^A - 2x'^A - 4\tau)^2}{36} - \frac{\gamma}{2}2\hat{x}^{A2} - F; \quad (5)$$

$$\text{where } \hat{x}^A = \frac{4(4 - 3\gamma) - 12\gamma\tau}{32 + 9\gamma(3\gamma - 8)}.$$

$$\Pi'^A = \frac{(1 + 4x'^A - 2\hat{x}^A + 2\tau)^2}{36} - \frac{\gamma}{2}2x'^{A2} \text{ where } x'^A = \frac{2(3\gamma - 8) + 4(3\gamma - 4)\tau}{32 + 9\gamma(3\gamma - 8)}. \quad (6)$$

2. The value of  $F = F'$ , which satisfies  $\Pi_i^c = \hat{\Pi}^A$ , is

$$\frac{1}{((8 - 9\gamma)^2(32 + 9\gamma(3\gamma - 8))^2)} [((c-1)\gamma(10240 + 3\gamma(81\gamma(128 + \gamma(-56 + 9\gamma)) - 10240)) - 8192\tau + 6\gamma(7168 + \gamma(27\gamma(464 + 3\gamma(9\gamma - 62))))\tau - 14080)(-4\tau + 3\gamma(c - 1 + 2\tau))]$$

The value of  $F = F'$  which satisfies  $\Pi^N = \hat{\Pi}^A$  is

$$\frac{1}{2(2\gamma - 1)(32 + 9\gamma(3\gamma - 8))^2} [(-(c - 1)^2\gamma^2(3\gamma(-8 + 3\gamma)(-52 + 9\gamma) - 448) + 12(c - 1)\gamma^2(2\gamma - 1)(3\gamma - 4)(9\gamma - 16)\tau + 2(2\gamma - 1)(512 + 9\gamma(\gamma(384 + \gamma(-224 + 45\gamma))))\tau^2 - 256)]$$

3. Redrawing Figure 1 with a higher value of  $\gamma = 3$ , the zone of asymmetric equilibrium shrinks almost to nothing

