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Radial Vibration Measurements Directly from Rotors using Laser Vibrometry: Uncertainty due to Surface Roughness

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Nomenclature

\vec{a}	vector vibration displacement
k	frequency ratio
x, \dot{x}	x-direction components of vibration displacement and velocity
y, \dot{y}	y-direction components of vibration displacement and velocity
U_x, U_y	measured vibration velocities in x- and y-directions
\tilde{U}_x, \tilde{U}_y	ac coupled measured vibration velocities in x- and y-directions
$W(\omega)$	weighting function used in post-processing algorithm
$\Omega, \bar{\Omega}$	angular rotation frequency and mean value
ω	angular vibration frequency
$\theta_x, \dot{\theta}_x$	pitch vibration displacement and velocity
$\theta_y, \dot{\theta}_y$	yaw vibration displacement and velocity
$\dot{\Theta}_x, \dot{\Theta}_y$	measured pitch and yaw vibration velocities
$\dot{X}(\omega), \dot{Y}(\omega)$	resolved spectral components of and x- and y-direction radial vibration

Abstract

Radial vibration measurements taken directly from rotors using Laser Vibrometry are known to show a significant cross-sensitivity to the orthogonal radial vibration component. A process for resolving the individual components is now well established and is suitable for both radial measurements and pitch / yaw measurements which show an equivalent cross-sensitivity. All of the work conducted in the development of this system was done using surfaces treated with retro-reflective tape, a common surface treatment for Laser Vibrometer measurements. In this paper investigations have been conducted on untreated surfaces with roughness close to or less than the wavelength of light. These investigations have highlighted an inherent uncertainty for measurements in this roughness range related to changes in the effective centre of the incident laser beam. This uncertainty is shown to be influenced by surface roundness and incident beam diameter as well as by the surface roughness itself. At even lower surface roughness on rotors with low roundness error, it is possible that the cross-sensitivity may be negligible.

Introduction

Since the advent of the laser in the early 1960's, optical metrology has continued to provide innovative techniques for remote and unobtrusive measurement in the most challenging of environments. Laser (Doppler) Vibrometry (LDV) is now an established technique for vibration measurement, complementing use of traditional transducers

such as the accelerometer in situations where remote or non-contact operation is beneficial or essential. The principle of operation of the Laser Vibrometer is detection of the Doppler frequency shift that occurs when light is scattered by a moving particle. This Doppler frequency shift is directly proportional to the velocity of the particle. Laser Vibrometers are technically well suited to general application but offer special benefits where certain measurement constraints are imposed, for example by the context, which may demand high frequency operation, high spatial resolution or remote transducer operation, or by the structure itself, which may be hot, light or rotating.

On rotating machines, it is most common to measure the vibration transmitted into a non-rotating part of the machine using a contacting transducer such as an accelerometer but low vibration transmission can make this unreliable. For translational or angular vibrations, non-contact transducers capable of measuring directly from any location on a structure, but especially directly from the rotor itself, would be valuable assets and Laser Vibrometers offer this possibility. Measurements on rotating structures are often cited as important applications of LDV and rotor vibration measurements are the particular focus of this paper.

Almost 40 years ago, one of the first reported LDV applications was for axial vibration measurement directly from a rotating turbine blade [1]. Investigations of magnetic discs [2,3], bladed discs [4,5] and modal analysis on rotating discs [6] are typical and more recent examples of measurements that can be made using a single probe laser beam. Parallel beam arrangements, for angular vibration measurements, have been used for assessment of torsional damper health [7] and crankshaft bending vibration [8].

Cross-sensitivity in radial and pitch / yaw vibration measurements

A feature of much early work was prediction of acceptable performance but only in the presence of a single vibration component, neglecting the effects of other components present in the more complex motions likely to be encountered in practice. The presence of a velocity component due to the rotation itself, however, is now known to have an important effect on the measured velocity. The effect was first reported for radial vibration measurements [9] where sensitivity to both speed fluctuation (including torsional oscillation) and in-plane motion (i.e. that perpendicular to the intended measurement) was demonstrated. Although the effect has now been more comprehensively described [10,11], it remains useful to review the original 2D analysis as a means to explain the fundamental issue. With reference to figure 1, consider an axial element of a rotating shaft, of arbitrary cross-section, undergoing a vector vibration displacement \vec{a} . Point P is the instantaneous point of the incidence of the laser beam on the shaft.

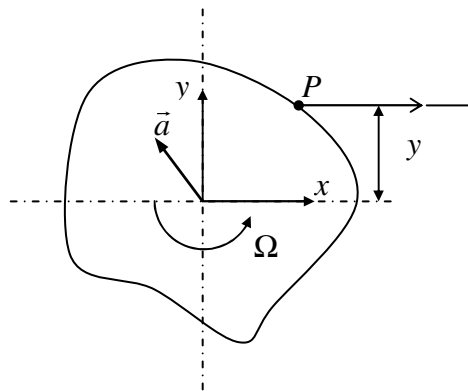


Figure 1 – Rotating shaft geometry

If the target vibrates without rotating then the velocity in the direction of the incident laser beam is simply \dot{x} , the time derivative of the x-radial component of \vec{a} . When the target rotates, however, the velocity in the direction of the incident laser beam also contains a component due to the tangential velocity. Moreover, even for a perfectly circular cross-section, the tangential velocity component sensed by the laser beam will be influenced by any y-radial vibration (perpendicular to the direction of the beam) and by any variation in the speed of the illuminated element. This will be the case even if the initial laser beam alignment is directly through the rotation centre of the shaft. While the intended measurement is of \dot{x} , the measured vibration velocity, U_x , will be:

$$U_x = \dot{x} + \Omega y \tag{1a}$$

and if a measurement were made in the y-radial direction such that the intended measurement was of \dot{y} , the measured vibration velocity, U_y , would be:

$$U_y = \dot{y} - \Omega x \quad (1b)$$

The use of parallel beam arrangements allows direct measurements of the angular vibration components, such as pitch and yaw, but as the fundamental measurement principle is the same it is not surprising to see that these measurements are subject to the same type of cross-sensitivity that affects the radial measurements. An intended pitch measurement, $\dot{\theta}_x$, will result in a measured velocity, $\dot{\Theta}_x$, that is affected by yaw:

$$\dot{\Theta}_x = \dot{\theta}_x + \Omega \theta_y \quad (2a)$$

and, similarly, an intended yaw measurement, $\dot{\theta}_y$, will result in a measured velocity, $\dot{\Theta}_y$, that is affected by pitch:

$$\dot{\Theta}_y = \dot{\theta}_y - \Omega \theta_x \quad (2b)$$

The same post-processing solution used for radial vibration can therefore be used for pitch / yaw vibrations too. A solution to resolve steady-state, non-synchronous radial and pitch / yaw rotor vibrations, in the absence of speed fluctuations, requires simultaneous orthogonal measurements to be combined with a speed measurement, followed by post-processing. The current resolution algorithm [8] is implemented in LabVIEW and uses the real and imaginary parts of the Fourier Transforms of the ac-coupled measured signals to give the spectra of the genuine vibration velocities. A second algorithm, not described here, is able to correct initial estimates from measurements made in the presence of speed fluctuations or torsional vibrations.

For resolution of the radial vibrations, $\dot{X}(\omega)$ and $\dot{Y}(\omega)$:

$$\begin{aligned} \dot{X}(\omega) &= W(\omega) \left(FT[\tilde{U}_x] - \frac{\bar{\Omega}}{j\omega} FT[\tilde{U}_y] \right) \\ &= W(\omega) \left(\left\{ \Re[\tilde{U}_x] - \frac{\bar{\Omega}}{\omega} \Im[\tilde{U}_y] \right\} + j \left\{ \Im[\tilde{U}_x] + \frac{\bar{\Omega}}{\omega} \Re[\tilde{U}_y] \right\} \right) \end{aligned} \quad (3a)$$

$$\begin{aligned} \dot{Y}(\omega) &= W(\omega) \left(FT[\tilde{U}_y] + \frac{\bar{\Omega}}{j\omega} FT[\tilde{U}_x] \right) \\ &= W(\omega) \left(\left\{ \Re[\tilde{U}_y] + \frac{\bar{\Omega}}{\omega} \Im[\tilde{U}_x] \right\} + j \left\{ \Im[\tilde{U}_y] - \frac{\bar{\Omega}}{\omega} \Re[\tilde{U}_x] \right\} \right) \end{aligned} \quad (3b)$$

where the weighting $W(\omega) = (k^2/k^2 - 1)$ and $k = (\omega/\bar{\Omega})$. The same algorithm is used for resolution of the pitch and yaw vibrations. Inspection of these equations shows that the weighting term is infinite and the bracketed term is zero for synchronous vibrations, i.e. $\omega = \bar{\Omega}$, which makes this technique unsuitable for synchronous vibration measurement. This leads to a gap in the resolved data at the synchronous frequency and a velocity-time trace cannot be reconstructed. This is not a limitation of the post-processing technique but a limitation on the use of Laser Vibrometers for both translational and angular vibration measurements on rotors. A measured synchronous component can only be considered as a conservative estimate of the sum of the two (either radial or angular) velocities. Notwithstanding this limitation, the post-processing technique has been used to resolve the pitch / yaw and radial vibrations from simultaneous measurements on the crankshaft pulley of a running engine [8].

Surface Treatments

In many applications, it is convenient, and occasionally essential, to coat the rotor surface with a retro-reflective tape or paint and the work of the author to date on rotor vibration measurements has almost exclusively used such surface treatment. The tape or paint acts to concentrate the scattered light in a narrow cone back in the direction of the incident beam, facilitating the collection of sufficient light intensity without the need for additional optics or careful alignment. With a growth in the number of applications of LDV, however, there has been increased interest in taking radial measurements, in particular, directly from untreated rotor surfaces. The specific application of interest is measurement from polished surfaces. The author is grateful to Karl Bendel of Robert Bosch GmbH who prompted consideration of this application, followed by further discussion with researchers from Lulea University who have recently published work on spindle vibrations [12, 13].

When a coherent laser beam is incident on a surface that is optically rough, i.e. the surface roughness is large on the scale of the laser wavelength, the component wavelets of the scattered light become dephased. This condition is satisfied by many of the surfaces likely to be encountered in engineering structures. The dephased, but still coherent, wavelets interfere constructively and destructively, thus resulting in a chaotic distribution in backscatter of high and low intensities, referred to as a “speckle pattern”. The presence of a fully-developed speckle pattern is indicative of scattering from a diffuse surface which means that the light collected through the aperture of the Laser Vibrometer has been scattered from all points in the incident beam in proportions that are directly related only to the intensity profile of the beam. In such cases, the point of incidence of the beam is regarded as being the same as the (unchanging) geometric centre of the beam, as if the beam has no extent and is incident on a single point.

If the laser beam is incident on the circumference of an optically smooth surface with circular cross-section, the light incident away from the centre-line of the rotor is likely to be reflected away from the collecting aperture. When the incident beam is aligned through the rotor centre, the aperture collects light from a portion of the incident beam centred on the beam’s geometric centre. In the presence of a motion perpendicular to the direction of the incident laser beam (and assuming perfect roundness), the region of the incident beam from which light is collected moves away from the geometric centre and tends towards the portion of the beam that passes through the centre of the rotor. An example of this is shown in exaggerated form in figure 2. If the surface roughness of the rotor is between these optically rough and optically smooth extremes, it is reasonable to expect that the effective centre of the beam will be a function of that roughness and progress from the geometric centre of the beam (optically rough) towards the rotor centre-line (optically smooth) but this may occur in a manner that is not easy to define.

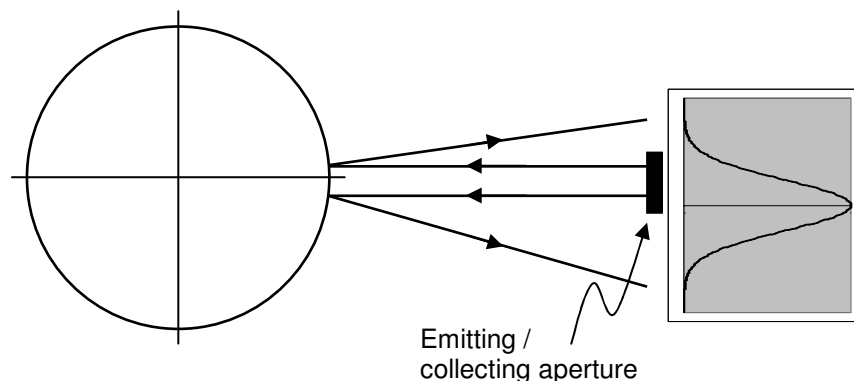


Figure 2: Reflection of light from an optically smooth, round surface

The notion of changes in the effective centre of an incident laser beam as a function of motion perpendicular to the direction of laser beam incidence is not one that has been incorporated in any published mathematical model of velocity sensitivity. This factor raises important questions about the validity of LDV measurements directly from rotating surfaces which must be answered and, while this paper does not yet answer all of the questions in sufficient detail, it does attempt to begin the process.

Experimental Results

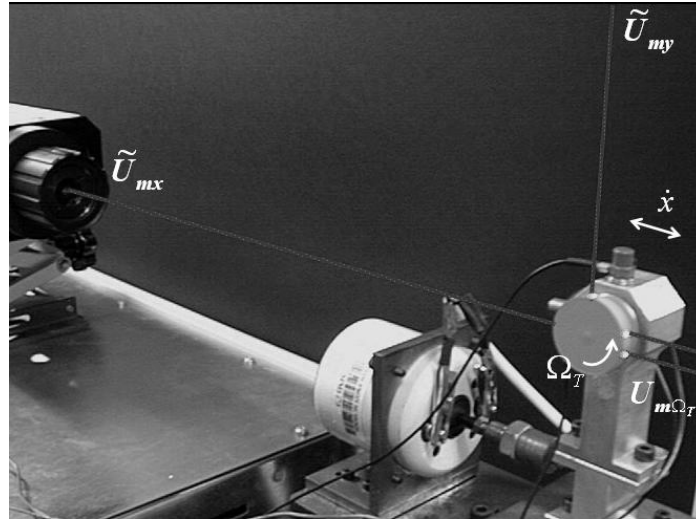


Figure 3: Experimental Rig

A first set of experiments was conducted using the arrangement shown in Figure 3. The small rotor system is mounted on a linear guideway such that, when excited by an electromagnetic shaker, the resulting motion is limited, as far as reasonably practicable, to the x-direction. In these special circumstances, the x-direction Vibrometer should register the correct measurement under both rotating and non-rotating conditions. The genuine y-direction velocity is zero under both conditions but, as described by equation 1(b), cross-sensitivity will give a measurement Ωx from a diffusely scattering surface under the rotating condition.

Three rotors were included in this experiment, each with an axial element several mm wide with an untreated surface between two axial elements coated in retro-reflective tape. The surface roughness (Ra) values were between $1\mu\text{m}$ and $0.15\mu\text{m}$, covering the significant range from slightly above the laser wavelength to well below it. The rotor systems were excited at 30Hz with a rotation speed around 20Hz. These frequencies were chosen deliberately to avoid other contributions, such as speckle noise, to the measured data at the excitation frequency. Five measurements of the 30Hz vibration were taken at each location for each rotor and the ratio of the measured value (y-Vibrometer) to the expected value (from Ωx) was calculated. The mean of the ratios is plotted in figure 4.

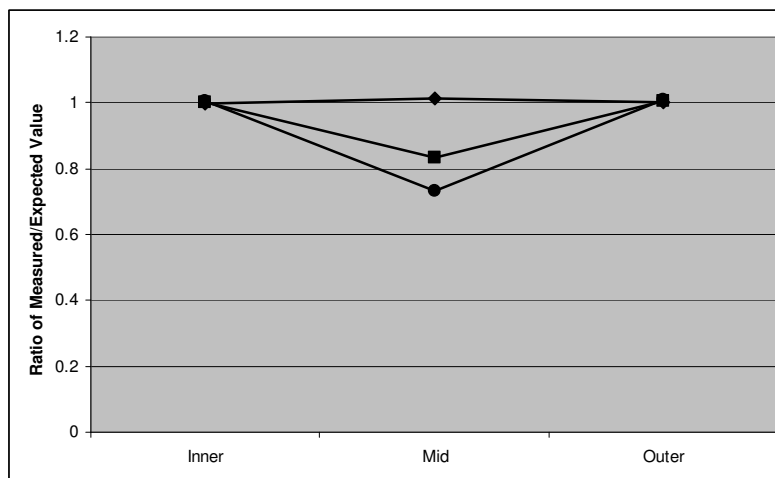


Figure 4:

Experimental data showing the effect of surface roughness on cross-sensitivity in radial vibration measurements using an unfocused beam (Rotor A, $Ra=1\mu\text{m}$ (◆); Rotor B, $Ra=0.85\mu\text{m}$ (■); Rotor C $Ra=0.15\mu\text{m}$ (●))

As expected, the y-direction measurements from the taped axial elements are very close to the expected values. The values from the untreated surface, however, diminish with decreasing surface roughness as a result of the changing effective centre of the incident beam. In the absence of any cross-sensitivity, the value of this ratio would be zero. The cross-sensitivity encountered for the smoother rotors falls part way between zero and Ωx and so it appears that there is some significant ambiguity in measurements from surfaces with roughness in this range.

The data in figure 4 was taken using a Polytec OFV400 Rotational Vibrometer (2 parallel laser beams) with one of the beams capped to configure the instrument for translational vibration measurements. Significantly, the incident beam is unfocussed with this instrument while a measurement with a conventional single beam Vibrometer would have a much smaller focussed beam incident on the target. The corresponding data using the focussed beam is shown in figure 5. Note how no reduction in the measured value is apparent in this data. When defocussing the beam, the measurement on Rotor B ($Ra=0.85\mu m$) was relatively unaffected but the measurement on Rotor C ($Ra=0.15\mu m$) produced lower values, typical of those encountered with the unfocussed beam as shown in figure 4.

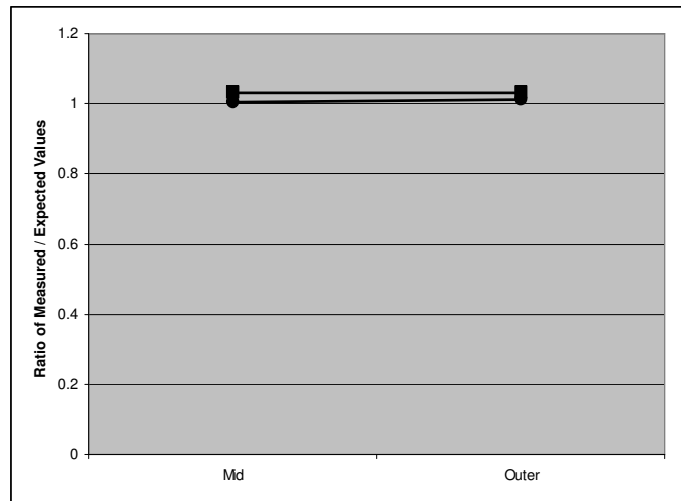


Figure 5: Experimental data showing the effect of surface roughness on cross-sensitivity in radial vibration measurements using a focussed beam (Rotor B $Ra=0.85\mu m$ (■); Rotor C $Ra=0.15\mu m$ (●))

A second point of note in these tests is the increase in levels encountered at the rotational frequency. The rotors in these tests did suffer from a visible runout (estimated to be up to 0.25mm rms) because they were mounted fairly crudely on the motor output shaft. This will have resulted in some measurable unbalance and there will also be speckle noise present at this frequency. Levels at this frequency were 5 to 10 times higher in measurements from all of the untreated surfaces compared to the levels from the measurements from the taped axial elements. This is believed to be the result of the combination of surface finish and (excessive) runout.

In many Laser Vibrometer measurements, it is possible to see the backscattered light on the body of the Vibrometer around the collecting aperture. The backscattered light gives a useful clue to the specular or diffuse nature of the reflection / scattering from the target surface. For diffuse scatter, a speckle pattern will be visible across the collecting aperture. The following set of figures were obtained when using the OFV400 i.e. unfocussed beam.

With retro-reflective tape, the characteristic bright centre of the speckle pattern will sit over the collecting aperture as seen in figure 6. This image was obtained by placing a screen across the front of the Vibrometer body with a central hole over the emitting / collecting aperture, visible as a bright central region in the figure. (The image has also been converted to black and white for ease of viewing). Collection of a speckle pattern with this appearance is consistent with collection of light from all points on the incident beam i.e. the effective centre of the beam is the same as the geometric centre of the beam. When the target rotates the speckle pattern changes rapidly but maintains its position centred on the collecting aperture.

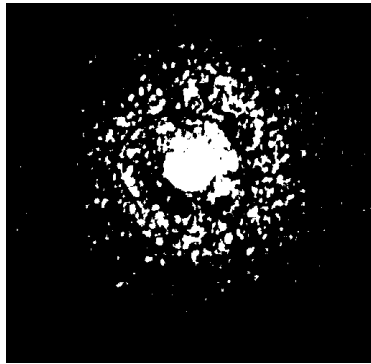


Figure 6: Speckle pattern incident on the Vibrometer collecting aperture due to scattering from retro-reflective tape

The light collected from a more polished surface takes a different appearance and figure 7 shows five images of the light backscattered / reflected from the rotor surface at approximately equally spaced angularly positions during one full rotation of the target rotor. These individual figures differ markedly from the image shown in figure 6 in a number of ways. Firstly there is a significant specular component to the light in the image that does not exist in the preceding figure and the dominance of specular reflection from the target is further emphasised by the off-centre position of the light on the screen relative to the collecting aperture. (The aperture is at the centre of each image and identifiable as a bright circular region as a result of emitted light). The light collected by the aperture originates, at different times in the rotation cycle, from different areas of this specular reflection. This means that the light collected originates from different parts of the incident beam at different times in the rotation cycle i.e. the effective centre of the beam changes during the rotation cycle. The variation in the effective centre of the beam repeats with each cycle and this is believed to make a significant contribution to the raised levels found earlier at the rotational frequency.

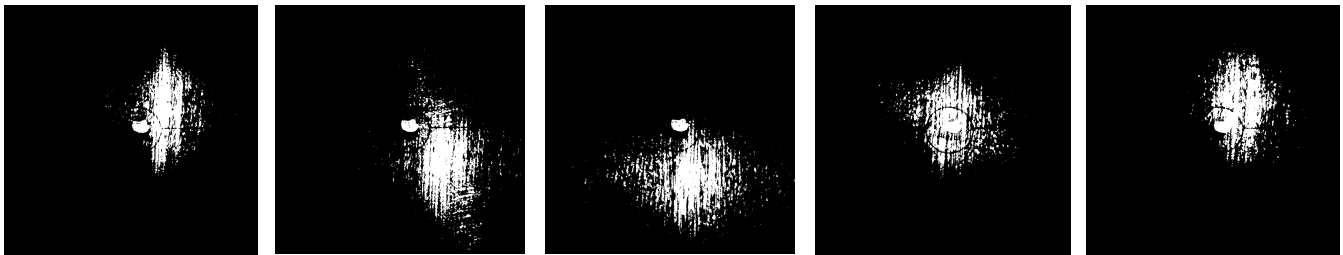


Figure 7: Scattered light incident on the Vibrometer collecting aperture due to scattering from a polished surface ($R_a=0.15\mu\text{m}$)

At some moments in the rotation cycle, the collected intensity is low (second and third images in the sequence) and dependent entirely on a diffuse component to the scattered light that is also identifiable in the images as a 'background' speckle pattern. When this is the case, light is being collected from all points in the incident beam and the effective centre of the beam is now the geometric centre of the beam. Further evidence for this is shown in figure 8 which shows several rotation cycles of the Vibrometer output (no vibration). In each cycle of the obviously periodic signal, there are regions of relatively high frequency noise with zero mean and low rms, typical of speckle noise from a diffusely scattering target, and regions where there are larger amplitude but lower frequency changes in measured velocity, consistent with collection of light from different regions within a dominant specular reflection. For a 0.7mm diameter beam and 20Hz target rotation, pk-pk changes in measured velocity of around 50mm/s would be reasonable.

These figures provide an interesting insight into the detailed behaviour of the light collected in such measurements but it must be remembered that these rotors exhibited larger than normal runout levels. For this reason, a second set of tests was conducted on a rotor with low runout. This rotor was set into oscillation by the attachment of unbalanced masses and vibration was measured at the synchronous frequency. In this set-up, accelerometers were mounted to the bearing housing closest to the laser measurement location but they were sufficiently far from this measurement location that they could only serve as a qualitative check.

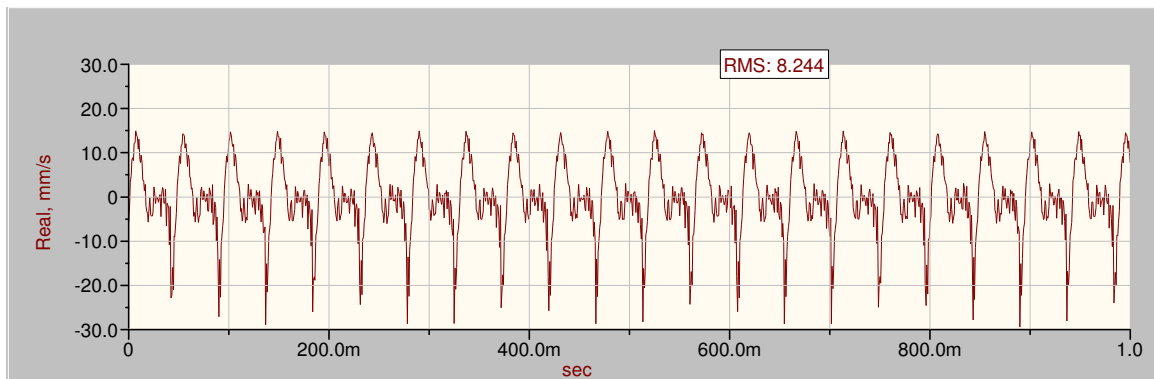


Figure 8: Laser Vibrometer output for radial measurement (no vibration) on rotating polished surface ($R_a=0.15\mu\text{m}$)

The first measurement was conducted on a region of the shaft with R_a of $1.8\mu\text{m}$ using the OFV400 (unfocused beam). Observation of the backscattered light revealed a fairly well-developed speckle pattern but with very rapidly decaying intensity in a direction perpendicular to the rotation axis of the shaft suggesting a significant element of specular reflection from which one might expect a tendency for the effective centre of the incident beam to move towards the shaft centre line and away from the geometric centre of the beam during radial vibration perpendicular to the line of incidence. Comparison of measurements taken from the untreated surface either side of an axial element covered in retro-reflective tape revealed measurements that were considerably higher than the genuine radial velocity in the direction of the incident beam and around 10% down on the measurements taken from the tape. This was consistent with detection of the cross-sensitivity term but with reduced sensitivity to the radial vibration perpendicular to the line of incidence.

This measurement was then repeated after polishing of the shaft which reduced the R_a value to $0.2\mu\text{m}$. In these measurements, the measured velocities from the untreated surface were around 40% down on the measurements taken from the tape, nor did they correspond at all closely to the best available estimate of the genuine radial velocity in the direction of the incident beam. On this occasion the measurement from the untreated surface could not be explained in terms of a reduced sensitivity to the radial vibration perpendicular to the line of incidence. It was, however, the case that there was some evidence of the kind of behaviour depicted in figure 7 in the backscattered light. Such a contribution to the measured velocity at the synchronous frequency could account for this anomaly.

Finally, the measurement on the polished surface was repeated with the focussed laser beam of the OFV300 Laser Vibrometer. Comparison over several measurements from the untreated surface and from a surface covered in retro-reflective tape showed agreement to within 1%, on average, although the standard deviation was greater in the data from the untreated surface. This is consistent with data shown in figure 5 where no reduction in cross-sensitivity was apparent at these roughness values when using a smaller (focussed) beam.

Concluding Comments

The data presented in this paper has suggested that there is some merit in the suggestion that Laser Vibrometer measurements of radial vibration from polished rotor surfaces might not exhibit the cross-sensitivity known to affect such measurements when taken from diffusely scattering surfaces such as those coated with retro-reflective tape. The data presented, however, have only shown reduced cross-sensitivity rather than zero cross-sensitivity. In a recently published paper [13], Tatar *et al* showed that, on a rotor of $R_a=21\text{nm}$, there was convincing evidence that the cross-sensitivity was not present in radial vibration measurements. These data combined with those in this paper suggest a roughness range, from approximately $2\mu\text{m}$ to 20nm , in which a Laser Vibrometer based on a 633nm laser will progress from a situation in which there is measurable and definable cross-sensitivity in radial vibration measurements to one in which there is negligible cross-sensitivity. This range must be defined more precisely in future work but, even when the range is more precisely defined, there may not be any practical way to define the level of cross-sensitivity likely to be encountered within it. Factors such as incident beam diameter and surface roundness also require further investigation because they have been shown in this paper to affect data measurably, adding to the uncertainty associated with measurements taken from rotors with surface roughness in this range.

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