

# Ultra-broadband dual-branch optical frequency comb with $10^{-18}$ instability: supplementary material

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Published 30 August 2018

This document provides supplementary information to "Ultra-broadband dual-branch optical frequency comb with  $10^{-18}$  instability, <https://doi.org/10.1364/optica.5.001070>. We present the characterization of the visible branch when used in the single-branch configuration. We give a more detailed version of the electrical detection. Finally we talk about how the measurement has been acquired and how the data are interpreted.

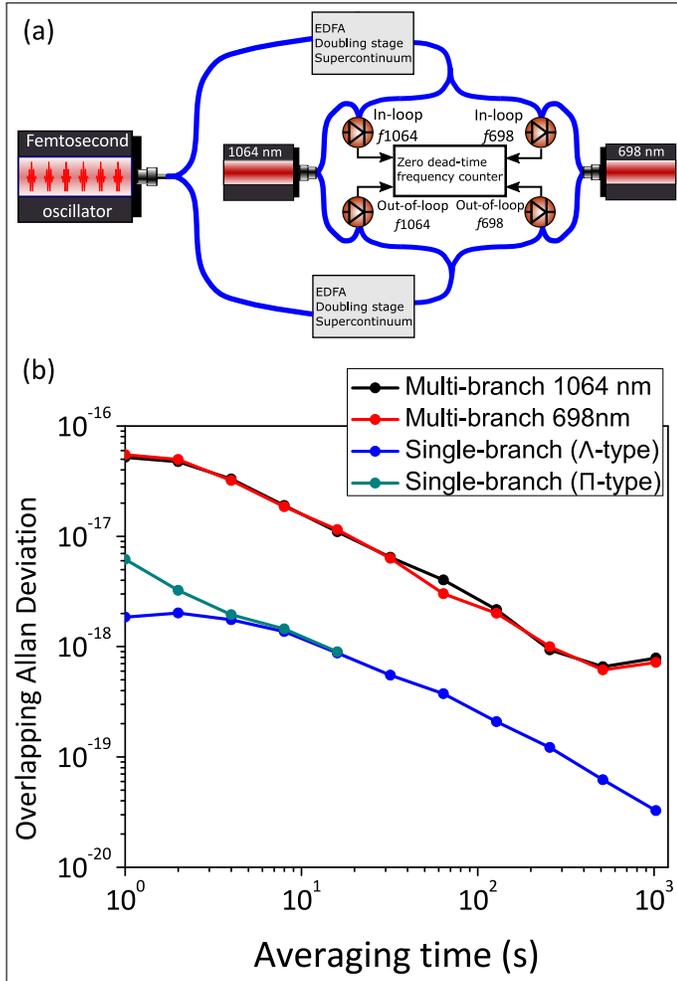
## 1. SINGLE-BRANCH VISIBLE FREQUENCY COMB

We describe in this first section the performance of the visible comb generation in dual- and single-branch operation. Using an f-2f interferometer generated through the visible comb, we detect the  $f_{ceo}$  with a signal-to-noise ratio (SNR) of > 35 dB in 100 kHz RBW in the visible domain. As an additional check of coherence we further detect two beatnotes at 698 nm (wavelength corresponding to the strontium lattice clock transition) and at 1064 nm. In order to measure the residual instability of the visible comb, we set up a dual branch system (Figure S1 (a)) using two broadband visible continua seeded with one common comb oscillator. The first branch is configured for stabilization of the comb with the 1064 nm reference laser and stabilization of the 698 nm laser with the comb simultaneously. The 698 nm laser is then phase locked to the 1064 nm laser through the frequency comb. The second branch is used for out-of-loop measurements. We have measured a SNR higher than 40 dB for both wavelengths in 100 kHz RBW. The power per mode at 698 nm is 20 nW leading to a white phase noise floor that is shot noise limited at -104 dBc/Hz. We have measured, in post-processing, the phase noise of the ratio (virtual beatnote between the two CW lasers) on both branches. The out-of-loop phase noise exhibits a phase noise lower than -60 dBc/Hz at 1 s which is far below the phase noise of the best local oscillator based on a cryogenic optical cavity. We then measure the beatnote with a multi-channel zero-dead-time frequency counter (K+K FXE counter) in rectangular and triangular weighing functions ( $\Pi$  and  $\Lambda$  mode). We obtain a residual instability of  $6 \times 10^{-17}$  at 1 s integration

time reaching  $2 \times 10^{-18}$  for both wavelengths at 10 s (accuracy of the best optical clock). This level is limited to first order by differential phase noise between the two non-common optical path, induced by acoustic noise and temperature fluctuations. In the context of comparing two optical clocks as well as spectral purity transfer in the visible domain we report data of a single branch frequency comb modified for operation in the visible. As the comb branch noise is correlated for both wavelengths, the residual noise of the frequency ratio (comb agreement) is reduced to  $1.8 \times 10^{-18}$  at 1 s ( $\Lambda$  mode) and reaches  $3 \times 10^{-20}$  after only 1000 s. The high SNRs achieved here leads to an instability limit of  $6 \times 10^{-18}$  at 1 s integration time with a  $\Pi$ -type counting (see Figure S1 (b)). A statistical analysis of repeated measurements leads to a ratio agreement lower than  $1 \times 10^{-19}$  (more than an order of magnitude below the best optical lattice clock uncertainty).

## 2. FREQUENCY COUNTING

An Allan deviation is defined for a frequency counter using a rectangular weighting function ( $\Pi$ -type frequency counter). In optical frequency standards the atomic reference typically contributes information only over times larger than a few seconds, corresponding to a bandwidth < 1 Hz. An enhanced-resolution  $\Lambda$ -type frequency counter is then preferable in our case. The measurement bandwidth with a triangular weighting function is reduced drastically to the inverse of the gate time (0.5 Hz measurement bandwidth for a 1 s gate time) rejecting the white phase noise. The theoretical limit of the 1 s instability against



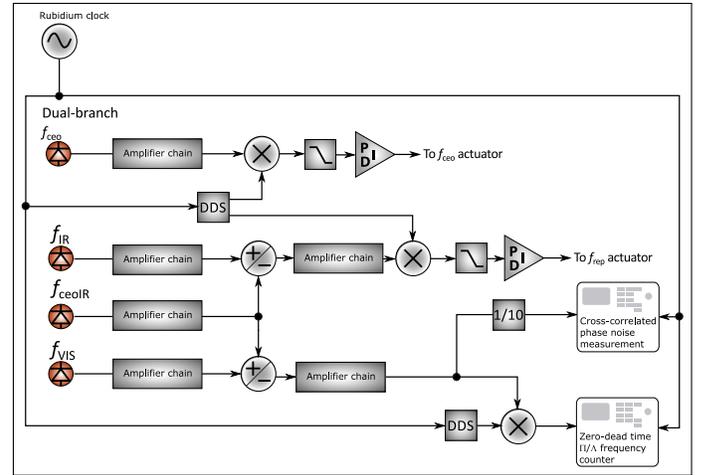
**Fig. S1.** (a) Experimental setup used to characterize the visible optical frequency comb generation in dual- and single-branch configuration. EDFA: erbium doped optical amplifier. (b) Out-of-loop fractional frequency instability of the 698 nm and 1064 nm beatnote with the visible comb in dual-branch configuration (red and black curve) and the frequency ratio (blue and green curve,  $\Lambda$  and  $\Pi$ , respectively).

the white phase noise with a  $\Lambda$ -type frequency counter is then reduced by a factor of  $\sqrt{n}$ ,  $n$  being the sampling rate of the averaging within the interval of the counter. A sampling rate of 1 kHz (1 ms) in 1 s gate time will then reduce the instability by a factor of  $\sqrt{1000}$ . We experimentally checked this reduction by counting an RF signal dominated by white phase noise. The experimental numbers follow well the theoretical one within the frequency counter resolution. It is very important to stress on those aspects when reporting instability. However, we think that the figure of merit of a spectral purity transfer is the power spectral density of phase noise, which really illustrates phase differences. In the paper, we report the fractional instability in terms of Allan Deviation. We use the  $\Lambda$ -type frequency counter. The intrinsic nature of a triangular weighing function is that the phase difference is being low-pass filtered with a bandwidth that depends on the sampling rate as well as the gate time of the counter. Note that a low signal-to-noise ratio on the optical beatnotes will limit the achievable bandwidth for the spectral purity transfer and pre-stabilization of a slave laser would be

required.

### 3. ELECTRICAL PATH

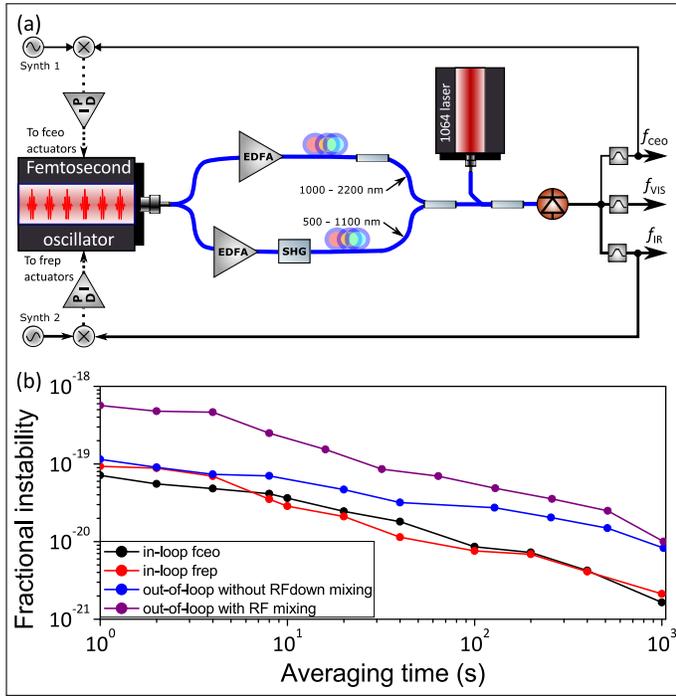
At instability levels  $< 1 \times 10^{-18}$  non common electrical path can limit the measurement. We show on figure S2 the electrical detection and processing of the four optical beatnotes used for the spectral purity transfer from infrared to visible. The beatnotes are independently filtered and amplified. The infrared and visible optical beatnotes are down/up mixed with the infrared  $f_{ceoIR}$  and the output signal is re-filtered and re-amplified. The two errors signals are generated with a common direct digital synthesizer minimizing greatly the servo locking errors. The visible out-of-loop beatnote is measured using two measurement setups. We divide by 10 the beatnotes to measure the beatnote with a cross correlated phase noise analyzer setup (we then need to add 20 dB to the measured phase noise). We use the Microsemi 5125A phase noise test set. The noise floor was around  $5 \times 10^{-14}$  at 10 MHz and  $1 \times 10^{-21}$  when scaled to an optical frequency at 1 s averaging time.



**Fig. S2.** Setup of the electrical detection and processing of the optical beatnote comprising the generation of error signals, and the noise characterization of the out-of-loop beatnote.

To validate the concept, we use a 1064 nm CW reference laser that we combine with the two branches on a common photodetector in order to only be limited by the new technique (see Fig. S3 (a)). The photodetector provides three RF signals:  $f_{IR}$ ,  $f_{VIS}$ , and  $f_{ceo}$ , which are distinguished via selection of different RF frequencies. We use the  $f_{IR}$  beatnote to phase lock the comb to the 1064 nm CW laser with an intracavity electro-optic modulator. The in-loop instability is  $9 \times 10^{-20}$  at 1 s integration time. We then analyzed in the time domain the additional out-of-loop beatnote between the visible branch and the 1064 nm CW laser (note that this beatnote is equal to  $f_{IR} + f_{ceo}$ ). We choose, at first, to eliminate the RF down-mixing as it is possible due to the fact that mode numbers are equal. The instability level is  $1 \times 10^{-19}$  at 1 s and averages down to  $9 \times 10^{-21}$  at 1000 s (see Fig. S3 (b)). It is worthwhile to notice that the instability degradation here is due to the electrical non common path leading to a lower limit of optical synthesis. We have used great care at making the non common path very short and especially by not using any cables. We then introduce the down-mixing to fully characterize the reported technique even in the case of equal mode numbers. Hence, the present technique shows an improvement by more

than 3 orders of magnitude. We can notice that the instability increase to  $6 \times 10^{-19}$  at 1 s averaging time and around  $1 \times 10^{-20}$  at 1000 s. The RF mixing induces conversion loss that will ultimately limit the instability level. Finally, higher order noise (dispersion, refractive index difference), counter resolution and electronics will now need to be considered to establish the new performance limits of this new comb.



**Fig. S3.** (Experimental setup to estimate the limits induced by the electrical detection and processing of the coherent stitching of the two supercontinua. (b) Fractional instability in terms of Allan deviation of the in-loop servo locking errors of  $f_{ceo}$  and  $f_{rep}$ ), the out-of-loop measurement with RF down mixing (purple curve) and without RF down mixing (blue curve). See text for details.