

# **The phenotype control kernel of a biomolecular regulatory network**

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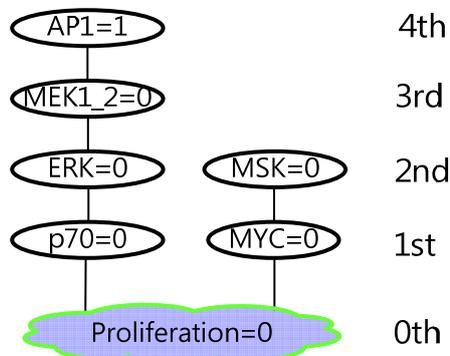
## **Supporting information**

### **Completion of construction of the converging tree of the simplified MAPK network**

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We have constructed the following converging tree of the simplified MAPK network up to Step 4 in the main text:



Step 5. Find children sets that directly generate the parent set  $\{AP1=1\}$  in the 4<sup>th</sup> level.

The candidates for control nodes are

JUN, FOS and ATF2

because of

$$AP1^* = JUN \& (FOS | ATF2).$$

Then the steady state value

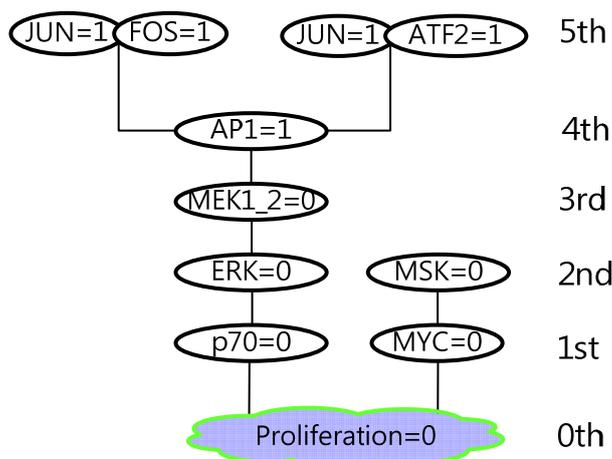
AP1=1

is directly generated by one of the perturbations

$$\{(JUN, FOS)=(1,1)\}, \{(JUN, ATF2)=(1,1)\}.$$

Applying the two removal rules, we find that no control set is removed up to the present level.

Therefore the two control sets consist of the 5<sup>th</sup> level. We have the following converging tree:



Step 6. Find children sets that directly generate each parent set in the 5<sup>th</sup> level.

The 5<sup>th</sup> level contains two control sets  $\{(JUN, FOS)=(1,1)\}$  and  $\{(JUN, ATF2)=(1,1)\}$ . We follow the two steps:

Step 6-1. Parent set  $\{(JUN, FOS)=(1,1)\}$  in the 5<sup>th</sup> level.

The candidates for control nodes are

JUN, FOS, JNK, ERK, RSK, ELK1 and CREB

because of the system of the two equations

$$JUN^* = JNK,$$

$$FOS^* = ERK \& RSK \& (ELK1 | CREB).$$

Then the steady state values

$$(JUN, FOS) = (1, 1)$$

are directly generated by one of the perturbations

$$\{(JUN, ERK, RSK, ELK1) = (1, 1, 1, 1)\},$$

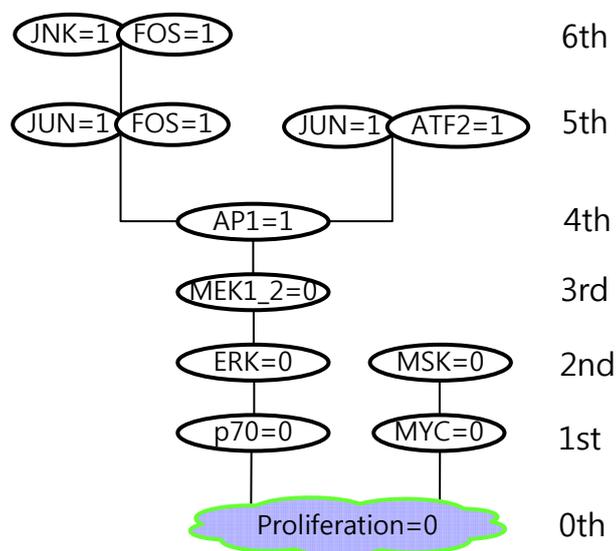
$$\{(JUN, ERK, RSK, CREB) = (1, 1, 1, 1)\},$$

$$\{(JNK, FOS) = (1, 1)\},$$

$$\{(JNK, ERK, RSK, ELK1) = (1, 1, 1, 1)\},$$

$$\{(JNK, ERK, RSK, CREB) = (1, 1, 1, 1)\},$$

which are removed except  $\{(JNK, FOS)=(1,1)\}$  when applying the second removal rule with  $\{ERK=0\}$  in the 2<sup>nd</sup> level. Therefore we have the following converging tree:



Step 6-2. Parent set  $\{(JUN, ATF2)=(1,1)\}$  in the 5<sup>th</sup> level.

The candidates for control nodes are

JUN, JNK, ATF2 and p38

because of the system of the two equations

$$JUN^* = JNK,$$

$$ATF2^* = JNK|p38.$$

Then the steady state values

$$(JUN, ATF2) = (1,1)$$

are directly generated by one of the perturbations

$$\{(JUN, JNK) = (1,1)\}, \{(JUN, p38) = (1,1)\},$$

$$\{(ATF2, JNK) = (1,1)\} \text{ and } \{JNK=1\},$$

where the control sets included in  $\{JNK=1\}$  are removed by the first removal rule and

so the remaining control sets are

$$\{(JUN, p38) = (1,1)\} \text{ and } \{JNK=1\}.$$

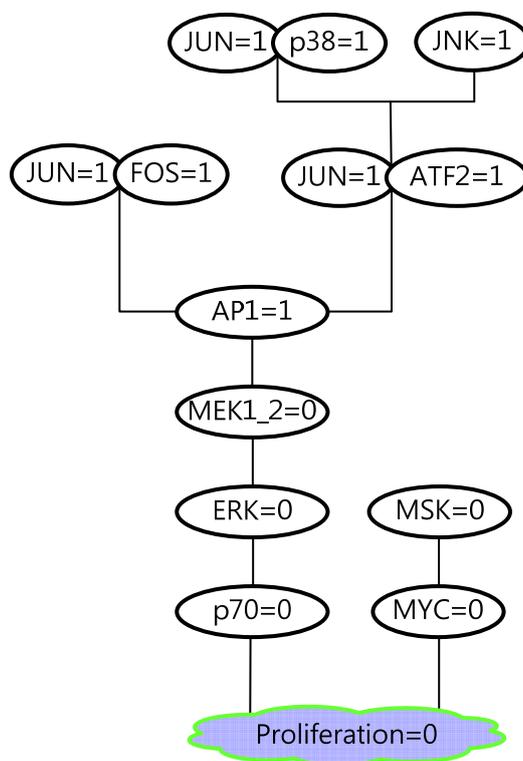
Since the child set  $\{(JNK, FOS)=(1,1)\}$  of the parent set  $\{(JUN, FOS)=(1,1)\}$  in Step 6-1 is also removed because of  $\{JNK=1\}$ ,

$$\{(JUN, FOS)=(1,1)\} \text{ becomes a leaf set in the 5}^{\text{th}} \text{ level.}$$

It follows from Step 6-1 and Step 6-2 that the control sets in the 6<sup>th</sup> level are

$$\{(JUN, p38) = (1,1)\} \text{ and } \{JNK=1\}.$$

Therefore we have the following converging tree:



Step 7. Find children sets that directly generate each control set in the 6<sup>th</sup> level.

Since there exist two control sets in the 6<sup>th</sup> level, we follow the two steps:

Step 7-1. Parent set  $\{(JUN, p38)=(1,1)\}$  in the 6<sup>th</sup> level.

The candidates for control nodes are

JUN, p38, JNK and DUSP1

because of the system of the two equations

$$\begin{aligned} \text{JUN}^* &= \text{JNK}, \\ \text{p38}^* &= !\text{DUSP1}. \end{aligned}$$

Then the steady state values

$$(\text{JUN}, \text{p38}) = (1, 1)$$

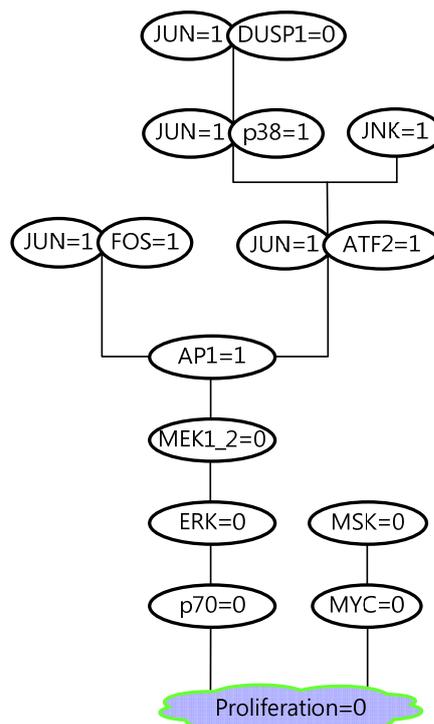
are directly generated by one of the perturbations

$$\begin{aligned} \{(\text{JUN}, \text{DUSP1}) = (1, 0)\}, \\ \{(\text{p38}, \text{JNK}) = (1, 1)\}, \\ \{(\text{JNK}, \text{DUSP1}) = (1, 0)\}, \end{aligned}$$

where the last two are removed when applying the first removal rule with  $\{\text{JNK}=1\}$  in the 6<sup>th</sup> level. Then the remaining control set is

$$\{(\text{JUN}, \text{DUSP1}) = (1, 0)\}.$$

Therefore we have the following converging tree:



Step 7-2. Parent set  $\{JNK=1\}$  in the 6<sup>th</sup> level.

The possible control node is

$$DUSP1$$

because of the equation

$$JNK^* = !DUSP1.$$

Then the steady state value  $JNK = 1$  is directly generated by  $\{DUSP1=0\}$ .

Applying the first removal rule, we find that the control set  $\{(JUN, DUSP1)=(1,0)\}$  in

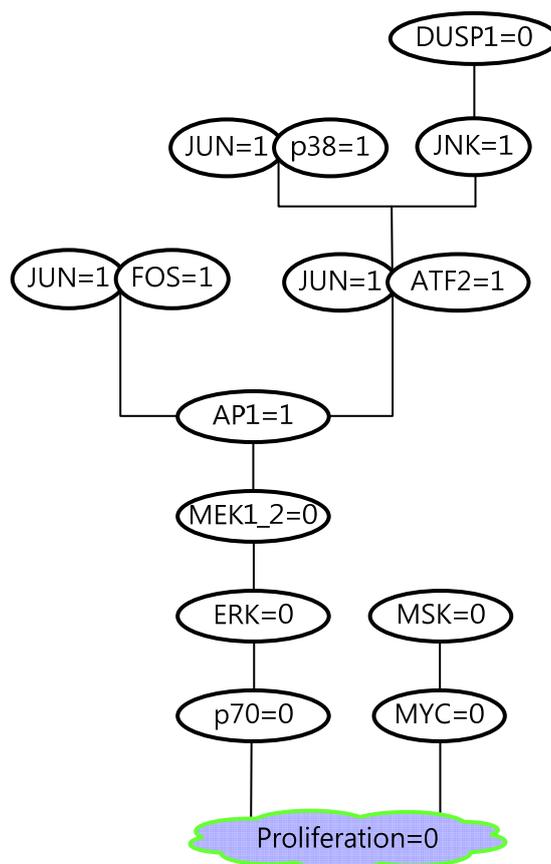
Step 7-1 is removed because of  $\{DUSP1=0\}$  and then

$$\{(JUN, p38)=(1,1)\} \text{ becomes a leaf set in the 6}^{\text{th}} \text{ level.}$$

It follows from Step 7-1 and Step 7-2 that the control set in the 7<sup>th</sup> level is

$$\{DUSP1=0\}.$$

Therefore we have the following converging tree:



Step 8. Find children sets that directly generate the parent set  $\{DUSP1=0\}$  in the 7<sup>th</sup> level.

The 7<sup>th</sup> level has only one parent set  $\{DUSP1=0\}$  with  $DUSP1 \neq CREB$  and then the possible control node is

CREB.

Then the steady state value

$DUSP1=0$

is directly generated by

$\{CREB=0\}$ ,

which is contradictory to the mutated value  $CREB=1$ . Then there exist no children sets of the parent  $\{DUSP1=0\}$  and hence

$\{DUSP1=0\}$  becomes a leaf set in the 7<sup>th</sup> level.

Finally, the converging tree is completed since any control set in the 7<sup>th</sup> level is a leaf set.

