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## **The simulation of aerial movement—III. The determination of the angular momentum of the human body**

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# THE SIMULATION OF AERIAL MOVEMENT - III. THE DETERMINATION OF THE ANGULAR MOMENTUM OF THE HUMAN BODY

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## Abstract

A method is presented for determining the angular momentum of the human body about its mass centre for general three-dimensional movements. The body is modelled as an 11 segment link system with 17 rotational degrees of freedom and the angular momentum of the body is derived as a sum of 12 terms, each of which is a vector function of just one angular velocity. This partitioning of the angular momentum vector gives the contribution due to the relative segmental movement at each joint rather than the usual contribution of each segment. A method of normalizing the angular momentum is introduced to enable the comparison of rotational movements which have different flight times and are performed by athletes with differing inertia parameters. Angular momentum estimates were calculated during the flight phases of nine twisting somersaults performed on trampoline. Errors in film digitization made large contributions to the angular momentum error estimates. For individual angular momentum estimates the relative error is estimated to be about 10% whereas for mean angular momentum estimates the relative error is estimated to be about 1%.

## INTRODUCTION

In cases where the effects of air resistance can be neglected, the rotational motion of human airborne movement is governed by the conservation of angular momentum. As a consequence the angular momentum equation forms the basis of a simulation model of aerial movement. The ability to determine the angular momentum from film of an aerial movement permits the use of angular momentum values as input parameters for simulations of such movements. Angular momentum values may also be used as measures of the rotational takeoff requirements for somersaulting movements and as measures of the twist initiated prior to takeoff in twisting somersaults.

Methods for determining the angular momentum about a transverse axis for planar movements have been developed by Miller (1970); Ramey (1973) and Hay et al. (1977). Dapena (1978) used a model in which the limbs were represented by thin rods, to determine the angular momentum for general three dimensional movements. In this study it was concluded that there was only a small error in the calculated angular momentum of a Fosbury flop high jump arising from the assumption of zero values for the moments of inertia of the limb segments about their longitudinal axes.

While such an assumption may introduce little error when the limbs are abducted, consider the case of a twisting somersault in which the body is straight with arms and legs adducted. If each leg is modelled as a cylinder of radius  $r$  and mass  $m$  the moment of inertia of a leg about its longitudinal axis will be  $0.5mr^2$  and the moment of inertia about a parallel axis on the surface of the cylinder will be  $1.5mr^2$ . Thus, using the theorem of parallel axes, the two leg unit will have a moment of inertia about its longitudinal axis equal to  $3mr^2$  of which  $mr^2$  is due to the local terms and  $2mr^2$  is due to the transfer terms. If thin rods rather than cylinders are used to model the legs the local terms will be zero so that the moment of inertia of the two leg system will be estimated to be  $2mr^2$ . This large difference in moment of inertia estimates indicates that the thin rod assumption will lead to substantial errors in the calculated angular momentum of movements in which there is twisting about the longitudinal axis of the body.

This paper presents a method for determining the angular momentum of the human body about its mass centre during a twisting somersault.

## THE HUMAN BODY MODEL

The body is modelled as a system of 11 linked rigid segments. The segmental inertia parameters are calculated from anthropometric measurements, using the inertia model described in part II of this series (Yeadon, 1990b), and the time histories of the angles which define the orientations of the body segments are calculated from film data, as described in part I (Yeadon, 1990a). This inertia and orientation information is used as input to the model which calculates the time histories of the angular momentum components about three orthogonal axes.

### Segmentation and orientation angles

Figure 1 shows the 11 segments of the model and the 10 joint centres linking the segments. The relative orientations of two adjacent segments are defined by one, two or three angles using the system described in part I (Yeadon, 1990a). The number of angles used at each joint centre is shown in Table 1.

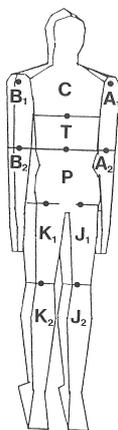


Figure 1: The 11 segments of the model. Joint centres linking the segments are represented by the symbol ●.

In total there are 18 angles defining the body configuration. The number of degrees of freedom for body configuration is less than this, however, since certain relationships are assumed to exist between the angles. The legs are assumed to move symmetrically relative to the pelvis but with relative abduction being permitted. As a consequence the thigh flexion angles are taken to be equal and the knee angles are taken to be equal. These symmetry assumptions are consistent with the requirements of good form in twisting somersaults. The two angles describing the orientation of the thorax  $T$  relative to the pelvis  $P$  are assumed to be functions of the orientations of the thighs relative to the pelvis, as described in Appendix 1. The reason for this assumption is that it is difficult to identify points of the thorax from film and so the orientation of the thorax relative to the pelvis cannot be determined accurately from film data. These relationships between configuration angles reduce the number of degrees of freedom by 4 so that body configuration is defined by 14 independent internal orientation angles.

The orientation of the whole body in space is defined by the angles  $\phi$ ,  $\theta$  and  $\psi$ , corresponding to somersault, tilt and twist, which give the orientation of a reference frame  $f$  of the 11 segment system relative to a non-rotating frame  $i$ , as described in part I of this series (Yeadon, 1990a). The system frame  $f$  of a multi-segment model is usually chosen to be fixed in one particular segment (Dapena, 1981; Van Gheluwe, 1981; Ramey and Yang, 1981). Such a procedure has the disadvantage that a change in body configuration produces a large apparent change in whole body orientation. In the present model an attempt has been made to minimize such changes by defining the system frame  $f$  using a number of segments as described in Appendix 2.

Table 1: Number of angles used at each joint centre

Joint centre	Number of angles
Junction of C and T	2
Junction of T and P	2
Each hip centre	2
Each knee centre	1
Each shoulder centre	3
Each elbow centre	1

## Angular velocities

The angular velocity of the system frame  $f$  defined in Appendix 2, relative to a non-rotating reference frame  $i$  is expressed in terms of the first derivatives  $\dot{\phi}$ ,  $\dot{\theta}$  and  $\dot{\psi}$  of the orientation angles  $\phi$ ,  $\theta$ ,  $\psi$  in the following way.

As described in part I (Yeadon, 1990a) the orientation of the system frame  $f$  relative to the non-rotating frame  $i$  is given by the somersault angle  $\phi$ , the tilt angle  $\theta$  and the twist angle  $\psi$ . If  $f$  is initially aligned with frame  $i$ , then successive rotations through  $\phi$  about  $\mathbf{f}_1$ ,  $\theta$  about  $\mathbf{f}_2$ , and  $\psi$  about  $\mathbf{f}_3$ , bring frame  $f$  into its final orientation. The rotation matrices  $R_1(\phi)$ ,  $R_2(\theta)$ ,  $R_3(\psi)$  transform the coordinates of a vector in one frame to the next. The columns of these matrices use the new coordinates of the previous directions of the unit vectors  $\mathbf{f}_1$ ,  $\mathbf{f}_2$ ,  $\mathbf{f}_3$ , so that:

$$R_1(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix}$$

$$R_2(\theta) = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

and

$$R_3(\psi) = \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The angular velocity of  $f$  relative to  $i$  is given by:

$$\boldsymbol{\omega}_{fi} = \dot{\phi}\mathbf{i}_1 + \dot{\theta}\mathbf{f}'_2 + \dot{\psi}\mathbf{f}_3$$

where  $\mathbf{f}'_2$  is a unit vector with the direction of  $\mathbf{f}_2$  after the first rotation through  $\phi$ .

If  $(\boldsymbol{\omega}_{fi})_f$  denotes the evaluation of the angular velocity  $\boldsymbol{\omega}_{fi}$  in frame  $f$  then:

$$(\boldsymbol{\omega}_{fi})_f = R_3(\psi) \cdot R_2(\theta) \cdot R_1(\phi) \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + R_3(\psi) \cdot R_2(\theta) \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}$$

so that

$$(\boldsymbol{\omega}_{fi})_f = R_\omega \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

where

$$R_\omega = \begin{bmatrix} \cos \theta \sin \psi & \sin \psi & 0 \\ -\cos \theta \sin \psi & \cos \psi & 0 \\ \sin \theta & 0 & 1 \end{bmatrix}$$

The relative angular velocities of body segments are expressed in terms of the first derivatives of the internal orientation angles in the following way.

Consider the movement of the frame  $a$  of the left upper arm relative to the frame  $c$  of the chest-head segment. As described in part I, frame  $a$  is brought from initial alignment with frame  $c$  into its final orientation by successive rotations through  $-\delta_a$  about  $\mathbf{a}_1$ ,  $-\varepsilon_a$  about  $\mathbf{a}_2$  and  $\psi_a$  about  $\mathbf{a}_3$ . If  $S_{ac}$  is the matrix which transforms the  $a$ -coordinates of a free vector into  $t$ -coordinates then

$$S_{ac} = R_1(\delta_a) \cdot R_2(\varepsilon_a) \cdot R_3(-\psi_a)$$

The angular velocity of frame  $a$  relative to frame  $c$  in  $c$ -coordinates is given by:

$$\begin{aligned} (\boldsymbol{\omega}_{ac})_c &= \begin{bmatrix} \dot{\delta}_a \\ 0 \\ 0 \end{bmatrix} + R_1(\delta_a) \begin{bmatrix} 0 \\ -\dot{\varepsilon}_a \\ 0 \end{bmatrix} + R_1(\delta_a) \cdot R_2(\varepsilon_a) \begin{bmatrix} 0 \\ 0 \\ -\dot{\psi}_a \end{bmatrix} \\ &= \begin{bmatrix} -\dot{\delta}_a - \dot{\psi}_a \sin \varepsilon_a \\ -\dot{\varepsilon}_a \cos \delta_a + \dot{\psi}_a \sin \delta_a \cos \varepsilon_a \\ \dot{\varepsilon}_a \sin \delta_a + \dot{\psi}_a \cos \delta_a \cos \varepsilon_a \end{bmatrix}. \end{aligned}$$

The angular velocity  $\boldsymbol{\omega}_{ac}$ , is then obtained in frame  $f$  as  $(\boldsymbol{\omega}_{ac})_f = S_{cf}(\boldsymbol{\omega}_{ac})_c$  where  $S_{cf}$  is the rotation matrix which transforms  $c$ -coordinates into  $f$ -coordinates.

It should be noted that the symbol  $a$ , rather than  $a_1$  has been used in the above derivation for quantities associated with the left upper arm  $A_1$  in order to simplify the notation.

The same procedure is used to express all relative segmental angular velocities in frame  $f$  in terms of the first derivatives of internal orientation angles.

## Inertia tensors

The inertia tensor of any segment  $S$  in its local reference frame  $S$  is given by:

$$(\mathbf{I}_{ss})_s = \begin{bmatrix} IS_1 & 0 & 0 \\ 0 & IS_2 & 0 \\ 0 & 0 & IS_3 \end{bmatrix}$$

where the principal moments of inertia  $IS_1$ ,  $IS_2$ ,  $IS_3$  of the segment referred to its mass centre are obtained using the inertia model described in part II (Yeadon, 1990b).

$\mathbf{I}_{ss}$  is then evaluated in frame  $f$  as:

$$(\mathbf{I}_{ss})_f = S_{sf} \cdot (\mathbf{I}_{ss})_s \cdot S_{fs}$$

where  $S_{sf}$  is the rotation matrix which transforms  $s$ -coordinates of a free vector into  $f$ -coordinates and  $S_{fs}$  is the inverse (and transpose) of  $S_{sf}$ .

The inertia tensor of the whole body is given by  $\mathbf{I}_{ff} = \sum \mathbf{I}_{sf}$  where  $\mathbf{I}_{sf}$  is the inertia tensor of segment  $S$  referred to the mass centre  $F$  of the system.

$\mathbf{I}_{sf}$  is given by the following generalized form of the theorem of parallel axes:

$$\mathbf{I}_{sf} = \mathbf{I}_{ss} + \mathbf{I}_{msf},$$

where  $\mathbf{I}_{msf}$  is the inertia tensor of a point mass  $m$ , situated at the mass centre of the segment  $S$ , referred to the mass centre  $F$  of the system.

In frame  $f$ :

$$(\mathbf{I}_{msf})_f = m_s \begin{bmatrix} s_2^2 + s_3^2 & -s_1 s_2 & -s_1 s_3 \\ -s_2 s_1 & s_1^2 + s_3^2 & -s_2 s_3 \\ -s_3 s_1 & -s_3 s_2 & s_1^2 + s_2^2 \end{bmatrix}$$

where  $[s_1, s_2, s_3]^T$  is the position vector in frame  $f$  of the mass centre of  $S$  relative to the whole body mass centre  $F$ .

The same procedure is used to obtain the inertia tensors used in the next section.

## The angular momentum equation

In this section it will be shown that the angular momentum of the system about its mass centre may be expressed as a sum of terms, each of which is a function of just one of the relative angular velocities. As a consequence of the linearity of this expression, each term may be obtained as the angular momentum associated with a particular angular velocity. This powerful result provides a simple expression for the angular momentum of a multi-link system.

Suppose that the system comprises  $n$  rigid segments  $S_k (k = 1, n)$  which are linked by  $(n - 1)$  joint centres  $O_k (k = 2, n)$ . The link system emanates from segment  $S_1$  and continues out to the extremities in the following way.

Let each segment  $S_k$ , which shares a joint centre  $O_k$  with  $S_1$ , be known as an immediate successor of  $S_1$ . The immediate successors of  $S_k$  will be those segments which share a joint centre with  $S_k$  other than  $O_k$ . Provided that each segment shares a joint centre with some other segment and that two segments share at most one joint centre and that there are no structural closed loops, the link system will enter  $S_k$  at exactly one point  $O_k (k = 2, n)$ .

The orientation of the system will be given by the three orientation angles  $\alpha_1, \alpha_2, \alpha_3$  which specify the orientation of  $S_1$  relative to the non-rotating frame  $i$ . The orientation of  $S_k$  relative to its predecessor  $S_1$ , which shares the joint centre  $O_k$ , will be specified by the three orientation angles  $\alpha_{3k-2}, \alpha_{3k-1}, \alpha_{3k}$  ( $k = 2, n$ ). The system has  $3n$  degrees of freedom, with the orientation and configuration specified by  $3n$  independent orientation angles  $\alpha_j (j = 1, m$  where  $m = 3n)$ . If a joint is required to have less than three degrees of freedom this may be accommodated at a later stage by considering motions for which certain of the  $\dot{\alpha}_j$  are zero.

For the segment  $S_k$  let:

$\mathbf{h}_k$  = the angular momentum of  $S_k$  about the mass centre  $F$  of the system

$\mathbf{I}_k$  = the inertia tensor of  $S_k$  referred to its mass centre  $G_k$

$\boldsymbol{\omega}_k$  = the angular velocity of  $S_k$  relative to frame  $i$

$m_k$  = the mass of  $S_k$ ,

$\mathbf{r}_k$  = the position vector of the mass centre  $G_k$  relative to  $F$ ,

$\mathbf{v}_k$  = the velocity of the mass centre  $G_k$  relative to  $F$ .

Greenwood (1965) shows that:

$$\mathbf{h}_k = \mathbf{I}_k \boldsymbol{\omega}_k + m_k \mathbf{r}_k \times \mathbf{v}_k$$

where  $\mathbf{r}_k$  is a vector function of  $\alpha_j (j = 1, m)$  (Kane, 1968) so that:

$$\mathbf{r}_k = r_{k1} \mathbf{i}_1 + r_{k2} \mathbf{i}_2 + r_{k3} \mathbf{i}_3$$

where  $\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3$  are orthogonal unit vectors of frame  $i$  and  $r_{kl} (l = 1, 3)$  are scalar functions of  $\alpha_j (j = 1, m)$ .

Kane (1968) shows that:

$$\mathbf{v}_k = \dot{\mathbf{r}}_k^i = \dot{r}_{k1} \mathbf{i}_1 + \dot{r}_{k2} \mathbf{i}_2 + \dot{r}_{k3} \mathbf{i}_3$$

where

$$\dot{r}_{kl} = \sum_{j=1}^m (\partial r_{kl} / \partial \alpha_j) \dot{\alpha}_j \quad (l = 1, 3)$$

Thus:

$$\mathbf{v}_k = \sum_{j=1}^m \mathbf{v}_{kj} \dot{\alpha}_j \quad \text{where} \quad \mathbf{v}_{kj} = (\partial r_{k1} / \partial \alpha_j) \mathbf{i}_1 + (\partial r_{k2} / \partial \alpha_j) \mathbf{i}_2 + (\partial r_{k3} / \partial \alpha_j) \mathbf{i}_3 \quad (j = 1, m),$$

so that  $\mathbf{v}_{kj} (j = 1, m)$  are vector functions of  $\alpha_p (p = 1, m)$  and are independent of  $\dot{\alpha}_p (p = 1, m)$ .

The angular velocity  $\boldsymbol{\omega}_k$  may also be expressed in the form:

$$\boldsymbol{\omega}_k = \sum_{j=1}^m \boldsymbol{\omega}_{kj} \dot{\alpha}_j$$

where  $\boldsymbol{\omega}_{kj}$  are vector functions of  $\alpha_p$  ( $p = 1, m$ ) (Kane, 1968). Thus:

$$\begin{aligned}\mathbf{h}_k &= \mathbf{I}_k \boldsymbol{\omega}_k + m_k \mathbf{r}_k \times \mathbf{v}_k \\ &= \mathbf{I}_k \left( \sum_{j=1}^m \boldsymbol{\omega}_{kj} \dot{\alpha}_j \right) + m_k \mathbf{r}_k \times \left( \sum_{j=1}^m \mathbf{v}_{kj} \dot{\alpha}_j \right) \\ &= \sum_{j=1}^m \mathbf{h}_{kj} \dot{\alpha}_j\end{aligned}$$

where

$$\mathbf{h}_{kj} = \mathbf{I}_k \boldsymbol{\omega}_{kj} + m_k \mathbf{r}_k \times \mathbf{v}_{kj} \quad (j = 1, m)$$

so that each  $\mathbf{h}_{kj}$  is a vector function of  $\alpha_p$  ( $p = 1, m$ ) and is not dependent upon  $\dot{\alpha}_p$  ( $p = 1, m$ ).

The total momentum may now be written as:

$$\begin{aligned}\mathbf{h} &= \sum_{k=1}^m \mathbf{h}_k = \sum_{k=1}^n \left( \sum_{j=1}^m \mathbf{h}_{kj} \dot{\alpha}_j \right) \\ &= \sum_{j=1}^m \left( \sum_{k=1}^n \mathbf{h}_{kj} \right) \dot{\alpha}_j \\ &= \sum_{j=1}^m \mathbf{h}'_j \dot{\alpha}_j\end{aligned}$$

where  $\mathbf{h}'_j = \sum_{k=1}^n \mathbf{h}_{kj}$  are vector functions of  $\alpha_p$ .

The three orientation angles which govern movement at the joint centre  $O_k$  are  $\alpha_{3k-2}$ ,  $\alpha_{3k-1}$  and  $\alpha_{3k}$  so that  $\mathbf{h}$  may be written as:

$$\begin{aligned}\mathbf{h} &= \sum_{k=1}^n (\mathbf{h}'_{3k-2} \dot{\alpha}_{3k-2} + \mathbf{h}'_{3k-1} \dot{\alpha}_{3k-1} + \mathbf{h}'_{3k} \dot{\alpha}_{3k}) \\ &= \sum_{k=1}^n \mathbf{h}''_k\end{aligned}$$

where  $\mathbf{h}''_k$  is dependent on  $\dot{\alpha}_{3k-2}$ ,  $\dot{\alpha}_{3k-1}$  and  $\dot{\alpha}_{3k}$  but is independent of the remaining  $\dot{\alpha}_p$ .

Thus  $\mathbf{h}$  has been expressed as a sum of terms  $\mathbf{h}''_k$ , each of which is a function of the relative movement at the joint centre  $O_k$  but is independent of the movement elsewhere. Note that  $\mathbf{h}''_1$  is a function of the movement of  $S_1$  relative to the non-rotating frame  $i$ .

When all the internal orientation angles are held fixed the total angular momentum will equal  $\mathbf{h}''_1$  since  $\mathbf{h}''_k = 0$  ( $k = 2, n$ ). In this situation the system moves as a rigid body with angular velocity  $\boldsymbol{\omega}_1$ , so that:

$$\mathbf{h}''_1 = \mathbf{I}_{ff} \boldsymbol{\omega}_1$$

where  $\mathbf{I}_{ff}$  is the inertia tensor of the system referred to the mass centre  $F$ .

When movement occurs only at the joint centre  $O_k$  the total angular momentum will equal  $\mathbf{h}''_k$  since  $\mathbf{h}''_p = 0$  ( $p \neq k$ ). In this situation the system comprises two supra-segments  $U$  and  $L$  which are linked at the joint centre  $O = O_k$ .  $U$  comprises the segment  $S_k$  and all its successors in the link system whilst  $L$  comprises the remaining segments.  $L$  will include the segment  $S_1$  and so will maintain a fixed orientation relative to frame  $i$ .

The angular momentum of the system will be:

$$\mathbf{h}''_k = \mathbf{I}_{uu} \boldsymbol{\omega}_{ui} + m_u \mathbf{u}_f \times \dot{\mathbf{u}}_f^i + \mathbf{I}_{ll} \boldsymbol{\omega}_{li} + m_l \mathbf{l}_f \times \dot{\mathbf{l}}_f^i$$

where

$\mathbf{I}_{uu}, \mathbf{I}_{ll}$  are the inertia tensors of  $U$  and  $L$  referred to their mass centres  
 $\boldsymbol{\omega}_{ui}, \boldsymbol{\omega}_{li}$  are the angular velocities of  $U$  and  $L$  relative to frame  $i$   
 $m_u, m_l$  are the masses of  $U$  and  $L$   
 $\mathbf{u}_f, \mathbf{l}_f$  are the position vectors of the mass centres of  $U$  and  $L$  relative to the mass centre  $F$  of the system  
 $\cdot^i$  denotes a vector derivative in frame  $i$ .

Since  $L$  maintains a fixed orientation relative to frame  $i$  the angular velocity  $\boldsymbol{\omega}_{li}$  will be zero and  $\boldsymbol{\omega}_{ui}$  will equal the angular velocity  $\boldsymbol{\omega}_{ul}$  of  $U$  relative to  $L$ .

Since  $F$  is the mass centre of the system  $m_l \mathbf{l}_f = -m_u \mathbf{u}_f$  so that:

$$m_u \mathbf{u}_f \times \dot{\mathbf{u}}_f^i + m_l \mathbf{l}_f \times \dot{\mathbf{l}}_f^i = m_u \mathbf{u}_f \times \dot{\mathbf{u}}_f^i.$$

Now  $\dot{\mathbf{u}}_f^i = \dot{\mathbf{u}}_o^i + \dot{\boldsymbol{\phi}}_i^i = \dot{\mathbf{u}}_o^i$  since  $O = O_k$  is fixed in  $L$  and  $\dot{\mathbf{u}}_o^i = \dot{\mathbf{u}}_o^u + \boldsymbol{\omega}_{ul} \times \mathbf{u}_o = \boldsymbol{\omega}_{ul} \times \mathbf{u}_o$  since  $O$  is fixed in  $U$  (Greenwood, 1965). Thus  $\mathbf{h}_k''$  takes the form:

$$\mathbf{h}_k'' = \mathbf{I}_{uu} \boldsymbol{\omega}_{ul} + m_u \mathbf{u}_f \times (\boldsymbol{\omega}_{ul} \times \mathbf{u}_{ok}) \quad (k = 2, n)$$

and is a function of the relative angular velocity  $\boldsymbol{\omega}_{ul}$  at the joint centre  $O_k$  but is independent of all other angular velocities.

The total angular momentum of the system about its mass centre is:

$$\mathbf{h} = \mathbf{h}_1'' + \sum_2^n \mathbf{h}_k''$$

where  $\mathbf{h}_1'' = \mathbf{I}_{ff} \boldsymbol{\omega}_1$  and  $\mathbf{h}_k''$  is given by the above expression ( $k = 2, n$ ).

The first term arises from the motion of the system as a whole and the remaining terms are associated with the internal movement of the system.

In the present 11 segment model the segment  $S_1$  from which the link system emanates is the pelvis  $P$  and the angular velocity  $\boldsymbol{\omega}_1$  of  $P$  in space may be written as  $\boldsymbol{\omega}_1 = \boldsymbol{\omega}_{pf} + \boldsymbol{\omega}_{fi}$  where  $\boldsymbol{\omega}_{pf}$  is the angular velocity of the pelvis relative to the system frame  $f$  and  $\boldsymbol{\omega}_{fi}$  is the angular velocity of frame  $f$  relative to the nonrotating frame  $i$ .

Thus  $\mathbf{h}_1'' = \mathbf{I}_{ff} \boldsymbol{\omega}_{pf} + \mathbf{I}_{ff} \boldsymbol{\omega}_{fi}$  and the total angular momentum  $\mathbf{h}$  is given by:

$$\begin{aligned} \mathbf{h} = & \mathbf{h}_{\boldsymbol{\omega}_{fi}} + \mathbf{h}_{\boldsymbol{\omega}_{pf}} + \mathbf{h}_{\boldsymbol{\omega}_{tp}} + \mathbf{h}_{\boldsymbol{\omega}_{ct}} \\ & + \mathbf{h}_{\boldsymbol{\omega}_{a_1c}} + \mathbf{h}_{\boldsymbol{\omega}_{b_1c}} + \mathbf{h}_{\boldsymbol{\omega}_{a_2a_1}} + \mathbf{h}_{\boldsymbol{\omega}_{b_2b_1}} \\ & + \mathbf{h}_{\boldsymbol{\omega}_{j_1p}} + \mathbf{h}_{\boldsymbol{\omega}_{k_1p}} + \mathbf{h}_{\boldsymbol{\omega}_{j_2j_1}} + \mathbf{h}_{\boldsymbol{\omega}_{k_2k_1}} \end{aligned}$$

where  $\mathbf{h}_{\boldsymbol{\omega}}$  is the angular momentum associated with the angular velocity  $\boldsymbol{\omega}$  and  $\boldsymbol{\omega}_{tp}, \boldsymbol{\omega}_{ct}, \boldsymbol{\omega}_{a_1c}, \boldsymbol{\omega}_{b_1c}, \boldsymbol{\omega}_{a_2a_1}, \boldsymbol{\omega}_{b_2b_1}, \boldsymbol{\omega}_{j_1p}, \boldsymbol{\omega}_{k_1p}, \boldsymbol{\omega}_{j_2j_1}, \boldsymbol{\omega}_{k_2k_1}$  are the relative angular velocities of the 11 segments shown in Fig. 1.

This expression for the total angular momentum forms the basis of the computer simulation model described in part IV of this series (Yeadon et al., 1990).

## EVALUATION OF THE METHOD

In order to evaluate the above method for calculating the angular momentum of the human body, nine performances of twisting somersaults on trampoline were filmed. The movements were performed by three subjects of varied ability. Subject A was a novice trampolinist, subject B was an experienced springboard diver and subject C was an elite competitive trampolinist. The movements performed ranged from a single somersault with one twist to a double somersault with two twists.

Table 2: Standard errors of the angular momentum estimates

Subject	Total somersault (revolutions)	Total twist (revolutions)	Angular momentum (ss/f.t.)	Standard error (ss/f.t.)	Flight time (s)
A	1	1	0.71	0.006	0.93
A	1	$1\frac{1}{2}$	0.81	0.010	1.02
B	1	1	0.90	0.011	1.23
B	1	$1\frac{1}{2}$	0.86	0.013	0.99
C	1	1	0.90	0.010	1.59
C	1	1	0.76	0.009	1.24
C	2	1	1.37	0.011	1.54
C	2	$1\frac{1}{2}$	1.39	0.014	1.51
C	2	2	1.49	0.014	1.55

Note: if  $I_s$  denotes the moment of inertia about the lateral axis for a straight body configuration of subject  $S$ , then:  $I_A = 8.58 \text{ kgm}^2$ ,  $I_B = 9.72 \text{ kgm}^2$  and  $I_C = 10.56 \text{ kgm}^2$ .

Prior to filming, anthropometric measurements were taken on each subject so that segmental inertia parameters could be calculated using the inertia model described in part II (Yeadon, 1990b). The movements were filmed using two Bolex spring driven cameras, the films were digitized and quintic spline coefficients of the orientation angles were calculated as described in part I (Yeadon, 1990a). Using the segmental inertia values and the time histories of the 17 orientation angles as input to the model described in this paper, angular momentum values for each movement were calculated at 101 equally spaced times.

The angular momentum values were normalized in the following manner. The values were divided by the moment of inertia of the body about the lateral axis for a straight body configuration; rotations were measured in revolutions rather than radians and the unit of time was taken to be the flight time of a movement. As a consequence, the unit of momentum became the ‘straight somersault per flight time.’ Thus for each movement the normalized momenta values represent the equivalent number of straight somersaults. This procedure permits a direct comparison of the rotational takeoff requirements for movements performed by subjects with differing inertias and differing flight times.

From the 101 estimates of each component of the normalized angular momentum, mean values and standard errors were calculated. The mean values and standard errors of the three normalized components were summed vectorally to give a mean angular momentum vector and a standard error vector. Since the angular momentum vector during the flight phase of a movement remains constant, if the effects of air resistance can be neglected, the magnitude of this standard error vector gives a measure of the accuracy of the method.

Table 2 lists the normalized angular momentum magnitude for each movement together with the magnitude of the corresponding standard error vector. The flight time of each movement and lateral moment of inertia of each subject are also given in Table 2 so that the angular momentum values may be transformed from normalized units into SI units if desired.

The standard error values are around 1% of the mean angular momentum values and so the mean angular momentum values may be considered to be quite accurate. On the other hand the standard deviation of the 101 angular momentum estimates will be  $101^{\frac{1}{2}} = 10.05$  times larger than the standard error estimate so that the individual estimates are distributed about the mean value with a standard deviation of about 10%.

In order to compare the accuracy of the present method with two other methods, the momentum estimates of a double somersault with  $1\frac{1}{2}$  twists will be taken to be representative of the present method. The standard deviations of the three components of angular momentum of this movement, which is depicted in Fig. 2, were each 0.08 straight somersaults per flight time (ss/f.t.). Since the mean angular momentum estimate of this movement was 1.39 ss/f.t. (Table 2), the estimates of one component had

a standard deviation of about 6% of the total angular momentum. In the planar technique (Hay et al., 1977) the angular momentum estimates of a tucked forward somersault also had a standard deviation of 6% of the mean angular momentum.

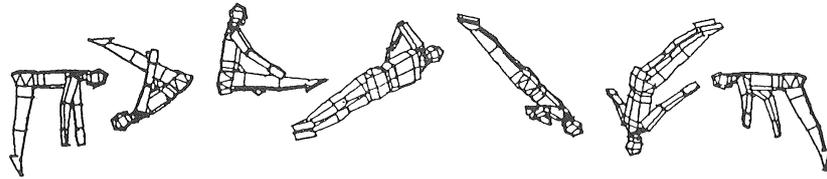


Figure 2: Graphics sequence generated from film of a double somersault with  $1\frac{1}{2}$  twists using the SAM-MIE model of Kingsley et al. (1981).

The standard deviations of the three components of the angular momentum of the movement shown in Fig. 2 were summed vectorally to produce a standard deviation vector whose magnitude was 10% of the total angular momentum. The corresponding value obtained for a Fosbury flop high jump using the three-dimensional method of Dapena (1978) was 20%.

The above comparisons suggest that the method described in this paper has a relative accuracy which is comparable with or better than other methods of calculating angular momentum. It should be recognized, however, that the normalized angular momentum values for the forward somersault and high jump were only 0.6 ss/f.t. and 0.3 ss/f.t. compared to 1.4 ss/f.t. for the double somersault with  $1\frac{1}{2}$  twists. In order to make a proper comparison of different methods the same film of the same movements would have to be used.

### The effect of anthropometric measurement errors

In order to determine the effect of anthropometric measurement errors on the calculated angular momentum estimates, subject C was measured twice. This produced two sets of segmental inertia parameter values which were input to the model together with the angle data obtained from film of the movement shown in Fig. 2. The two sets of angular momentum estimates were used to find the standard deviation of each set and the standard deviation of the differences between the sets. It was found that each set had a standard deviation of 10% of the mean momentum estimates while the difference between sets had a standard deviation of 3%. This indicates that anthropometric measurement errors have only a small effect on the calculated angular momentum.

### The effect of film digitization errors

The film from each of the two cameras was digitized twice to produce four combinations of film data in order to obtain four estimates of each orientation angle as described in part I. These four combinations comprise two sets of independent film data pairs. One of these sets was used to produce angular momentum estimates from each of the two film data pairs. Figure 3 shows the graphs of the three angular momentum components for the two pairs. It can be seen that there is considerable variation between the angular momentum estimates obtained from the two pairs. The standard deviations of the angular momentum estimates of the two pairs were 7% and 10% while the standard deviation of the difference between them was 7% of the mean momentum estimates. The same results were also obtained using the other set of independent film pairs. These measures indicate that much of the error in the momentum estimates arises from film digitization errors. Such errors in the location of joint centres introduce errors into the estimated orientation angles which are magnified when the first derivatives are calculated using quintic splines as described in part I (Yeadon, 1990a).

## SUMMARY

A method for the calculation of the angular momentum of the human body about its mass centre using film data has been described. A measure of the accuracy of the method is given by the standard error

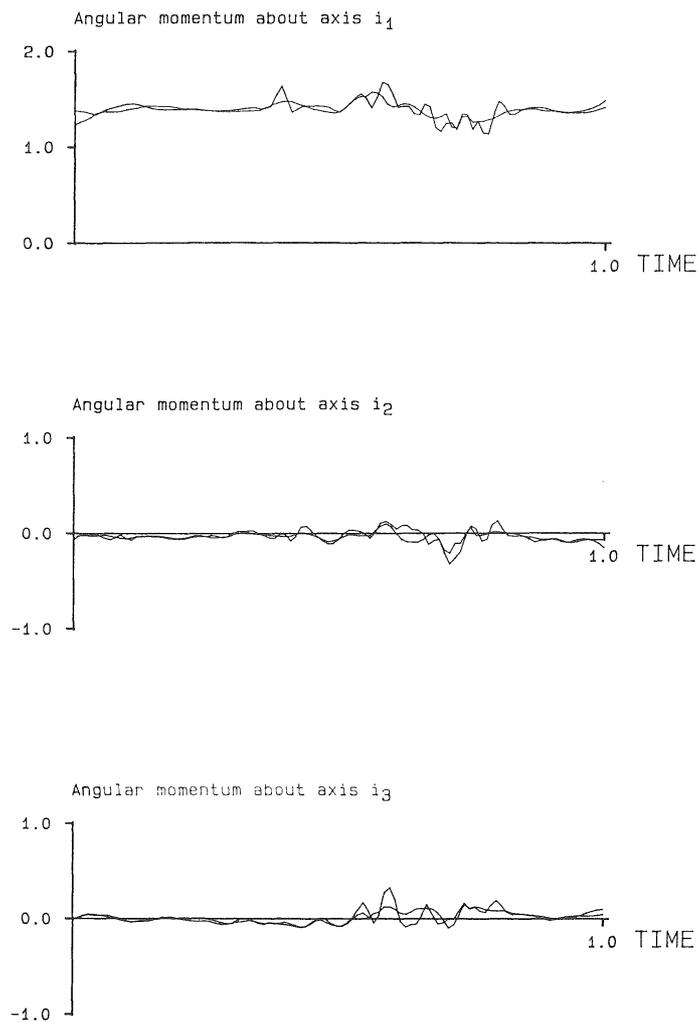


Figure 3: Estimates of the three components of angular momentum for the movement depicted in Fig. 2 using two independent sets of film data.

of the mean of the angular momentum estimates. This standard error is of the order of 1% of the mean angular momentum. It has been shown that much of the error in the angular momentum estimates arises from errors in the film digitizations. Such errors are probably due to the difficulty in identifying joint centres from 16 mm film of general three-dimensional movements since these joint centres are often obscured by body segments.

In addition to such measurement errors there are probably significant systematic errors arising from the assumptions used in modelling the human body. Theoretically it is possible to use parameter variation for such quantities as the segmental inertias to determine parameter values which minimize the variance in the calculated angular momentum estimates. Whether such a procedure is feasible with the present level of errors arising from the film data is unknown. The assumption that the shoulder centres are fixed in a rigid chest segment may introduce significant errors. Analytically it is not difficult to model independent shoulder movement but obtaining the corresponding angle values from film does pose a problem.

If the method is to be used for the calculation of the time history of the angular momentum in movements for which the conservation of angular momentum does not hold, the accuracy is of the order of 10%. If, on the other hand, the method is to be used to obtain a mean estimate of the angular momentum in movements for which the angular momentum remains constant, the accuracy is of the order of 1%. In the case of aerial movement angular momentum is conserved so that the method may

be considered to produce accurate estimates for input to the simulation model described in part IV of this series (Yeadon et al., 1990).

## References

- Dapena, J. (1978). A method to determine the angular momentum of a human body about three orthogonal axes passing its centre of gravity. *Journal of Biomechanics*, 11:251–256.
- Dapena, J. (1981). Simulation of modified human airborne movements. *Journal of Biomechanics*, 14:81–89.
- Greenwood, D. T. (1965). *Principles of Dynamics*. Prentice-Hall, Englewood Cliffs, NJ.
- Hay, J. G., Wilson, B. D., and Dapena, J. (1977). A computational technique to determine the angular momentum of a human body. *Journal of Biomechanics*, 10:269–277.
- Kane, T. R. (1968). *Dynamics*. Holt, Rinehard & Winston, New York.
- Kingsley, E. C., Schofield, N. A., and Case, K. (1981). SAMMIE: a computer aid for man machine modelling. *Computer Graphics*, 15:163–169.
- Miller, D. I. (1970). *A computer simulation model of the airborne phase of diving*. PhD thesis, Pennsylvania State University.
- Ramey, M. R. (1973). The use of angular momentum in the study of long jump take-offs. In Nelson, R. C. and Morehouse, C. A., editors, *Biomechanics IV*, pages 144–149. University Park Press, Baltimore.
- Ramey, M. R. and Yang, A. T. (1981). A simulation procedure for human motion studies. *Journal of Biomechanics*, 14:203–213.
- Van Gheluwe, B. (1981). A biomechanical simulation model for airborne twist in backward somersaults. *Journal of Human Movement Studies*, 7:1–22.
- Yeadon, M. R. (1990a). The simulation of aerial movement - I. The determination of orientation angles from film data. *Journal of Biomechanics*, 23:59–66.
- Yeadon, M. R. (1990b). The simulation of aerial movement - II. A mathematical inertia model of the human body. *Journal of Biomechanics*, 23:67–74.
- Yeadon, M. R., Atha, J., and Hales, F. D. (1990). The simulation of aerial movement - IV. A computer simulation model. *Journal of Biomechanics*, 23:85–89.

## Appendix 1 ORIENTATIONS OF THORAX AND THIGHS RELATIVE TO THE PELVIS

Flexion at the hip centres and flexion at the thorax-pelvis junction are assumed to be related in the following ways. Forward flexion occurs at the hips until an angle of  $90^\circ$  is reached, and beyond this angle the additional flexion is so that shared equally between the hip centres and thorax-pelvis junction. Sideways flexion and backwards flexion are shared equally at the two levels. The transition from one state to another is defined in such a way that there are no discontinuities in the flexion angles.

The orientations of the longitudinal axis  $\mathbf{t}_3$  of the thorax and the line  $\mathbf{l}_3$  (which is parallel to the line joining the midpoints of knee and hip centres) relative to the longitudinal axis  $\mathbf{p}_3$  of the pelvis are defined in terms of the hula angle  $\psi_p$ , and the pike angle  $\gamma$ . Thus there are two degrees of freedom in this part of the system.

The unit vectors  $\mathbf{p}_3$ ,  $\mathbf{t}_3$ ,  $\mathbf{l}_3$  of the pelvis, thorax and thighs are parallel to GX, OX and GO where G is the midpoint of the hip centres and X is the junction of the pelvis with the thorax (Fig. 4). The pike angle  $\gamma$  is the angle between GO and OX, that is between the thighs and thorax. The hula angle  $\psi_p$ , is

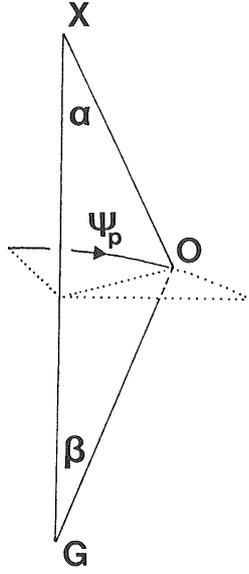


Figure 4: The pike angle GOX ( $\gamma$ ) and the hula angle  $\psi_p$ . (GX is the longitudinal axis of the pelvis, OX is an extension of the longitudinal axis of the thorax and GO is an extension of the mid-thigh line. )

the angle between the plane GOX and the plane which bisects the hip centres so that one cycle of  $\psi_p$  corresponds to one cycle of hip circumduction or hula-hoop movement.

The angle  $\alpha$  between  $\mathbf{p}_3$ , and  $\mathbf{t}_3$  is defined as a continuous function of  $\gamma$  and  $\psi_p$  as follows.

If  $\cos \psi_p > 0$  and  $\gamma > \frac{1}{2}\pi$  then  $\alpha = \frac{1}{2}(\pi - \gamma) \sin^2 \psi_p = \alpha_1$  so that

$$\dot{\alpha} = \frac{1}{2}\dot{\gamma} \sin^2 \psi_p + (\pi - \gamma) \sin \psi_p \cos \psi_p (\dot{\psi}_p) = \dot{\alpha}_1$$

If  $\cos \psi_p > 0$  and  $\gamma < \frac{1}{2}\pi$  then  $\alpha = \alpha_1 + \frac{1}{2}(\frac{1}{2}\pi - \gamma) \cos^2 \psi_p$ , so that

$$\dot{\alpha} = \dot{\alpha}_1 - \frac{1}{2}\dot{\gamma} \cos^2 \psi_p - (\frac{1}{2}\pi - \gamma) \cos \psi_p \sin \psi_p (\dot{\psi}_p).$$

If  $\cos \psi_p < 0$  then  $\alpha = \frac{1}{2}(\pi - \gamma)$  so that  $\dot{\alpha} = -\frac{1}{2}\dot{\gamma}$ .

The angle  $\beta$  between  $\mathbf{p}_3$  and  $\mathbf{l}_3$  is given by  $\beta = \pi - \gamma - \alpha$  so that  $\dot{\beta} = -\dot{\gamma} - \dot{\alpha}$ . Note that although  $\alpha$  and  $\beta$  are continuous across the boundaries  $\gamma = \frac{1}{2}\pi$  and  $\cos \psi_p = 0$  their derivatives  $\dot{\alpha}$  and  $\dot{\beta}$  are discontinuous at  $\gamma = \frac{1}{2}\pi$ .

The orientation of  $\mathbf{t}_3$  relative to the pelvis frame  $p$  may be expressed in terms of  $\alpha$  and  $\psi_p$ . The orientation of  $\mathbf{l}_3$ , relative to frame  $p$  may be expressed in terms of  $\beta$  and  $\psi_p$ .

## Appendix 2 THE REFERENCE FRAME $f$ OF THE LINK SYSTEM

The whole body orientation in space is specified by the orientation of a reference frame  $f$  in the body relative to a non-rotating frame  $i$ . The reference frame  $f$  comprises three orthogonal unit vectors  $\mathbf{f}_1$ ,  $\mathbf{f}_2$ ,  $\mathbf{f}_3$  which are defined as follows:

$\mathbf{f}_3$  is a unit vector parallel to the line joining the midpoint of knee centres to the midpoint of shoulder centres.

$\mathbf{f}_2$  is a unit vector parallel to the vector product  $\mathbf{f}_3 \times \mathbf{p}_1$  where  $\mathbf{p}_1$  is parallel to the line joining right and left hip centres.

$$\mathbf{f}_1 = \mathbf{f}_2 \times \mathbf{f}_3$$

The advantage of defining the system frame  $f$  in the above manner rather than choosing frame  $f$  to be fixed in a single body segment is illustrated in Fig. 5. This figure depicts a piked jump in which the total angular momentum is zero. If the axis  $\mathbf{f}_3$  were to be defined as the longitudinal axis of the upper trunk then the somersault angle would increase from  $0^\circ$  to  $90^\circ$  during the jump. By defining axis  $\mathbf{f}_3$  to be parallel to the knee-shoulder line the somersault angle remains much closer to zero during the jump. In this sense the system frame  $f$  is representative of the orientation of the whole body rather than one particular segment.

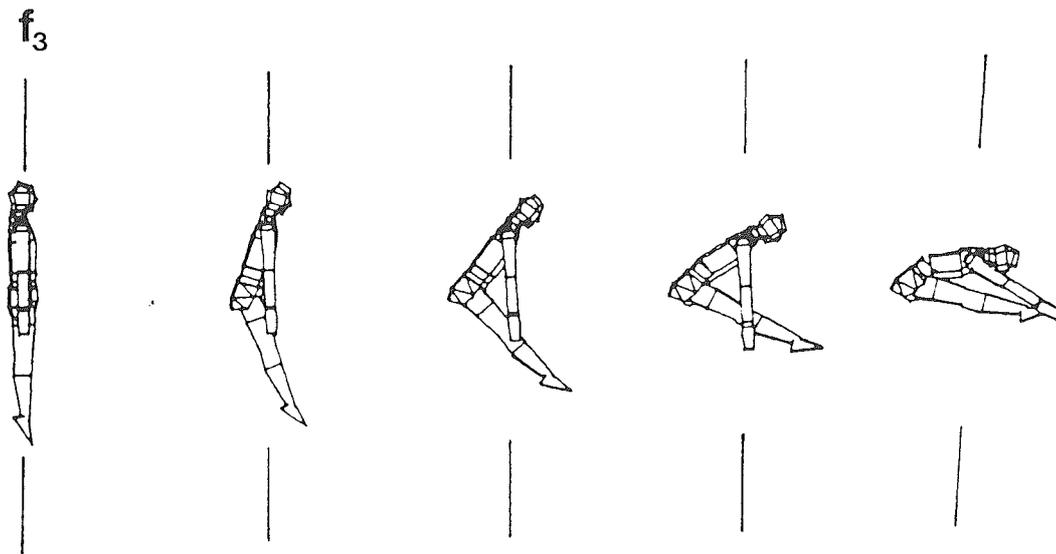


Figure 5: Orientation of the axis  $\mathbf{f}_3$  during a piked jump.