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# Nonlinear energy harvesting from base excitation in automotive applications

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**Abstract:** Energy harvesting is an emerging technological field, aiming, among others, to harvest kinetic energy from mechanical oscillations and converting the same to useful electrical energy. Usually, the reclaimed energy is quite small, acquired by lightweight devices which can be positioned in confined spaces and do not significantly add to the system mass and inertia. This potential use is in line with the vehicular powertrain development principle of high output power-to-light weight ratio as this concept has progressively led to increasing vibratory energy. These devices would potentially supply a few mW of power, which can typically power automotive sensors, potentially reducing the ever-increasing vehicle wiring network. A multitude of such devices positioned strategically can recover some of the vibratory energy of a plethora of vibration phenomena. The paper outlines some of these phenomena and proposes the use of a magnetic translational harvester. A preliminary study based on an experimental set up and a devised non-linear model show good potential across a range of frequencies, typical of engine order vibration at engine idling condition. However, the potential exists for both the optimisation of the demonstrated observer and increased energy recovery for suitable location(s) in the powertrain system.

*Keywords: Energy harvesting, magnetic levitation, non-linear dynamics, powertrain NVH*

## Nomenclature

Symbol	Description
B	coil residual flux density (T)
$d_w$	wire diameter (m)
$Z_a$	excitation amplitude ( $m/s^2$ )
$F_{\text{bottom}}$	bottom-middle magnetic force (N)
$F_{\text{net}}$	net restoring force (N)

$F_{top}$	top-middle magnetic force (N)
$h$	coil height (m)
$I$	current (A)
$k$	linear stiffness coefficient (N/m)
$k_3$	cubic stiffness coefficient (N/m <sup>3</sup> )
$L$	coil inductance (H)
$m$	magnet mass (kg)
$N$	number of turns
$R_c$	coil resistance ( $\Omega$ )
$R_l$	load Resistance ( $\Omega$ )
$2r_1$	coil inner diameter (m)
$2r_2$	coil outer diameter (m)
$V_{oc}$	open circuit voltage (V)
$x$	magnet relative displacement (m)
$x_0$	static magnet displacement (m)
$x_1$	Amplitude of magnet oscillations (m)
$y$	magnet absolute displacement (m)
$z$	housing displacement (m)
$z_1$	bottom coil edge (m)
$z_2$	top coil edge (m)
$\alpha$	electromagnetic coupling correction factor (-)
$\beta$	normalized cubic coefficient (N/kg m <sup>3</sup> )
$\zeta$	damping ratio
$\hat{\Theta}$	electromagnetic coupling (Vs/m)
$\xi$	fill factor
$v$	volume of the magnet (m <sup>3</sup> )
$\phi$	magnet oscillations phase (rad)
$\omega_n$	linear natural frequency (rad/s)
$\Omega$	excitation frequency (rad/s)

## 1-Introduction

Mechanical power transmission often involves excess energy which contributes to energy losses in the forms of heat, friction and vibration. Quite often, there is balance between these sources of energy loss. Therefore, implementing palliative measures to counter one can lead to exacerbation of another. For instance, light-weight philosophy for modern vehicle powertrains to mitigate errant

out-of-balance dynamics can lead to vibration of lightly damped structures such as the elasto-acoustic response of modern hollow driveshaft tubes in driveline systems; a phenomenon which is onomatopoeically referred to as clonk [1,2]. Furthermore, often loss of friction due to contact separation leads to excess vibration, which is a common occurrence in many noise, vibration and harshness (NVH) phenomena, such as in gear whine in vehicular differentials [3-5]. Therefore, when a balance is envisaged between transmission efficiency (reduced parasitic frictional loss) and NVH refinement, a certain degree of excess vibration still exists which is usually dissipated from the system, often in forms which are of concern to the practicing engineers (such as impacts, heat from inherent damping in vibrating components etc.). Depending on the energy input level, a sizeable portion invariably leads to structural elastic deformation of system components, which upon its release further vibration occurs, such as the case of the driveline clonk phenomenon. At least a portion of this energy in the form of kinetics can be harvested in the form of electrical output to power suitably designed operators.

Whilst the transient powertrain phenomena are short-lived and its capture and storage present a significant challenge, those of a fairly repetitive or longer duration (lower frequency content) create the opportunity for Targeted Energy Transfer (TET). There is a plethora of such NVH phenomena in powertrain systems of lightly damped nature, such as inertial dynamics' responses in vehicle shuffle and shunt and clutch judder [6-8] and transmission rattle in various operational modes; low energy idle rattle [9-12], as well as under various driving conditions, such as drive rattle [13, 14].

Energy Harvesting (EH) usually involves techniques, methods and technologies that harness ambient energy and convert it to useful electricity [15, 16]. The state-of-the-art technology relates to small amounts of harvested energy which, nevertheless, are sufficient to power low-demand electronic devices, such as personal gadgets or wireless sensors [17]. This approach can be applied in many industrial applications, including the automotive sector [18-20]. A typical vehicle uses in excess of 70 sensors for its reliable and safe operation and for many driver-assisted functions [21]. The downside of this increasing use of sensor technology is the extensive use of cabling and the associated harnessing. This represents the third heaviest component in a typical modern vehicle, right behind the chassis and engine [22]. Not only the number of sensors is increasing but the in-use time is also increasing. For instance, parking sensors are also used at higher vehicle speeds to warn of blind spot detection and proximity of external hazards, therefore increasing energy

demand. Harvesting the excess vibration energy to power up at least some of these devices would not only alleviate some untoward effects of various aforementioned NVH phenomena, but also reclaim some of the otherwise dissipated energy.

As far as vibration energy in mechanical systems is concerned, an abundance of studies is progressively dedicated to electromagnetic harvesters. The premise is that vibrations of the hosting structures or mechanisms can excite a suspended magnet in the proximity of a coil [23,24]. These oscillations induce voltage to the coil which can potentially be used to power an electrical load (e.g. sensors, actuators). Yet, previous research has shown that the performance of these harvesters depends highly upon proper tuning of the system's natural frequency to the expected vibration frequencies, such that the system would operate in or in the vicinity of resonant conditions. Evidently, mechanical systems experience varying operating conditions, leading a linear harvester to operate away from resonance, thus deteriorating its performance.

A potential solution to this problem involves the introduction of non-linearities in the system, thus broadening the useful range of operation of an oscillator, acting as a harvester. The modes of non-linear systems vary with the input energy as already noted above, particularly for broad band structural NVH phenomena such as those caused by impulsive action (clonk, transmission rattle, etc). This provides the opportunity of a broader range of frequencies for a non-linear oscillator operating at higher response amplitudes. The introduction of non-linearity and its beneficial effects are discussed in [25]. Many authors have explored different ways to implement non-linearities for harvesting vibration energy [26]. A widely accepted concept, involving non-linear restoring force applied to the oscillations of a magnet was initially proposed by Mann and Sims [27]. Essentially, a levitating magnet experiences non-linear oscillations in response to base excitations, due to its interaction with two other magnets, diametrically positioned with respect to its motion. This is the concept also used in this paper.

The Harmonic Balance Method (HBM) is used to approximately describe the response of the base-excited oscillator with a cubic nonlinearity, augmented by numerical integration. It is sought to demonstrate the suitability of this harvesting principle for typical vibrations encountered in automotive applications, and particularly for the case of engine vibrations. An experimental rig is

developed and preliminary results associated with engine idling conditions are presented. These conditions also serve for model validation.

## 2-Analytical Model

The dynamical system considered is shown in Figure 1(a). Two identical magnets are rigidly mounted at the ends of a tube. A third magnet is free to levitate in the axial  $y$ -direction. The polarity of the magnets is so arranged such that their edges restore any axial displacement of the levitating magnet with the net applied force. Since the edges of the magnets are identical, it would be sufficient to only consider the force between the top and the middle magnet and predict the net restoring force.

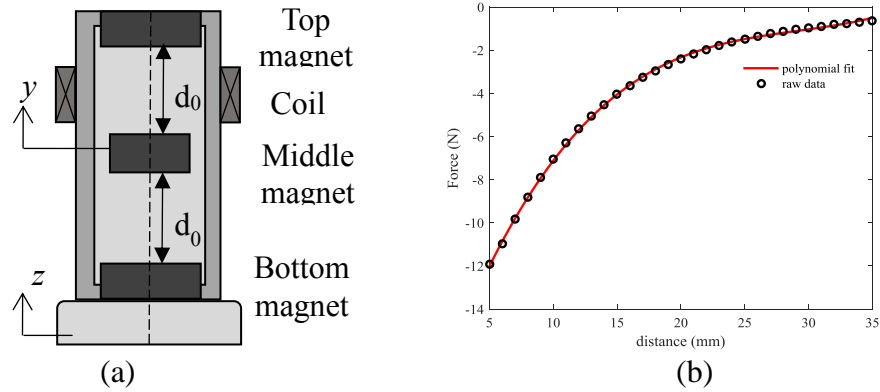


Figure 1: (a) Schematic representation of the proposed energy harvester;  $y$  is the displacement of the magnet and  $z$  of the housing, (b) Experimentally obtained repulsive force between the levitating magnet and an edge magnet (noted by  $\circ$ ), and its 3<sup>rd</sup> order polynomial fit (solid line).

Mann and Sims [27] introduced a procedure for predicting the system stiffness, based on expressing the restoring force between a pair of magnets as a polynomial function (Figure 1(b)). The combined force exerted by the layout of the magnets can be reasonably approximated by the odd elements of a 3<sup>rd</sup> order power series, such that for a stationary housing:

$$F_{net} = F_{bottom} - F_{top} = ky + k_3y^3 \quad (1)$$

In fact, the exact expressions relating the parameters  $k$  and  $k_3$  with the coefficients of the fitting polynomial and the distance  $d_0$  can be readily derived [27]. Considering the relative displacement

of the middle magnet,  $x=y-z$ , and assuming idealised viscous damping and harmonic excitations, the equation of motion can be expressed as:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x + \beta x^3 = -g + Z_a \cos \omega t \quad (2)$$

where  $\zeta$  is the viscous damping ratio,  $\omega_n^2=k/m$  is the linear natural frequency,  $\beta=k_3/m$  is the cubic force coefficient,  $g$  is the gravitational acceleration,  $F$  is the amplitude of the acceleration and  $\omega$  is the excitation frequency. Equation (2) is in the form of a Duffing oscillator. Of particular interest is the analytical treatment of such systems that have been developed [28,29]. The HBM is a well-established technique [28], capable of approximately describing the dynamics of non-linear systems, dominated by a limited number of frequencies. Here, the HBM is used with any derivations shown to be trivial. A harmonic solution of the following form is assumed:

$$x = x_0 + x_1 \cos(\omega t + \phi) \quad (3)$$

where  $x_0$  is the static displacement of the magnet,  $x_1$  the amplitude of the assumed solution and  $\phi$  its phase. Substituting equation (3) into equation (2) and equating similar terms containing  $\sin(\omega t)$ ,  $\cos(\omega t)$  and constant terms from both sides of the equation, one arrives at the following non-linear system for the newly introduced variables as:

$$\begin{aligned} x_0 \left( \beta x_0^2 + \frac{3}{2} \beta x_1^2 + \omega_n^2 \right) &= -g \\ x_1 \omega^2 + x_1 \left( 3\beta x_0^2 + \frac{3}{4} \beta x_1^2 + \omega_n^2 \right) &= Z_a \cos \phi \\ -2\zeta \omega_n \omega x_1 &= Z_a \sin \phi \end{aligned} \quad (4)$$

The equation set (4) can be solved for the variables uniquely defining  $x$ . Of those, the amplitude of the oscillations of the middle levitating magnet is of particular interest for the purpose of this paper. Solving equation (4) is rather useful in the next section for the identification of the mechanical properties of the experimental apparatus.

The induced voltage in the electric circuit (wire-wound coil and resistive load, where the harvested power is consumed) is the result of the varying magnetic flux which crosses each turn of the coil. The electromechanical system is then governed by the following set of equations:

$$\begin{aligned} \ddot{x} + 2\zeta\omega_n\dot{x} - \frac{\widehat{\Theta}}{m}I + \omega_n^2x + \beta x^3 &= -g + Z_a \cos \omega t \\ \dot{I} + \frac{R_c + R_l}{L}I + \frac{\widehat{\Theta}}{L}\dot{x} &= 0 \end{aligned} \quad (5)$$

Careful observation of equation (5) shows that the induced voltage for a given electrical load depends on the levitating magnet's velocity  $\dot{x}$  and the electromechanical coupling coefficient,  $\widehat{\Theta}$ . Quite often the latter parameter is treated as a constant, especially when the magnet is oscillating inside the coil. However, this approximation produces errors when the magnet oscillates at higher strokes. Thus, the following non-linear expression is used here [30]:

$$\widehat{\Theta} = \frac{aNbv\xi}{2A_c} \sum_{i,j=1}^2 (-1)^{i+j} \left[ \ln(r_{ij} + Z_{ij}) - \frac{r_i}{Z_{ij}} \right] \quad (6)$$

with  $A_c = (r_2 - r_1)(z_2 - z_1)$  being the coil cross-sectional area and  $Z_{ij}^2 = r_i^2 + (z_j - x)^2$ . Equation (6) inhibits further analytical development of the system's equations. However, numerical integration would suffice to calculate the induced voltage.

### 3-Experimental Verification

The excitation characteristics (frequency and amplitude) considered here correspond to typical engine vibrations of a 1999 model 4-cylinder 4-stroke 1.6 litre Ford Focus engine under idling condition [31]. The vibration response under these conditions comprises mainly of engine order vibration [32].

#### 3.1-Apparatus description

The experimental set up is shown in Figure 2, including the employed instrumentation. The EH device is rigidly mounted onto a LDS vibration shaker via a large aluminium attachment to prevent any significant interaction between the shaker's magnetic field and the levitating magnet. The shaker provides the input vibrations, simulating those measured at the engine mount of the engine. A laser vibrometer is used to measure the instantaneous velocity of the levitating magnet through an orifice drilled through the top cap of the assembly as shown in the figure. An accelerometer is used to monitor the shaker's input acceleration signal. This is filtered by a 6<sup>th</sup> order high-pass Butterworth filter in order to facilitate numerical integration, which is required to obtain the



magnet's relative velocity. Direct measurement of resistant load voltage is obtained through connecting the same to the data acquisition system.

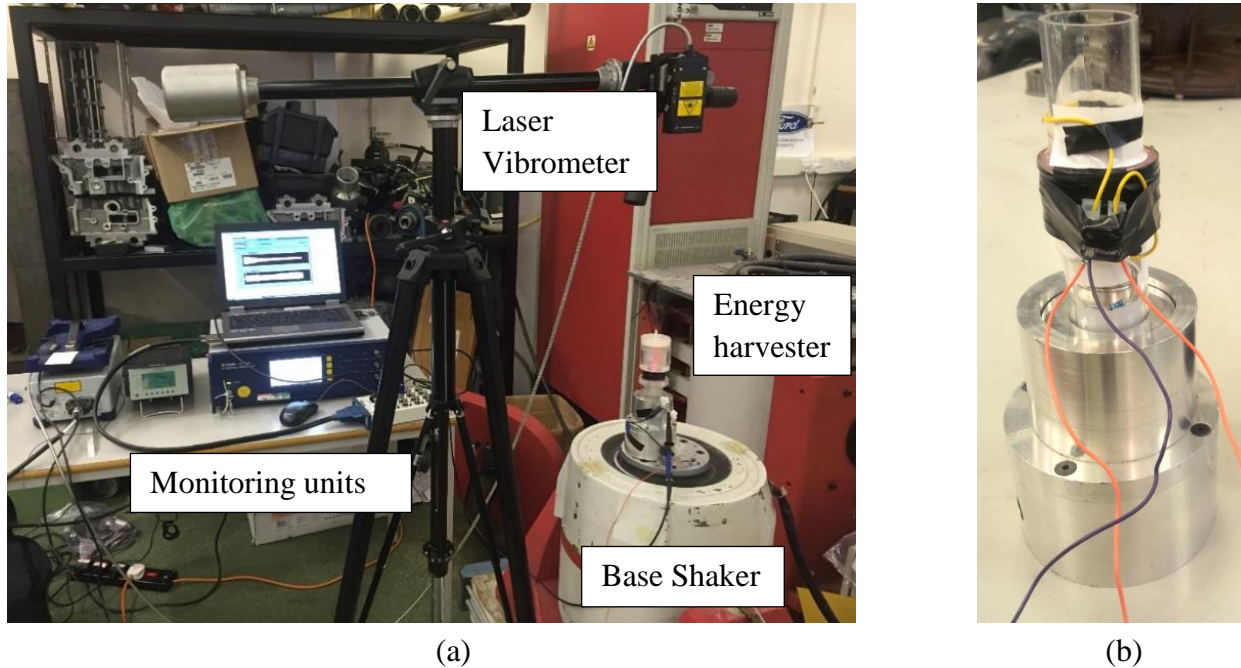


Figure 2: (a) General overview of the experimental set up, (b) close view of the energy harvester

### 3.2-System identification

In order to validate the devised model against experimental results, the physical parameters of the set up need to be quantified. Most of these are readily measurable and are listed in Table 1. The correction factor,  $\alpha$ , is measured using a free fall test of a single magnet.

Table 1: Parameters of the coil and the oscillating magnet

Parameter	Value
$m$	0.0211 Kg
$v$	2670 mm <sup>3</sup>
$h$	0.0155 M
$2r_1$	0.0283 M
$2r_2$	0.0405 M
$\xi$	0.545 -
$d_w$	0.2 Mm
$L$	0.070 H
$R_c$	94 $\Omega$

$R_1$	100 $\Omega$
$B$	1.31 T
$N$	1602 -
$\alpha$	2.4 -

However, the mechanical parameters of the device are not as easy to quantify. Therefore, the electric circuit was decoupled from the mechanical system and the harvester was excited at selected range of frequencies and amplitudes. The recorded response can then be compared with the corresponding numerical solution. An optimization algorithm is used to minimize the root-mean-square error between the experimental and the numerical responses. In this manner, the mechanical parameters are approximated and their values are listed in Table 2.

Table 2: Mechanical parameters of the experimental set up

Parameter	Value
$\zeta$	0.016
$k$	87 N/m
$k_3$	$3 \times 10^5$ N/m <sup>3</sup>

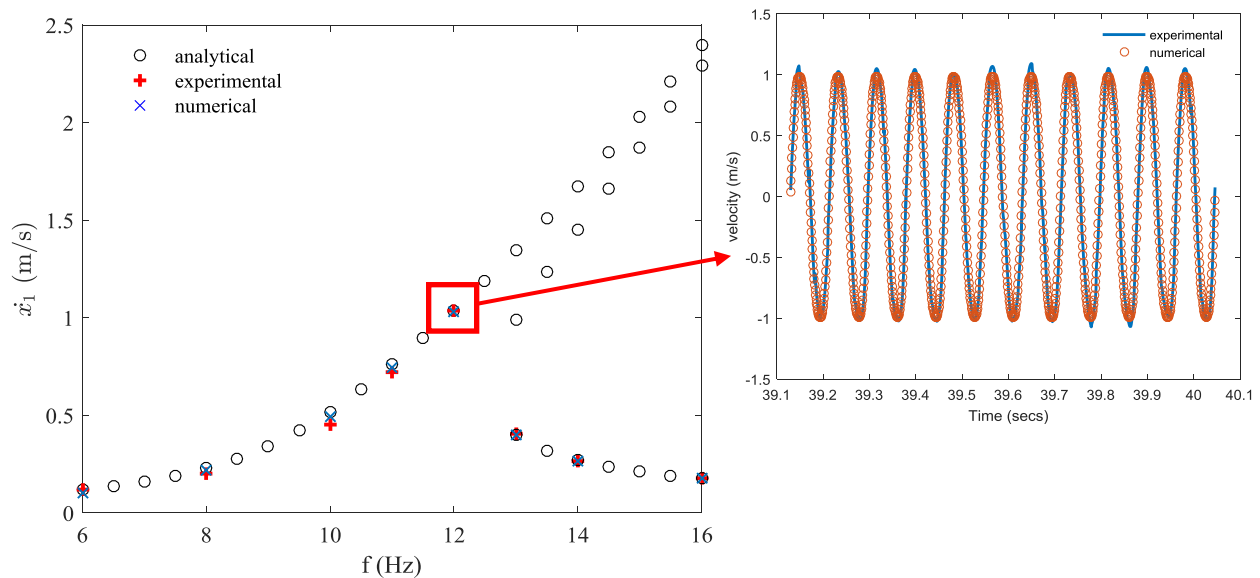


Figure 3: Comparison of the response of the decoupled mechanical system with analytical solution (equation (4)) and a numerical solution for the input amplitude of  $9.8 \text{ m/s}^2$  (free fall)

In total, 8 different points of the Frequency Response Curve (FRC) were used to achieve a good degree of confidence in the identified parameters. The post-processed experimental measurements are used to extract the amplitude of the magnet's relative velocity. The experimental response amplitudes are plotted against an analytical FRC and a corresponding numerical solution in Figure 3. It can be seen that the identified mechanical parameters provide a sufficiently accurate model.

### 3.3-Frequency response curves

Following the identification procedure, the harvesting device is subjected to a frequency sweep. Due to the limitations of the equipment, the input acceleration amplitude increases linearly with frequency sweep. In fact, the start frequency is selected at 8Hz and terminates at 18Hz (merely encompassing the first and second engine order vibration at the equivalent engine idling speed). The input amplitudes corresponding to the start and stop points are  $5\text{m/s}^2$  and  $15\text{m/s}^2$  respectively. Therefore, this input time history was recorded and then used as the input to the model as well. In this manner, the experimental results are directly comparable to the results of numerical integration predictions.

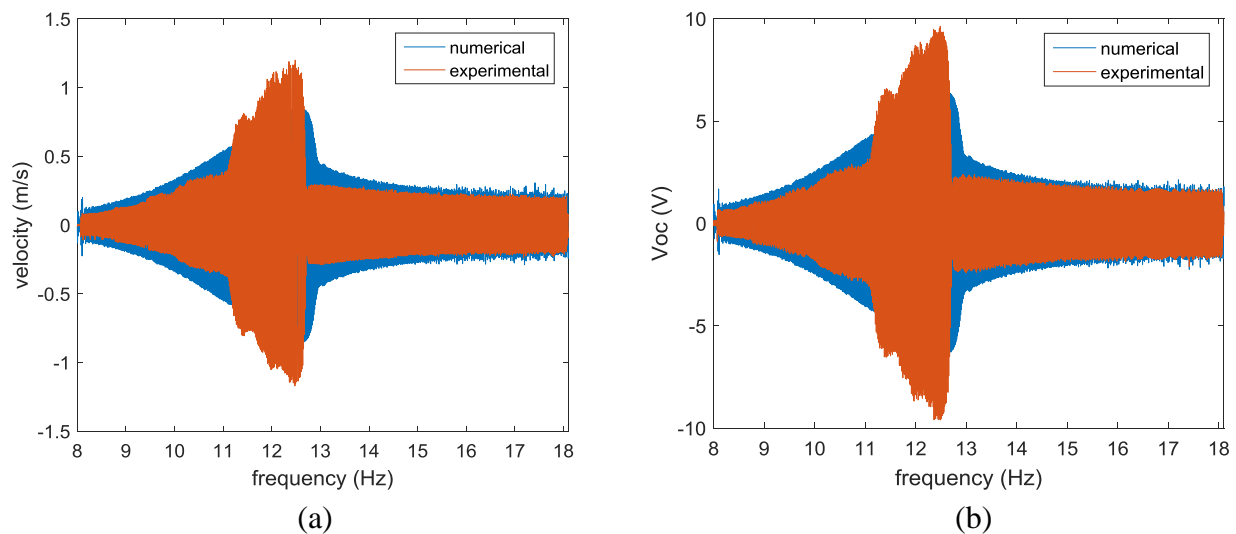


Figure 4: FRCs for numerical predictions for the parameters of Table 1 and 2 (a) middle magnet's relative velocity  $\dot{x}$  (b) open circuit voltage  $V_{oc}$ .

Figure 4 shows the model response to the frequency sweep. In Figure 4(a), the relative velocity is shown, while the open circuit voltage induced at the coil ends is shown in Figure 4(b). It is observed that the predictions of the developed model closely conform to the experimental measurements. Therefore, it is concluded that the adopted approach is appropriate for further investigation, encompassing higher frequencies and transient input.

### **Concluding Remarks**

The paper reflects upon the potential of non-linear energy harvesters in automotive applications. Simulated engine idling conditions are used to verify the mathematical model of magnetic levitating translational harvester. The results demonstrate the utility of such a device for powering remote, low-demand, electronic devices such as wireless sensors. The paper also highlights a plethora of NVH concerns which represent the potential sources for energy harvesting. The potential use of a multitude of energy harvesters has to be balanced against the prevailing light weight powertrain concept. Their utility will also depend on their performance across an increasing range of frequencies at high amplitudes, pointing to operation in the vicinity of resonant conditions. Therefore, non-linear behaviour and low damping would be essential features of such systems. This paper demonstrates a preliminary study, showing the utility of such devices for low to moderate energy recovery and further work would be required to optimise the performance of such devices, in terms of mass to energy recovery ratio.

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