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This Mathematica notebook is for checking the validity of the computation of detQ in Equation (13) in my paper

"Spectral analysis of the Integral operator arising from the beam deflection problem on elastic foundation: II eigenvalues".

1. Defining the functions e(x), c(x), s(x).

These functions in Section 3.1 will be here denoted by ee[x], cc[x], ss[x] respectively.

ee[x_] := Exp[κ α x]

cc[x_] := Cos[κ α x]

ss[x_] := Sin[κ α x]

Check to see they are well-defined:

ee[x]

$e^{x \alpha \kappa}$

cc[x]

$\text{Cos}[x \alpha \kappa]$

ss[x]

$\text{Sin}[x \alpha \kappa]$

2. Defining the matrix Q (denoted QQ here) in Section 3.1

**QQ = { { (κ^2 + Sqrt[2] κ + 1) ee[1], - (κ^2 - Sqrt[2] κ + 1) ee[-1],
- Sqrt[2] κ cc[1] + (κ^2 - 1) ss[1], - (κ^2 - 1) cc[1] - Sqrt[2] κ ss[1] },
{ (κ^2 - Sqrt[2] κ + 1) ee[-1], - (κ^2 + Sqrt[2] κ + 1) ee[1],
Sqrt[2] κ cc[1] - (κ^2 - 1) ss[1], - (κ^2 - 1) cc[1] - Sqrt[2] κ ss[1] },
{ (κ^3 - κ - Sqrt[2]) ee[1], - (κ^3 - κ + Sqrt[2]) ee[-1],
- Sqrt[2] cc[1] + (κ^3 + κ) ss[1], - (κ^3 + κ) cc[1] - Sqrt[2] ss[1] },
{ (κ^3 - κ + Sqrt[2]) ee[-1], - (κ^3 - κ - Sqrt[2]) ee[1],
Sqrt[2] cc[1] - (κ^3 + κ) ss[1], - (κ^3 + κ) cc[1] - Sqrt[2] ss[1] } }**

**{ { e^{1ακ} (1 + √2 κ + κ²), e^{-1ακ} (-1 + √2 κ - κ²), -√2 κ Cos[1 α κ] + (-1 + κ²) Sin[1 α κ],
(1 - κ²) Cos[1 α κ] - √2 κ Sin[1 α κ] }, { e^{-1ακ} (1 - √2 κ + κ²), e^{1ακ} (-1 - √2 κ - κ²),
√2 κ Cos[1 α κ] - (-1 + κ²) Sin[1 α κ], (1 - κ²) Cos[1 α κ] - √2 κ Sin[1 α κ] },
{ e^{1ακ} (-√2 - κ + κ³), e^{-1ακ} (-√2 + κ - κ³), -√2 Cos[1 α κ] + (κ + κ³) Sin[1 α κ],
(-κ - κ³) Cos[1 α κ] - √2 Sin[1 α κ] }, { e^{-1ακ} (√2 - κ + κ³), e^{1ακ} (√2 + κ - κ³),
√2 Cos[1 α κ] - (κ + κ³) Sin[1 α κ], (-κ - κ³) Cos[1 α κ] - √2 Sin[1 α κ] } }**

See QQ in matrix form:

MatrixForm[QQ]

$$\begin{pmatrix} e^{1\alpha\kappa} (1 + \sqrt{2} \kappa + \kappa^2) & e^{-1\alpha\kappa} (-1 + \sqrt{2} \kappa - \kappa^2) & -\sqrt{2} \kappa \text{Cos}[1 \alpha \kappa] + (-1 + \kappa^2) \text{Sin}[1 \alpha \kappa] & (1 - \kappa^2) \text{Cos}[1 \alpha \kappa] - \sqrt{2} \kappa \text{Sin}[1 \alpha \kappa] \\ e^{-1\alpha\kappa} (1 - \sqrt{2} \kappa + \kappa^2) & e^{1\alpha\kappa} (-1 - \sqrt{2} \kappa - \kappa^2) & \sqrt{2} \kappa \text{Cos}[1 \alpha \kappa] - (-1 + \kappa^2) \text{Sin}[1 \alpha \kappa] & (1 - \kappa^2) \text{Cos}[1 \alpha \kappa] - \sqrt{2} \kappa \text{Sin}[1 \alpha \kappa] \\ e^{1\alpha\kappa} (-\sqrt{2} - \kappa + \kappa^3) & e^{-1\alpha\kappa} (-\sqrt{2} + \kappa - \kappa^3) & -\sqrt{2} \text{Cos}[1 \alpha \kappa] + (\kappa + \kappa^3) \text{Sin}[1 \alpha \kappa] & (-\kappa - \kappa^3) \text{Cos}[1 \alpha \kappa] - \sqrt{2} \text{Sin}[1 \alpha \kappa] \\ e^{-1\alpha\kappa} (\sqrt{2} - \kappa + \kappa^3) & e^{1\alpha\kappa} (\sqrt{2} + \kappa - \kappa^3) & \sqrt{2} \text{Cos}[1 \alpha \kappa] - (\kappa + \kappa^3) \text{Sin}[1 \alpha \kappa] & (-\kappa - \kappa^3) \text{Cos}[1 \alpha \kappa] - \sqrt{2} \text{Sin}[1 \alpha \kappa] \end{pmatrix}$$

We split it in case you cannot see the whole:

MatrixForm[QQ[[All, 1 ;; 2]]]

$$\begin{pmatrix} e^{1\alpha\kappa} (1 + \sqrt{2}\kappa + \kappa^2) & e^{-1\alpha\kappa} (-1 + \sqrt{2}\kappa - \kappa^2) \\ e^{-1\alpha\kappa} (1 - \sqrt{2}\kappa + \kappa^2) & e^{1\alpha\kappa} (-1 - \sqrt{2}\kappa - \kappa^2) \\ e^{1\alpha\kappa} (-\sqrt{2} - \kappa + \kappa^3) & e^{-1\alpha\kappa} (-\sqrt{2} + \kappa - \kappa^3) \\ e^{-1\alpha\kappa} (\sqrt{2} - \kappa + \kappa^3) & e^{1\alpha\kappa} (\sqrt{2} + \kappa - \kappa^3) \end{pmatrix}$$

MatrixForm[QQ[[All, 3 ;; 4]]]

$$\begin{pmatrix} -\sqrt{2}\kappa \cos[1\alpha\kappa] + (-1 + \kappa^2) \sin[1\alpha\kappa] & (1 - \kappa^2) \cos[1\alpha\kappa] - \sqrt{2}\kappa \sin[1\alpha\kappa] \\ \sqrt{2}\kappa \cos[1\alpha\kappa] - (-1 + \kappa^2) \sin[1\alpha\kappa] & (1 - \kappa^2) \cos[1\alpha\kappa] - \sqrt{2}\kappa \sin[1\alpha\kappa] \\ -\sqrt{2} \cos[1\alpha\kappa] + (\kappa + \kappa^3) \sin[1\alpha\kappa] & (-\kappa - \kappa^3) \cos[1\alpha\kappa] - \sqrt{2} \sin[1\alpha\kappa] \\ \sqrt{2} \cos[1\alpha\kappa] - (\kappa + \kappa^3) \sin[1\alpha\kappa] & (-\kappa - \kappa^3) \cos[1\alpha\kappa] - \sqrt{2} \sin[1\alpha\kappa] \end{pmatrix}$$

3. Compute the determinant of Q.

Simplify[Det[QQ]]

$$4 e^{-21\alpha\kappa} \left((1 - 4\kappa^2 + \kappa^4) (1 - 2\sqrt{2}\kappa + 4\kappa^2 - 2\sqrt{2}\kappa^3 + \kappa^4 + e^{41\alpha\kappa} (1 + 2\sqrt{2}\kappa + 4\kappa^2 + 2\sqrt{2}\kappa^3 + \kappa^4)) \cos[21\alpha\kappa] + 2 \left(-e^{21\alpha\kappa} (1 + \kappa^4)^2 + \kappa (-1 + \kappa^2) (\sqrt{2} - 4\kappa + 4\sqrt{2}\kappa^2 - 4\kappa^3 + \sqrt{2}\kappa^4 + e^{41\alpha\kappa} (\sqrt{2} + 4\kappa + 4\sqrt{2}\kappa^2 + 4\kappa^3 + \sqrt{2}\kappa^4)) \right) \sin[21\alpha\kappa] \right)$$

$21\alpha \rightarrow L$:

Simplify[Det[QQ]] /. $\alpha \rightarrow L / (21)$

$$4 e^{-L\kappa} \left((1 - 4\kappa^2 + \kappa^4) (1 - 2\sqrt{2}\kappa + 4\kappa^2 - 2\sqrt{2}\kappa^3 + \kappa^4 + e^{2L\kappa} (1 + 2\sqrt{2}\kappa + 4\kappa^2 + 2\sqrt{2}\kappa^3 + \kappa^4)) \cos[L\kappa] + 2 \left(-e^{L\kappa} (1 + \kappa^4)^2 + \kappa (-1 + \kappa^2) (\sqrt{2} - 4\kappa + 4\sqrt{2}\kappa^2 - 4\kappa^3 + \sqrt{2}\kappa^4 + e^{2L\kappa} (\sqrt{2} + 4\kappa + 4\sqrt{2}\kappa^2 + 4\kappa^3 + \sqrt{2}\kappa^4)) \right) \sin[L\kappa] \right)$$

4. Comparing with Equation (13) in the paper.

Equation (13):

$$4 e^{L\kappa} \left(-2 e^{-L\kappa} (1 + \kappa^4)^2 + (1 + 2\sqrt{2}\kappa + 4\kappa^2 + 2\sqrt{2}\kappa^3 + \kappa^4 + e^{-2L\kappa} (1 - 2\sqrt{2}\kappa + 4\kappa^2 - 2\sqrt{2}\kappa^3 + \kappa^4)) \left((1 - 4\kappa^2 + \kappa^4) \cos[L\kappa] + 2\sqrt{2}\kappa (-1 + \kappa^2) \sin[L\kappa] \right) \right) + 4 e^{L\kappa} \left(-2 e^{-L\kappa} (1 + \kappa^4)^2 + (1 + 2\sqrt{2}\kappa + 4\kappa^2 + 2\sqrt{2}\kappa^3 + \kappa^4 + e^{-2L\kappa} (1 - 2\sqrt{2}\kappa + 4\kappa^2 - 2\sqrt{2}\kappa^3 + \kappa^4)) \left((1 - 4\kappa^2 + \kappa^4) \cos[L\kappa] + 2\sqrt{2}\kappa (-1 + \kappa^2) \sin[L\kappa] \right) \right)$$

Check that it is correct:

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Simplify[4 e^{L x} (-2 e^{-L x} (1 + x^4))^2 +
  (1 + 2 sqrt(2) x + 4 x^2 + 2 sqrt(2) x^3 + x^4 + e^{-2 L x} (1 - 2 sqrt(2) x + 4 x^2 - 2 sqrt(2) x^3 + x^4))
  ((1 - 4 x^2 + x^4) Cos[L x] + 2 sqrt(2) x (-1 + x^2) Sin[L x])] -
(Simplify[Det[QQ]] /. alpha -> L / (2 l)) ]

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