Parameter estimation and equalization techniques for communication channels with multipath and multiple frequency offsets

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Parameter Estimation and Equalization Techniques for Communication Channels With Multipath and Multiple Frequency Offsets

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Abstract—We consider estimation of frequency offset (FO) and equalization of a wireless communication channel, within a general framework which allows for different frequency offsets for various multipaths. Such a scenario may arise due to different Doppler shifts associated with various multipaths, or in situations where multiple basestations are used to transmit identical information. For this general framework, we propose an approximative maximum-likelihood estimator exploiting the correlation property of the transmitted pilot signal. We further show that the conventional minimum mean-square error equalizer is computationally cumbersome, as the effective channel-convolution matrix changes deterministically between symbols, due to the multiple FOs. Exploiting the structural property of these variations, we propose a computationally efficient recursive algorithm for the equalizer design. Simulation results show that the proposed estimator is statistically efficient, as the mean-square estimation error attains the Cramér–Rao lower bound. Further, we show via extensive simulations that our proposed scheme significantly outperforms equalizers not employing FO estimation.

Index Terms—Frequency offset (FO) estimation, minimum mean-square error (MMSE) equalizer and complexity, wireless multipath channels.

I. INTRODUCTION

WIRELESS transmission is impaired by signal fading, interference, and additive measurement noises. If present, frequency offsets (FOs) can further seriously degrade the performance of a communication system. There are two main causes for FOs; poor synchronization between the transmitter and receiver carrier frequencies, or motion-induced Doppler shift, due to the relative motion between the mobile station and the local scatterer. Frequency estimation and tracking is crucial to enable accurate decoding of the transmitted information, and the FO estimation techniques for communication channels have been widely studied (see, e.g., [1] and [2]). However, the FOs have been assumed to be common for all multipaths. The FO and channel-estimation problem has also been addressed for a multiple-input and multiple-output (MIMO) system with distinct FOs for each transmit and receive antenna, but only for the case of a flat-fading channel [3]. In contrast to previous work, we address the FO problem for a single transmit and receive antenna,

under a general framework allowing different FOs for each multipath. Such a scenario may arise when either the receiver or the transmitter moves at high speed, thereby resulting in distinct Doppler shifts for paths with different angles of arrival (AOAs). This effect can degrade the performance of the receiver significantly if not accounted for in the equalizer design. The organization of the paper is as follows. In the following section, the signal model is presented. Then, in Section III, we develop an approximate maximum-likelihood (AML) estimator for FOs and channel parameters, and compare its performance with the corresponding Cramér–Rao lower bound (CRLB). In Section IV, we study the minimum mean-square error (MMSE)-based equalizer and provide a computationally efficient algorithm for the equalizer design. Simulation results are given in Section V, followed by our conclusions in Section VI.

II. PROBLEM STATEMENT

Consider a communication system employing a single transmit and receive antenna, and assume that the signal has propagated through L different paths, each having different FO and complex channel gain (CG). The received baseband signal can be written as

$$r(n) = \bar{r}(n) + \eta(n) = \sum_{l=0}^{L-1} h_l x(n-l) e^{j2\pi f_l n} + \eta(n) \quad (1)$$

where h_l and f_l denote, respectively, the unknown complex CG and the FO for the lth channel tap. Herein, we assume that h_l and f_l are quasi-stationary, not changing significantly over the observed data frame, and only vary between data frames. Further, $x(n)$ is the transmitted signal with variance σ_x^2, and $\eta(n)$ is an additive circularly Gaussian distributed zero-mean white (complex) noise with variance σ_η^2. Let $\mathbf{X}_{l,n}$ denote the $N \times N$ diagonal matrix formed from the vector $\mathbf{x}_{l,n}$ along the diagonal, with

$$\mathbf{x}_{l,n} = [x(n-l) \quad \cdots \quad x(n-l-N+1)]^T \quad (2)$$

where N is the frame length, and $(\cdot)^T$ denotes the transpose. Further, let $\mathbf{e}_{l,n}$ model the FO for the lth path

$$\mathbf{e}_{l,n} = \left[e^{j2\pi f_l n} \quad \cdots \quad e^{j2\pi f_l(n-N+1)} \right]^T \quad (3)$$

and form the $N \times L$ matrix $\mathbf{V}_n$ as

$$\mathbf{V}_n \triangleq [\mathbf{v}_{0,n} \quad \cdots \quad \mathbf{v}_{L-1,n}] \quad (4)$$

where $\mathbf{v}_{k,n} = \mathbf{x}_{k,n} \odot \mathbf{e}_{k,n}$, with $\odot$ denoting the Schur–Hadamard (elementwise) product. Hence, the frame constructed from N consecutive received samples can be expressed as

$$\mathbf{r}_n \triangleq [r(n) \quad \cdots \quad r(n-N+1)]^T$$
$$= \sum_{l=0}^{L-1} h_l \mathbf{X}_{l,n} \mathbf{e}_{l,n} + \boldsymbol{\eta}_n$$
$$= \mathbf{V}_n \mathbf{h}_L + \boldsymbol{\eta}_n \quad (5)$$

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where $\mathbf{h}_L = [h_0 \ \cdots \ h_{L-1}]^T$ and $\boldsymbol{\eta}_n = [\eta(n) \ \cdots \ \eta(n - N + 1)]^T$. Here, the problem of interest is given $\mathbf{r}_n$ and $x(n)$, estimate the unknown parameter vector $\boldsymbol{\theta} = [h_0 \ \cdots \ h_{L-1} \ f_0 \ \cdots \ f_{L-1}]^T \triangleq [\mathbf{h}_L^T \ \mathbf{f}_L^T]^T$. We do not explicitly require the knowledge of the channel length, but knowledge on its upper bound is required. For an over-modeled system, the estimates of the additional channel coefficients would be close to zero, and in this way, the channel length can be determined. In the next section, we present a computationally efficient approach to estimate the unknown CGs and FOs.

III. CG AND FO ESTIMATION

In this section, we outline an AML estimator of the complex CGs and the FOs. Consider that the received signal, as expressed in (5), is only a function of the complex CGs and FOs. Thus, the log-likelihood function of the received signal frame can be expressed as (ignoring the constant terms)

$$\ln p(\mathbf{r}_n|\boldsymbol{\theta}_l) = -\frac{1}{\sigma_\eta^2}(\mathbf{r}_n - \mathbf{V}_n\mathbf{h}_L)^H(\mathbf{r}_n - \mathbf{V}_n\mathbf{h}_L) \quad (6)$$

where $(\cdot)^H$ denotes the conjugate transpose. Minimizing (6) with respect to $\mathbf{h}_L$ yields

$$\hat{\mathbf{h}}_L = (\mathbf{V}_n^H\mathbf{V}_n)^{-1}\mathbf{V}_n^H\mathbf{r}_n \triangleq \mathbf{V}_n^\dagger\mathbf{r}_n \quad (7)$$

where $\mathbf{V}_n^\dagger$ denotes the Moore–Penrose pseudoinverse. The FOs are estimated by minimizing the cost function, $J(\mathbf{f}_L)$, obtained by substituting (7) into (6)

$$J(\mathbf{f}_L) = \mathbf{r}_n^H\mathbf{r}_n - \mathbf{r}_n^H\boldsymbol{\Pi}_{\mathbf{V}_n}\mathbf{r}_n \quad (8)$$

where $\boldsymbol{\Pi}_{\mathbf{V}_n} = \mathbf{V}_n\mathbf{V}_n^\dagger$ is the projection onto the range space of $\mathbf{V}_n$. By choosing the training sequence, $x(n)$, such that $E\{x^*(n - l)x(n - p)\} = \delta_{p-l}$, where δ_k denotes the Kronecker delta and $(\cdot)^*$ the conjugate, the n-dimensional minimization problem in (8) may be decoupled into n one-dimensional problems, significantly reducing the complexity of the minimization. Note that $\mathbf{V}_n^H\mathbf{V}_n$ will be dominated by the large diagonal terms, with almost negligible contribution from the off-diagonal terms, if $x(n)$ is chosen as a pseudorandom sequence (as in the case of a training signal). Thus, $\mathbf{V}_n^H\mathbf{V}_n \approx \sum_{n=0}^{N-1} |x(n)|^2\mathbf{I} \triangleq \kappa\mathbf{I}$, where κ is constant over the frame considered, enabling us to approximate the minimum of (8) as the maximum of

$$J'(\mathbf{f}_L) = \mathbf{r}^H\mathbf{V}_n\mathbf{V}_n^H\mathbf{r}_n$$
$$= \sum_{p=0}^{L-1}\left|\sum_{n=0}^{N-1} r^*(n)x(n - p)e^{j2\pi f_p n}\right|^2. \quad (9)$$

This is a difficult joint multidimensional optimization problem. We therefore consider the FO of an arbitrary individual path s between a transmit and a receive antenna, for which the contribution to (9) is

$$\psi_s(n) = r^*(n)x(n - s)e^{j2\pi f_s n}$$
$$= Q_s + c_1(n) + c_2(n)$$

where

$$Q_s = h_s^*|x(n - s)|^2$$
$$c_1(n) = \eta^*(n)x(n - s)e^{j2\pi f_s n}$$
$$c_2(n) = \sum_{\substack{p=0 \\ p\neq s}}^{L-1} h_p^*(p)x^*(n - p)x(n - s)e^{j\Delta 2\pi f_p n}$$
$$\Delta f_p = f_s - f_p.$$

Moreover, as $E\{x(n - u)x(n - v)\} = 0$ for $u \neq v$, $|\sum_{n=0}^{N-1} \psi_s(n)|^2 \approx N^2|h_s|^2|x(n-s)|^4 + N\sum_{\substack{p=0 \\ p\neq s}}^{L-1} |h_p|^2|x(n-p)|^2|x(n-s)|^2 = N^2\rho_s + N\rho_i \approx N^2\rho_s$ for large N. Therefore, provided that the ratio between the signal component for which the parameters are to be estimated, $\rho_s = |h_s|^2|x(n-s)|^4$, to the interfering components $\rho_i = \sum_{\substack{p=0 \\ p\neq s}}^{L-1} |h_p|^2|x(n-p)|^2|x(n-s)|^2$, is greater than $1/N$, we could reduce the multidimensional joint maximization in (9) over all possible frequencies, to the maximization of the following for each individual frequency:

$$\hat{f}_s = \arg\max_f \left|\sum_{n=0}^{N-1} r^*(n)x(n - s)e^{j2\pi f n}\right|^2 \quad (10)$$

which can be efficiently evaluated using the fast Fourier transform (FFT). Once the FOs are estimated, the CGs, $\mathbf{h}_k$, can be estimated using (7).

IV. MMSE EQUALIZER DESIGN

A. Equalizer for Channels Without FOs

In this section, we initially consider the MMSE equalizer for the communication channel not affected by multiple FOs. Consider the received baseband signal in (1), for the case when $f_p \equiv 0$. For an equalizer of length M, the received signal vector can then be written as

$$\tilde{\mathbf{r}}_n = \mathbf{H}\tilde{\mathbf{x}}_n + \tilde{\boldsymbol{\eta}}_n \quad (11)$$

where $\tilde{\mathbf{r}}_n = [r(n) \ \cdots \ r(n-M+1)]^T$, $\tilde{\mathbf{x}}_n = [x(n) \ \cdots \ x(n-M-L+2)]^T$, $\tilde{\boldsymbol{\eta}}_n = [\eta(n) \ \cdots \ \eta(n-M+1)]^T$, and $\mathbf{H}$ is the $M \times (L + M - 1)$ channel convolution matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_L^T & 0 & \cdots & \mathbf{0} \\ 0 & \mathbf{h}_L^T & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ \mathbf{0} & \cdots & 0 & \mathbf{h}_L^T \end{bmatrix}. \quad (12)$$

The MMSE equalizer is then found as [4], [5]

$$\mathbf{w}_o = \left(\mathbf{H}\mathbf{H}^H + \frac{\sigma_n^2}{\sigma_x^2}\mathbf{I}\right)^{-1}\mathbf{H}\mathbf{z}_v \quad (13)$$

where $\mathbf{z}_v$ is a coordinate vector, only containing a nonzero component at position v, i.e., $\mathbf{z}_v = [0 \ \cdots \ 0 \ 1 \ 0 \ \cdots \ 0]^T$.

B. Equalizer for Channels With FOs

In the presence of FOs, the effective channel convolution matrix $\mathbf{H}$ will vary over time, and we modify (11) accordingly, yielding

$$\tilde{\mathbf{r}}_n = (\mathbf{H} \odot \boldsymbol{\Gamma}_L)\tilde{\mathbf{x}}_n + \tilde{\boldsymbol{\eta}}_n \triangleq \mathbf{H}_n\tilde{\mathbf{x}}_n + \tilde{\boldsymbol{\eta}}_n \quad (14)$$

where

$$\mathbf{\Gamma}_L = \begin{bmatrix} \boldsymbol{\gamma}_n^T & 0 & \cdots & \mathbf{0} \\ 0 & \boldsymbol{\gamma}_{n-1}^T & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ \mathbf{0} & \cdots & 0 & \boldsymbol{\gamma}_{n-M+1}^T \end{bmatrix} \quad (15)$$

$$\boldsymbol{\gamma}_k = [e^{j2\pi f_0 k} \quad \cdots \quad e^{j2\pi f_{L-1} k}]^T. \quad (16)$$

The optimum equalizer derived using (14), instead of (11), will thus yield

$$\mathbf{w}_o = \left(\mathbf{H}_n \mathbf{H}_n^H + \frac{\sigma_n^2}{\sigma_x^2}\mathbf{I}\right)^{-1} \mathbf{H}_n \mathbf{z}_v \triangleq \mathbf{R}_n^{-1}\mathbf{H}_n\mathbf{z}_v \quad (17)$$

which, due to the time-varying $\mathbf{H}_n$, will require the inversion of $\mathbf{R}_n$ for each symbol. Such an equalizer may be computationally infeasible. In the following, we propose a computationally efficient recursive scheme, not requiring the inversion of $\mathbf{R}_n$, by fully exploiting the movement of submatrices in $\mathbf{R}_n$. Generally for stationary channels, long equalizers provide better performance. However, for quasi-stationary channels, very long equalizers are not appropriate due to nonstationarity of the signal that enters into the equalizer. Here, for simplicity, we initially consider a simple example with only three channel taps and a length-three equalizer to illustrate the key ideas. Thus

$$\mathbf{R}_n = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{12}^* & \alpha_{22} & \alpha_{23} \\ \alpha_{13}^* & \alpha_{23}^* & \alpha_{33} \end{bmatrix} \triangleq \begin{bmatrix} \mathbf{A}_n & \alpha_{13} \\ & \alpha_{23} \\ \alpha_{13}^* & \alpha_{23}^* & \alpha_{33} \end{bmatrix}.$$

Note that, due to the structure of $\mathbf{R}_n$

$$\mathbf{R}_{n+1} = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{12}^* & \alpha_{11} & \alpha_{12} \\ \beta_{13}^* & \alpha_{12}^* & \alpha_{22} \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{12}^* & & \\ \beta_{13}^* & \mathbf{A}_n & \end{bmatrix}$$

the Hermitian matrix $\mathbf{A}_n$ will appear in both $\mathbf{R}_n$ and $\mathbf{R}_{n+1}$. This structural property will hold for any number of channel coefficients and equalizer taps. For the general case, we write $\mathbf{R}_n$ as

$$\mathbf{R}_n = \begin{bmatrix} \mathbf{A}_n & \mathbf{c}_n \\ \mathbf{c}_n^H & b_n \end{bmatrix} \quad (18)$$

where $\mathbf{A}_n \in \mathbb{C}^{(M-1)\times(M-1)}$, $\mathbf{c}_n \in \mathbb{C}^{(M-1)\times 1}$, and $b_n \in \mathbb{R}$. Provided that the relevant inverses exist, the inverse of a general block matrix can be expressed as (19) or (20) (see, e.g., [6] and [7])

$$\begin{bmatrix} \mathbf{A} & \mathbf{C} \\ \mathbf{D} & \mathbf{B} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \mathbf{A}^{-1}[\mathbf{I} \quad \mathbf{0}] + \begin{bmatrix} -\mathbf{A}^{-1}\mathbf{C} \\ \mathbf{I} \end{bmatrix}$$
$$\times (\mathbf{B} - \mathbf{D}\mathbf{A}^{-1}\mathbf{C})^{-1}[-\mathbf{D}\mathbf{A}^{-1} \quad \mathbf{I}] \quad (19)$$
$$= \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \mathbf{B}^{-1}[\mathbf{0} \quad \mathbf{I}] + \begin{bmatrix} \mathbf{I} \\ -\mathbf{B}^{-1}\mathbf{D} \end{bmatrix}$$
$$\times (\mathbf{A} - \mathbf{C}\mathbf{B}^{-1}\mathbf{D})^{-1}[\mathbf{I} \quad -\mathbf{C}\mathbf{B}^{-1}]. \quad (20)$$

Thus, given $\mathbf{A}_n^{-1}$, one can easily obtain $\mathbf{R}_n^{-1}$ and $\mathbf{R}_{n+1}^{-1}$, avoiding the matrix inversion. As $\mathbf{A}_n$ does not appear in $\mathbf{R}_{n+2}$, it can not be used to find $\mathbf{R}_{n+2}^{-1}$, and the scheme so far only

allows for a pairwise computational saving, still requiring the inversion of $\mathbf{A}_{n+1}$ to compute $\mathbf{R}_{n+2}^{-1}$ efficiently. However, further exploiting the structure of $\mathbf{R}_n$, one may compute $\mathbf{A}_{n+1}^{-1}$ efficiently from $\mathbf{R}_{n+1}^{-1}$ using the following lemma.

Lemma 1: Let

$$\begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}^{-1} \quad (21)$$

where $\dim\{\mathbf{H}_{kl}\} = \dim\{\mathbf{A}_{kl}\}$. Then, provided that the relevant inverses exist, the inverse of matrix $\mathbf{A}_{11}$ can be written as the Schur complement of $\mathbf{H}_{22}$, i.e.,

$$\mathbf{A}_{11}^{-1} = \mathbf{H}_{11} - \mathbf{H}_{12}\mathbf{H}_{22}^{-1}\mathbf{H}_{21}. \quad (22)$$

$\square$

The proof is given in the Appendix. At time n, let us find the inverse of submatrix $\mathbf{A}_n$, and use it, respectively, to find the inverses of $\mathbf{R}_n$ and $\mathbf{R}_{n+1}$ using (19) and (20). Since we know the inverse of $\mathbf{R}_{n+1}$, we find the inverse of the new top left-hand corner submatrix $\mathbf{A}_{n+1}$, using *Lemma 1*. Once found, the inverse of $\mathbf{A}_{n+1}$ can be used to compute the inverse of $\mathbf{R}_{n+2}$ using (20). We call this a forward and backward recursion method. Therefore, we need the explicit inverse of the matrix $\mathbf{A}_n$ only once, and thereafter, update the optimum equalizer for every symbol with the forward and backward recursions method using (20) and *Lemma 1*. We note that for our problem, $\mathbf{H}_{22}$ is a scalar.

V. SIMULATIONS

Initially, we assess the efficiency of the proposed AML estimator. A two-tap channel with FOs $f_0 = 0.003$ and $f_1 = 0.005$ is considered. This is reasonable, as the maximum Doppler shift for a vehicular speed of 250 km/h (RA250 channels as defined in global systems for mobile communications (GSM) standards) at a carrier frequency of 900 MHz is 1.3 KHz, which corresponds to 0.005 when normalized to the symbol rate of 270 KHz, as in GSM. The number of training samples is $N = 100$. Figs. 1 and 2 show that the mean-square estimation error (MSEE) as obtained from 10^4 Monte-Carlo simulations attains the CRLB, derived in the Appendix.

In order to demonstrate the benefits of employing FOs in equalization, we considered a two-path wireless communication channel and an equalizer of length 10. We assume a quasi-stationary channel. The FOs have been initially set to 0.003 and 0.005, and at every burst n, they have been changed according to the random-walk model, $f_l(n) = f_l(n-1) + 0.0001u(n)$, where $u(n)$ is a zero-mean real Gaussian noise with unity variance. The complex CGs h_0 and h_1 have been assumed to be constant throughout a burst, but change according to a Gaussian distribution between the frames. The training signal is assumed to have a binary alphabet, while the data symbols in the burst have been drawn from an 8-ary phase-shift keying (8-PSK) constellation. We considered three scenarios. In the first scenario, we set the FOs associated with the first and the second paths to $f_0 = 0.003$ and $f_1 = 0.005$, and designed a recursive equalizer, as explained previously. In the second scenario, we adopt a decision-directed scheme to equalize the channel effects, where the
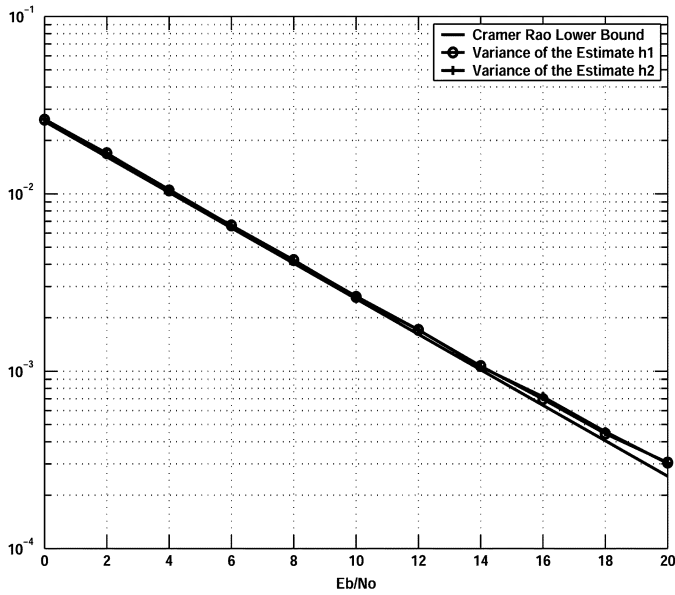
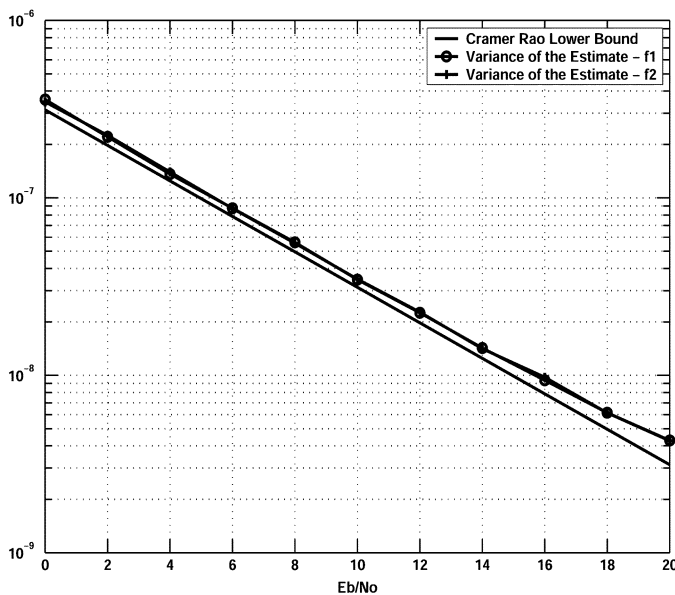Fig. 1.   Comparison of the MSEE of h_0 and h_1 with the CRLB.

Fig. 2.   Comparison of the MSEE of f_0 and f_1 with the CRLB.
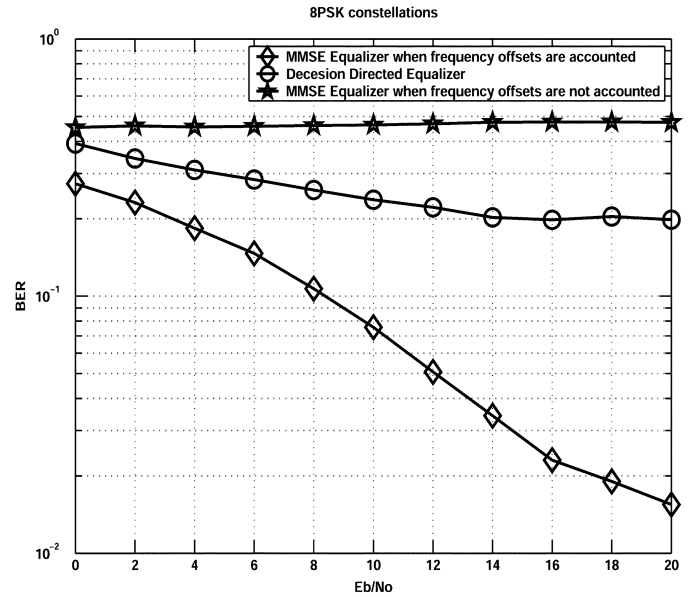

Fig. 3.   BER performance for MMSE equalizers with and without FO estimation, and a decision-directed adaptive equalizer.
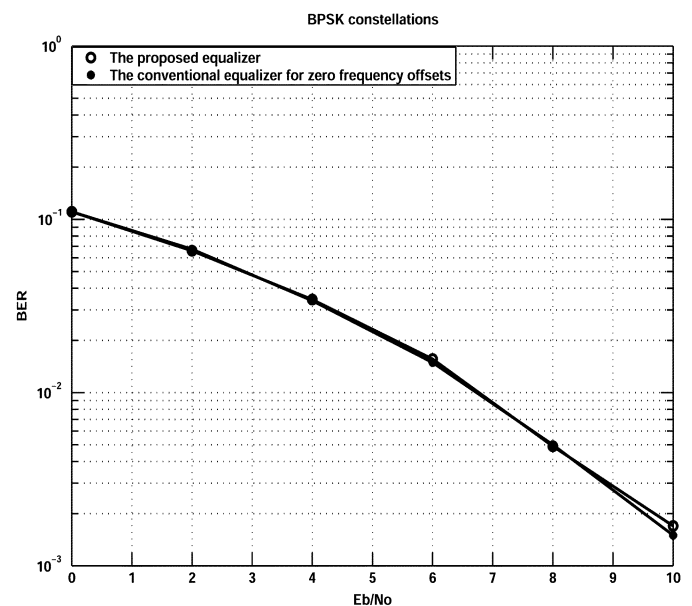

Fig. 4.   BER performance of MMSE equalizers for a GSM system.

least-mean square (LMS) algorithm is used to control the equalizer parameters. Here, the LMS equalizer is initialized with the correct MMSE solution, found at the end of the training interval. In the last scenario, an equalizer is designed ignoring the effect of FOs. The results depicted in Fig. 3 show the superior performance of our proposed scheme over the decision-directed scheme and an equalizer not considering the effect of FOs. In all the simulations, we considered a training signal of length 142 for the estimation of the channel parameters, including the FOs. The proposed scheme can also be employed in a GSM system. Here, the number of pilot symbols available in a burst is only 26. With this training length, we can estimate the channel very efficiently, but to estimate FO, this length is insufficient. However, the performance of the FO estimator can be enhanced, by concatenating the pilot symbols from a number of recent past

bursts. To illustrate this, we simulated GSM burst transmission through a two-path time-varying channel identical to that used in the previous simulation. A normal burst in GSM consists of 116 encrypted data symbols and 26 pilot symbols in the middle [8]. At every burst, the CGs h_0 and h_1 were estimated using the pilot symbols, but the FOs have been estimated using 142 symbols from the previous burst obtained with the retrieved data symbols together with the pilot symbols. In this way, to estimate frequency we used five previous successive bursts. To avoid occasional FO estimation error, we filtered the estimates through a moving median filter of length five. The bit-error rate (BER) performance of our proposed scheme is identical to the performance of a GSM communication channel without FOs (see Fig. 4).

VI. CONCLUSION

We considered estimation and equalization problems for a frequency-selective channel with distinct FOs for each path. This problem could arise when the receiver or transmitter moves with very high speed with different paths experiencing different Doppler shifts, due to different AOAs. By exploiting the correlation property of the transmitted pilot signal, we provided an AML estimator, and showed that the MSEE attains the CRLB. Since the effective channel convolution matrix varies between symbols due to multiple FOs, we provided a recursive scheme to design the optimum equalizer for every symbol. The simulation results demonstrate the superior performance of the proposed scheme over an equalization scheme that does not consider the effects of multiple FOs.

APPENDIX I
DERIVATION OF CRLB

In this appendix, we derive the CRLB for the problem at hand. Let $\bar{\mathbf{r}}(\boldsymbol{\theta}) = [\bar{r}(n) \cdots \bar{r}(n - N + 1)]^T$ and $\boldsymbol{\theta} = [\Re(\mathbf{h}_L)^T \ \Im(\mathbf{h}_L)^T \ \mathbf{f}_L^T]^T$. Under the assumption that $\eta(t)$ is complex white Gaussian with zero mean and variance σ_η^2, the CRLB can be found using the Slepian–Bangs formula (see, e.g., [6])

$$\left[\mathbf{P}_{\text{CRLB}}^{-1}(\boldsymbol{\theta})\right]_{l,p} = 2\sigma_\eta^{-2}\Re\left\{\frac{\partial \bar{\mathbf{r}}^H(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}_l} \frac{\partial \bar{\mathbf{r}}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}_p}\right\} \quad (23)$$

where $[\mathbf{A}]_{k,p}$ denotes the (k, p)th element of $\mathbf{A}$, and $\mathbf{P} \triangleq E\{(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^H\} \geq \mathbf{P}_{\text{CRLB}}$. We further note that $(\partial\bar{r}(n))/(\partial h_l^r) = e^{j2\pi f_l n}\bar{r}(n - l)$, $(\partial\bar{r}(n))/(\partial h_l^i) = je^{j2\pi f_l n}\bar{r}(n - l)$, and $(\partial\bar{r}(n))/(\partial f_l) = j2\pi n h_l e^{j2\pi f_l n}\bar{r}(n - l)$, where h_l^r and h_l^i are the lth element of $\Re(\mathbf{h}_L)$ and $\Im(\mathbf{h}_L)$.

APPENDIX II
PROOF OF LEMMA 1

Using the block-matrix inversion lemma, as expressed in (19) and (20), (21) can be written as (see, e.g., [6] and [7])

$$\mathbf{H}_{11} = \mathbf{A}_{11}^{-1} + \mathbf{A}_{11}^{-1}\mathbf{A}_{12}\boldsymbol{\Delta}_{\mathbf{A}}^{-1}\mathbf{A}_{21}\mathbf{A}_{11}^{-1} \quad (24)$$

$$\mathbf{H}_{12} = -\mathbf{A}_{11}^{-1}\mathbf{A}_{12}\boldsymbol{\Delta}_{\mathbf{A}}^{-1} \quad (25)$$

$$\mathbf{H}_{21} = -\boldsymbol{\Delta}_{\mathbf{A}}^{-1}\mathbf{A}_{21}\mathbf{A}_{11}^{-1} \quad (26)$$

$$\mathbf{H}_{22} = \boldsymbol{\Delta}_{\mathbf{A}}^{-1} \quad (27)$$

where $\boldsymbol{\Delta}_{\mathbf{A}} = \mathbf{A}_{22} - \mathbf{A}_{21}\mathbf{A}_{11}^{-1}\mathbf{A}_{12}$ is the Schur complement of $\mathbf{A}_{11}$. Substituting (27) into (26), and (25) into (24), yields

$$\mathbf{H}_{21} = -\mathbf{H}_{22}\mathbf{A}_{21}\mathbf{A}_{11}^{-1} \quad (28)$$

$$\mathbf{H}_{11} = \mathbf{A}_{11}^{-1} - \mathbf{H}_{12}\mathbf{A}_{21}\mathbf{A}_{11}^{-1}. \quad (29)$$

Substituting (28) into (29) yields

$$\mathbf{H}_{11} = \mathbf{A}_{11}^{-1} + \mathbf{H}_{12}\mathbf{H}_{22}^{-1}\mathbf{H}_{21} \quad (30)$$

and hence (22), which concludes the proof.

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