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<http://dx.doi.org/10.1007/s12283-010-0056-x>

PUBLISHER

Springer (© International Sports Engineering Association)

VERSION

AM (Accepted Manuscript)

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A computer simulation model of tennis racket/ball impacts

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Abstract

A forward dynamics computer simulation for replicating tennis racket/ball impacts is described consisting of two rigid segments coupled with two degrees of rotational freedom for the racket frame, nine equally spaced point masses connected by 24 visco-elastic springs for the string-bed and a point mass visco-elastic ball model. The first and second modal responses both in and perpendicular to the racket string-bed plane have been reproduced for two contrasting racket frames, each strung at a high and a low tension. Ball/string-bed normal impact simulations of real impacts at nine locations on each string-bed and six different initial ball velocities resulted in <3% RMS error in rebound velocity (over the 16-27 m/s range observed). The RMS difference between simulated and measured oblique impact rebound angles across nine impact locations was 1°. Thus careful measurement of ball and racket characteristics to configure the model parameters enables researchers to accurately introduce ball impact at different locations and subsequent modal response of the tennis racket to rigid body simulations of tennis strokes without punitive computational cost.

Keywords: ball, impact, model, racket, simulation, tennis.

Introduction

There have been few experiments to determine a racket's influence on injury (Brody, 1997; De Smedt et al., 2007). It has been suggested that off-centre impacts may be a major factor in the development of elbow injuries from tennis strokes (Hennig et al., 1992), string vibration dampers have been shown to be ineffective at reducing shock transmission to the forearm (Li et al., 2004) and discomfort (Stroede et al., 1999) and inappropriate grip size has been shown to be a less significant risk factor than commonly believed (Hatch et al., 2006). Studies have indicated active frame damping technology using piezo ceramic fibres reduces the symptoms of both acute and chronic tennis elbow sufferers (Cotney et al., 2006).

Experimental research in tennis has been hampered by the complexity of strokes and the short contact time of the ball on the string-bed (Schlarb et al., 1998). It is also difficult to isolate the effect of a single parameter perturbation as changing the moment of inertia of the racket, for example, may also change its mass (Mitchell et al., 2000). To overcome some of these problems, computer simulation models have been developed (Riek et al., 1999; Nesbit et al., 2006). An important consideration is whether the model has appropriate complexity so that the conclusions obtained apply to the real world and not just the model (Yeadon & Challis, 1994). Previous simulation models predicting loads on the human in tennis strokes (Nesbit et al., 2006) may be too simplistic as potentially important features such as the modal response of the racket frame have been omitted.

A weakness of the rigid body modelling approach is that the vibrations of the racket frame and their associated energy losses are not accounted for. Goodwill and Haake (2003) showed that for impacts along the longitudinal axis of the racket, close to the geometric string-bed

centre (GSC), a rigid body model agreed well with ball rebound velocity data determined experimentally. However, given a range of impact locations away from a node point near the GSC, a flexible beam model offered a closer match. The major difficulties in creating a more complex flexible beam model are the dynamic interaction with other bodies such as the human and the long computational processing time. This has often resulted in the adoption of unrealistic boundary conditions such as a clamped grip (Brannigan & Adali, 1981). These simulations have typically led to results that do not correspond well to the real stroke situation (Schlarb et al., 1998). Although it is widely accepted that the racket held in the human hand behaves, in response to impact, more nearly as if it were freely suspended than rigidly clamped, both rigidly clamped and freely suspended paradigms are inadequate when determining force transfer from the racket to the human hand during and after the impact event. The former over constrains and so predicts excessive force whereas the latter under constrains and predicts zero force. A better paradigm, sufficient for the intended purpose whilst avoiding punitive computational costs associated with unnecessary detail, is needed.

The aim of this study was to establish a more realistic approach to modelling tennis rackets and their interaction with a ball for use within biomechanical simulations of tennis strokes. The model is to be used to investigate the effects of racket properties and impact conditions on the player without introducing punitive computational costs. Two key weaknesses in previous rigid body biomechanical models have been addressed: the inaccuracy of the string-bed model for off-centre impacts and the lack of accurate and efficient reproduction of the racket frame modal response to impact and its subsequent effect on upper limb loading.

Methods

The AutolevTM 3.4 Professional software package was used to develop the equations of motion for the computer simulation model of tennis racket/ball impacts. To simulate the fundamental modes of vibration in and out of the plane of the non-rigid frame with rigid model segments the racket frame was represented using two rigid bodies connected by a frictionless pin-joint with two degrees of freedom. The joint was at the mean location of the antinodes for the fundamental mode shapes in and out of the plane of the racket [Fig. 1]. Two linear torsional spring-dampers provided a torque T_i and T_o to resist the angular motion θ_i and θ_o between the two bodies in and out of the plane of the racket respectively. The torques were given by the general equation $T = -k_f\theta - c_f\dot{\theta}$ with two sets of stiffness (k_{fi} and k_{fo}) and damping (c_{fi} and c_{fo}) parameters for in and out of plane motion.

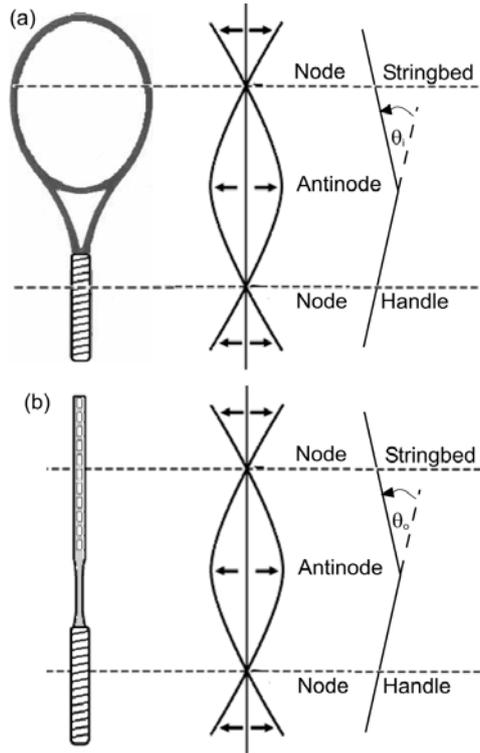


Figure 1. A two-segment rigid body approximation of the fundamental mode shapes of a tennis racket, (a) in the racket plane and (b) out of the racket plane.

The string-bed of the racket was represented by nine point masses connected using 24 linear elastic springs [Fig. 2]. One point mass was at the geometric string-bed centre and the other eight points were positioned around it so that adjoining points were the diameter of an ITF approved tennis ball (68.5 mm) away from each other. Each point mass had a single degree of freedom perpendicular to the plane of the string-bed in its resting position and a mass equal to one-ninth of the total mass of synthetic gut used to string the racket. The initial tension in all string-bed springs T_0 was equal so that the string-bed system was in equilibrium at the start of a simulation. The stiffness ks_i ($i=1-24$) of each spring varied across the string-bed due to differences in string-bed density (caused by the variations in main and cross string spacing) and the shape of the racket frame with the stiffness values of the springs towards the rim of the racket proportional to those of the centre of the racket [Fig. 2]. During a simulation, the force in each spring T_i was defined by the equation $T_i = -ks_i(L_i - L_{0i} + (T_0/ks_i))$, where L_{0i} is the initial length of the spring and L_i is the length of the spring during a simulation.

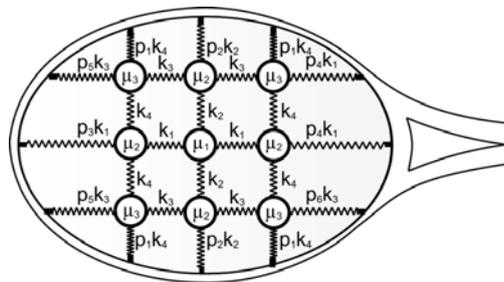


Figure 2. Nine point masses connected by 24 elastic springs to represent the string-bed of the tennis racket. Four stiffness parameters $k_1 - k_4$, six proportional parameters $p_1 - p_6$ and three friction coefficients $\mu_1 - \mu_3$ shown.

The ball was represented by a point mass m_b which could impact with any one of the nine point masses on the string-bed m_s in a simulation. The normal component of the force between the ball and string-bed point masses was modelled as a linear spring-damper ($N = -k_b x - c_b \dot{x}$ where x was the displacement of the ball relative to the point on the string-bed; k_b and c_b were the ball stiffness and damping parameters respectively). Oblique impact conditions were modelled with the addition of a sliding friction force $F = \mu_i N$ with the direction of F perpendicular to N and along a unit vector from m_s to m_b to oppose the motion of the ball. Three coefficient of friction parameters were used to account for both ball felt/string friction and variations in deformed ball/string-bed traction, due to local string spacing and string-bed stiffness, at the different impact locations on the string-bed [Fig. 2].

The simulation model of racket/ball impacts was customised by determining equipment specific parameters to represent two rackets each strung at two different tensions and one type of tennis ball. A 'tour' Head LM Prestige and a 'recreational' Head LM 8 racket frame which had contrasting inertia parameters were used. The tour racket was strung at a 'low' tension of 254 N (57 lbs) and a 'high' tension of 311 N (70 lbs) while the recreational racket was strung at 254 N (57 lbs) and 334 N (75 lbs) of tension. Due to the differences in head size for the two frame designs it was impossible to match resulting string-bed stiffness using the same string tension. It was possible to match the string-bed stiffness of the Prestige strung at low tension and the LM 8 strung at high tension whilst keeping within the manufacturer's safe limits. The alternate tensions for each frame design employed allowed assessment of the model's ability to accurately simulate realistic configuration change. Each racket was strung with Prince 15L synthetic gut (mass = 0.0149 kg), kept in a stable laboratory environment 24 hours prior to and throughout testing.

New Pro Penn Titanium tennis balls (mass = 0.059 kg) were used for all tests. The balls were allowed to acclimatise in a stable laboratory environment for 24 hours before preconditioning and selection based on adherence to the ITF Type 2 ball specification for size, mass, deformation and rebound and the relevant stipulated tests (International Tennis Federation 2004). Balls were discarded after 30 impacts to avoid performance variation due to wear effects. The stiffness k_b and damping c_b parameters for the linear spring damper representing the visco-elastic properties of the tennis ball (normal component), were determined by matching the force-time histories obtained from a range of perpendicular tennis ball impacts on a piezoelectric Kistler Type 9067 force plate sampling at 65 kHz. The balls were fired from a pneumatic air cannon at inbound ball velocities of 18 m/s to 33 m/s in increments of approximately 3 m/s (the range of relative velocities between the ball and the racket was obtained from digitising video data of one-handed backhand groundstrokes by an elite tennis player under simulated play conditions captured using twin genlocked Vision Research Inc. Phantom v4.1 high-speed digital cameras operating at 2500 Hz; Glynn et al., 2007). For each inbound ball velocity the stiffness and damping parameters of the tennis ball were determined by manually varying the two parameters in order to minimise the difference between simulation and measured force-time history for the duration of the ball contact phase. The resulting six stiffness and six damping parameters were then regressed against inbound ball velocity to obtain a linear relationship between each parameter and inbound ball velocity.

The location of the joint between the two segments of each racket was determined from a modal analysis of each racket (Hocknell et al., 1998) and defined as the mean location of the antinodes for the fundamental mode shapes. The racket frames were stimulated by an electromagnetic exciter attached to the frame via a stiff wire and force transducer [Fig. 3(a)]. The Doppler shift between the frequency of the transmitted and reflected light from reflective tape was measured using a Polytec laser vibrometer at arbitrary locations covering the surface of the racket frame and string-bed in and out of the plane of the racket [Fig. 3(b) and (c)].

The output from the force transducer and the laser vibrometer were input to a spectrum analyzer for analysis. The extreme linear displacements of the points on the racket frame from their resting positions were plotted against the distance from the butt end of the racket along its longitudinal axis and quadratic regression equations were fitted to the displacement data. The node and antinode positions were estimated for the fundamental mode shapes of each racket by finding the average points where minimum and maximum displacements occurred respectively during a complete oscillation.

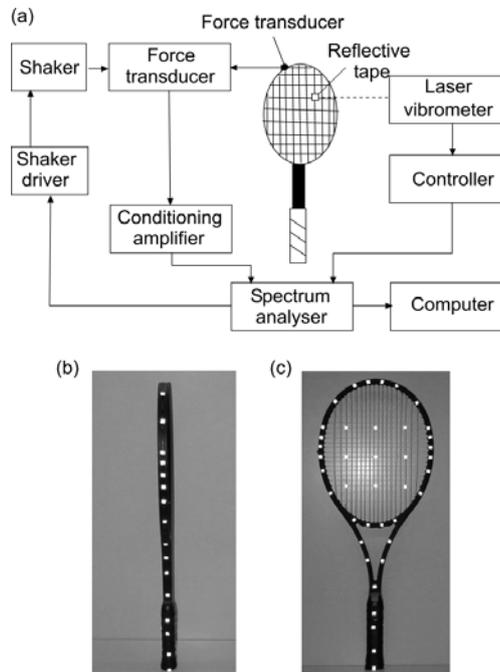


Figure 3. The experimental arrangement for the modal analysis of a tennis racket (a) and the location of reflective markers on the tennis rackets in (b) and out of the racket plane (c).

The principle moments of inertia (MOI) of the racket parts containing the handle and string-bed, about the frontal and transverse axes through the centres of mass, were determined from results of pendulum oscillation tests (Brody, 1985). For the MOI of the racket frame about its longitudinal axis, an Inertia Dynamics 005-104 industrial measurement device with a custom-built attachment was used to give a direct reading. A fixed knife-edge and an additional knife-edge on a set of electronic scales were used to balance each section of the racket and calculate its centre of mass along the longitudinal axis by taking moments about a fixed point.

The stiffness (k_{fi} and k_{fo}) and damping (c_{fi} and c_{fo}) parameters for the two torsional springs at the joint between the racket head and handle in the simulation model of each racket frame were determined by matching experimental acceleration time histories collected from impact hammer tests in and out of the plane of both rackets. To measure the acceleration decay trace of a racket due to an impact a uni-axial accelerometer was mounted half way up the racket handle on the freely suspended unstrung racket frame (a small cross-section of the polyurethane foam handle was cut away to enable the racket to be struck by the impact hammer and the accelerometer to be mounted directly opposite using beeswax). The impact hammer was connected to a charge amplifier to measure the impact force time history. The experimental force time history from each test was input to the two-segment racket frame model and the stiffness and damping parameters for each racket frame were determined by varying the parameters in order to minimize the root mean square (RMS) difference between

the simulated and measured linear acceleration time histories over a 100 ms period starting at the time of impact. The same procedure was applied for both rackets in and out of the racket plane with the location of the accelerometer and impact location adjusted accordingly.

To determine the eleven spring bed parameters for each racket/string-bed combination (T_0 , p_{1-6} , k_{1-4} , Fig. 2) optimisations were run to match ball string-bed simulations to experimental data. In an experiment, tennis balls were projected to impact perpendicular to each racket string-bed combination at the nine points of contact used in the model, with the heads rigidly clamped, at inbound ball velocities that ranged from 18 m/s to 33 m/s in increments of approximately 3 m/s (monitored to within ± 0.5 m/s using ballistic lightgates). Data from 5 impacts at each location were averaged to compensate for minor experimental fluctuations. The string bed patterns were monitored and maintained for each impact. Correct racket positioning was achieved using a custom alignment fixture initially mounted in the firing barrel during setup employing a laser beam to indicate ball trajectory/impact location. Outbound ball velocities for each trial were obtained by digitizing images from a Sensicam 2000 multiple exposure camera and flash unit operating at 250 Hz with 500 ns exposures yielding results to within ± 0.1 m/s. Actual ball impact location was monitored using the same data to ensure correct alignment to within ± 1 mm. The coefficients of restitution for each impact location on each string-bed were then calculated as the magnitude of the outbound ball velocity divided by the inbound ball velocity. The initial tension in the string-bed and the stiffness parameters (eleven parameters per racket/string-bed combination) were optimised using the Simulated Annealing Algorithm (Corana et al., 1987) by minimizing the RMS differences between the measured coefficients of restitution and simulation values (54 CORs per racket/string-bed combination due to 6 inbound velocities and 9 impact locations). Initially all eleven string-bed parameters were optimised for each racket at both the high and low initial string tensions. Subsequently, only the initial string tension parameter T_0 was optimised for the case of the two higher tension rackets, with the other ten parameters equal to those for the same racket frame strung at the lower tension.

To determine the three coefficients of friction required for each racket/string-bed/ball combination, optimisations were run to match computer simulations to experimental data. Experimentally, tennis balls were projected at two different oblique angle/ball velocity combinations onto the nine locations on each string-bed. The ball velocities and oblique angles used were chosen to be equivalent to the relative velocities and angle between the racket and ball measured from high-speed video data of typical topspin drive (relative velocity: 31 m/s; oblique angle of the racket 17° in front of the vertical when viewed from side-on at ball impact), and sliced one-handed backhand groundstrokes (relative velocity: 25 m/s; oblique angle of the racket 16° back from the vertical when viewed from side-on at ball impact) by an elite tennis player (Glynn et al., 2007). Rebound angle data for both impacts and each location on the string-bed were determined by digitizing images obtained from each trial using a Sensicam 2000 flash and camera unit operating at 250 Hz. The coefficients of friction for the specified points of impact were varied using Simulated Annealing until the RMS differences between simulated rebound angles and those determined experimentally, were minimised.

Results

A typical ball/force plate impact force time history and the corresponding model approximation are shown in Figure 4. The tennis ball spring stiffness and damping parameters (normal component) were found to be linearly related to inbound ball velocity [Fig. 5] and consistent with the findings of other studies (Dignall et al., 2000; Davies, 2005). Over a 15

m/s range in impact velocity the ball stiffness component increased by 52% while the damping parameter increased by 97%.

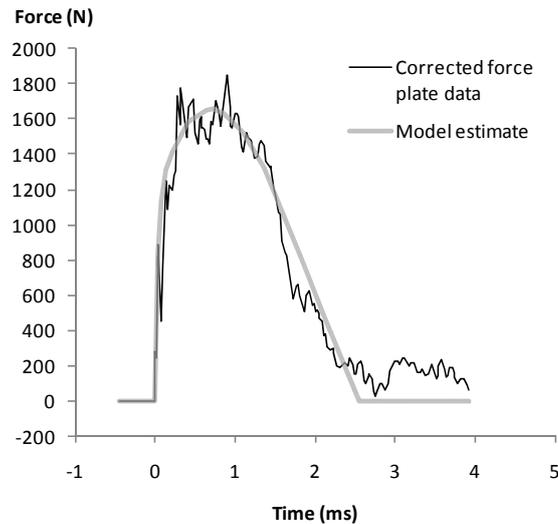


Figure 4. Typical ball/force plate impact force time history

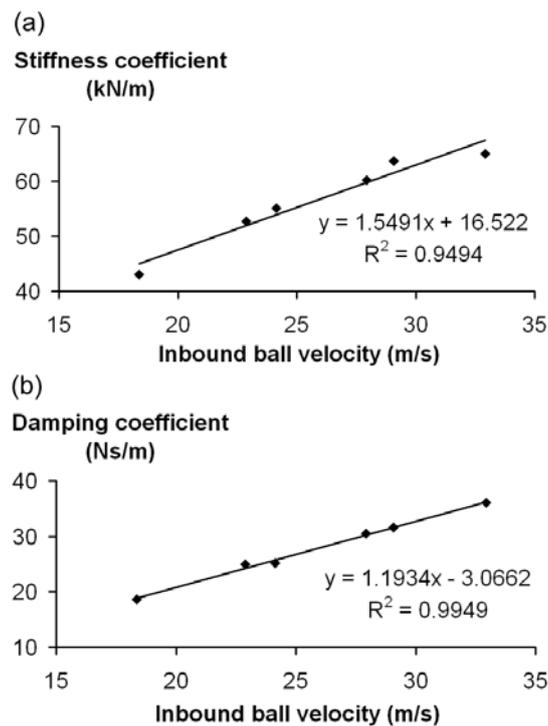


Figure 5. The relationship between spring stiffness (a) and spring damping (b) with inbound ball velocity for normal ball impacts onto a force plate.

The location of the antinodes for the fundamental modes of vibration in and out of the plane (Fig. 6) of the two rackets were found to be 0.366 m and 0.349 m for the ‘tour’ racket and 0.375 m and 0.350 m for the ‘recreational’ racket (measured from the butt end of the racket). Dividing each racket into two sections with the joint at the average location of the antinodes for that racket allowed the inertia parameters for each section to be calculated (Table 1).

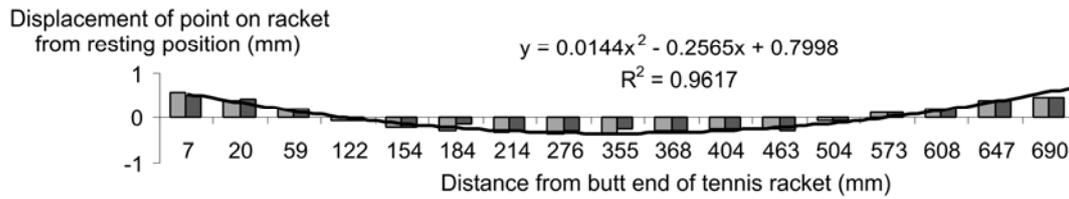


Figure 6. Example location of nodes and antinodes determination for the fundamental mode shapes of a tennis racket from quadratic regression equations fitted to experimental data.

Table 1 Inertia parameters of racket frame parts

racket frame	racket part	mass (kg)	length (m)	COM (m)	MOI about longitudinal axis (kgm ²)	MOI about frontal axis (kgm ²)	MOI about transverse axis (kgm ²)
tour	handle	0.13350	0.35600	0.11365	0.00004	0.00173	0.00172
	string-bed	0.16050	0.33000	0.15032	0.00137	0.00465	0.00331
recreational	handle	0.10690	0.35800	0.19543	0.00010	0.00203	0.00200
	string-bed	0.13360	0.33700	0.18702	0.00151	0.00392	0.00311

Nomenclature: COM = centre of mass location from end closest to the butt of the racket; MOI = moment of inertia about an axis through the mass centre of the racket section.

The fundamental modal frequencies of the recreational tennis racket frame, calculated by modal analysis, were 178 Hz and 150 Hz respectively for the in and out of plane mode shapes. While the corresponding values for the tour racket were 158 Hz and 132 Hz. Optimising the torsional stiffness and damping parameters for each racket both in and out of the plane resulted in good agreement between simulation and experimental acceleration time histories in terms of both frequency and amplitude (Table 2, Fig. 7). Both the tour and recreational racket were approximately 70% stiffer out of plane compared to in plane (due to the structure of a tennis racket frame) and the recreational racket was approximately 15% stiffer than the tour racket. Both rackets had comparable levels of damping with damping out of the plane 2 – 3 times higher than the in plane damping.

Table 2 Torsional stiffness and damping parameters in and out of the plane of each racket

racket frame	k_f (Nm/rad)	c_f (Nms/rad)
tour (in)	910	0.03
tour (out)	1650	0.09
recreational (in)	1075	0.04
recreational (out)	1800	0.10

Nomenclature: k_f = linear torsional stiffness coefficient; c_f = linear torsional damping coefficient.

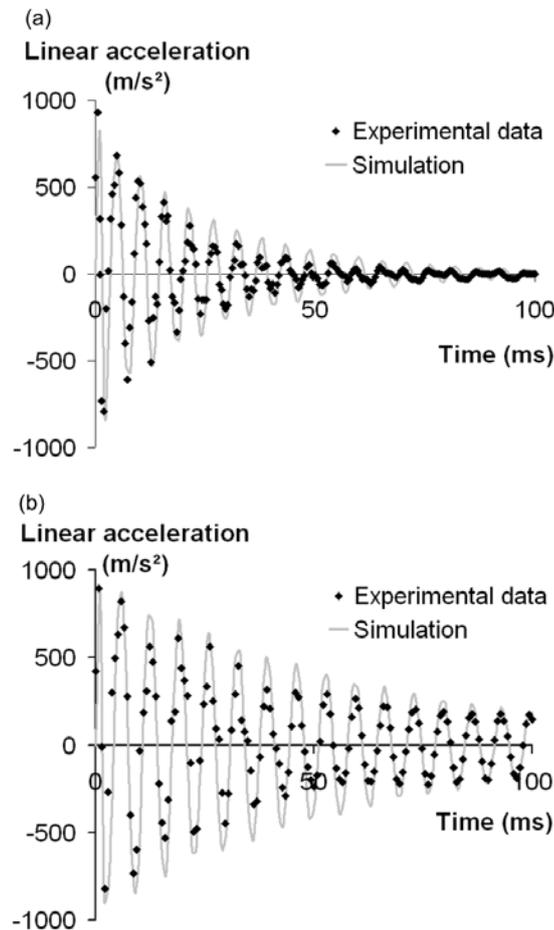


Figure 7. comparison of simulated and measured linear acceleration of a point in the middle of the handle in (a) and out of (b) the plane of the Head LM 8 racket.

Optimising the eleven string-bed parameters for the two low tension racket/string-bed combinations resulted in percentage RMS differences (average across all impact locations and velocities) between experimental COR values and simulations of 0.8% and 2.1% for the tour

and recreational rackets respectively (Table 3). For the higher tension string-beds where only T_0 was varied and the other ten parameters remained fixed at the values determined for the low tension string-beds, similar percentage RMS differences of 1.5% and 1.4% were found for the tour and recreational rackets respectively. These RMS percentage differences for the higher tension string-beds were approximately 0.4% higher than for optimisations where all 11 parameters were varied for the higher tension string-beds. The string-bed simulation model behaved well across all nine locations and all four string-beds with a maximum difference of 3% in COR which occurred for the LM 8 racket strung at 57 lbs tension.

Table 3 Optimised string-bed parameters for the two rackets and two string-beds

parameter	tour		recreational	
	low tension	high tension	low tension	high tension
k_1 (N/m x 10^5)	1.77	1.77	1.02	1.02
k_2 (N/m x 10^5)	2.00	2.00	1.03	1.03
k_3 (N/m x 10^5)	5.97	5.97	1.20	1.20
k_4 (N/m x 10^5)	7.91	7.91	1.13	1.13
p_1	0.53	0.53	0.44	0.44
p_2	0.77	0.77	0.38	0.38
p_3	1.00	1.00	0.95	0.95
p_4	0.83	0.83	0.32	0.32
p_5	0.74	0.74	0.40	0.40
p_6	1.40	1.40	1.58	1.58
T_0 (N)	648.2	772.9	735.8	967.7

Nomenclature: k (1-4)= linear spring stiffness coefficient; p (1-6) = proportional parameters, T_0 = initial string-bed tension.

Optimised values for the coefficient of friction for the four string-beds ranged from 0.26 to 0.38 (Table 4) with corresponding RMS differences between experimental and simulation rebound angles for the 18 trials conducted (9 ball impact locations and 2 inbound velocities) per racket string-bed) ranging from $0.7^\circ - 1.2^\circ$.

Table 4 Coefficients of friction for each racket string-bed

racket string-bed	coefficients of friction		
	μ_1	μ_2	μ_3
tour (low tension)	0.35	0.36	0.37
tour (high tension)	0.35	0.36	0.38
recreational (low tension)	0.26	0.29	0.32
recreational (high tension)	0.26	0.30	0.35

Discussion

A rigid body tennis racket/ball computer simulation model has been developed which has advantages over previous rigid racket/ball simulation models. The model accurately reproduces variation in off-centre string-bed characteristics important to improve confidence in eccentric impact simulation. It also accurately reproduces the first two modal responses of the racket frame, both in and perpendicular to the string-bed plane, important to simulate the contribution of racket frame vibrations to player limb loading when used in combination with an appropriate upper limb model and gripping condition simulation. Furthermore the model was developed using the Autolev software package which generates ready to compile source code. As a result, there are few restrictions on how the model can be used and the code can be interfaced with other programs (e.g. an optimisation routine) thereby reducing the time for a simulation.

The complexity of the simulation model was decided upon so that the model was sufficiently detailed to accurately represent the low order modal responses to normal and oblique impacts at a range of impact locations on the string-bed while being sufficiently simple to minimise computational overhead. A two-segment, two degree of freedom racket frame model with the location of the joint between the two racket parts at the mean location of the antinodes for the in and out of plane cases was found to be sufficiently complex to exhibit the racket response to impacts. This simplification is acceptable as simulations are in reasonably close agreement to experimental impact hammer data collected from both rackets even though the antinodes for the fundamental modes of vibration in and out of the plane of the racket were up to 25 mm apart. In the future, it may be desirable to increase the complexity of the racket model to incorporate higher order mode shapes since tennis ball impacts excite vibration modes across the full frequency spectrum. It is not inconceivable that higher order racket frame vibrations affect the forces experienced in the player's hand. However, it would appear that the higher order mode shapes are of appreciably smaller magnitude and therefore not clearly visible on Fig. 7(a) or Fig. 7(b). Higher order modal behaviour could be modelled by increasing the number of segments in the model. The addition of this level of complexity would add to the computational burden and was considered undesirable until the need for this refinement was proven. It is anticipated that either further experimental evidence or results from computer simulations incorporating appropriate ball, racket, grip and upper limb model elements will confirm the potential need,

if one exists, for additional complexity by first demonstrating that lower order vibration responses have a substantial effect on upper limb loading.

Nine points of contact were used so that impacts could occur at the centre of the string-bed and a variety of off centre impact locations. Reducing the number of contact points to less than nine would have reduced the usefulness of the model while more than nine would have been unnecessary and increased the time for simulations. Optimising only the string tension parameter (T_0) for each higher tension racket resulted in string-bed model parameters that can be used with confidence for a range of string tensions so that the effect of perturbing the initial string tension of a racket can be looked at in isolation. This is far more efficient than determining 11 string stiffness parameters for each string-bed tension.

A point mass representation for the ball, with a spring damper for the normal component of impacts and a frictional force for the oblique component, was sufficient to give good agreement with experimental data across a range of ball cannon tests for both normal and oblique impacts. Expressing the normal component spring parameters as a function of relative velocity at impact allows the model to be used for a range of impact velocities while using three coefficients of friction across the nine points of contact resulted in realistic rebound angles for both slice and topspin impact conditions.

Conclusions

A computer simulation model of tennis racket/ball impacts has been customised to two contrasting tennis racket frames, two string-beds for each and a tennis ball. This was achieved by determining equipment-specific parameters independently, by direct measurement where possible. Although the simulation model is a simplified representation of real tennis-racket ball impacts, the model shows good agreement between simulations and experimental results for each element of the model in isolation. Overall a simplified racket/string-bed/ball model agrees well with experimental data over a range of tests and so the assumptions made are reasonable and the model has been shown to be 'fit for purpose'.

The racket/ball impact model is of a sufficient level of complexity to accurately represent the responses of the ball, string-bed and racket frame in terms of outbound ball velocity and racket frame linear acceleration. As the racket/ball system is fully parameterised and the model code is compact and efficient, the predicted relative effects of features such as the stiffness of the racket frame and ball impact location on the loading on the human can be investigated. As part of a more complex, parameterised racket/player system, the model presented can be used to predict a racket's influence on the potential for injury to the performer during tennis strokes. It may also shed light on the possible mechanisms for injury to the tennis player and the relative merits of mass distribution and vibration damping.

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