Design of illumination optics with extended sources based on wavefront tailoring: supplementary material

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This document provides supplementary information to “Design of illumination optics with extended sources based on wavefront tailoring.” https://doi.org/10.1364/OPTICA.6.000966. Section 1 gives a proof of the polynomial forms used for the optical momentum functions \( p_{\min} \) and \( q_{\min} \). Section 2 provides a detailed explanation of the SMS calculation of a 2-surface lens, using the output wavefronts determined in the main article via the generalized wavefront tailoring approach.

1. POLYNOMIAL EXPRESSIONS OF OPTICAL MOMENTUM COMPONENTS \( p_{\min} \) AND \( q_{\min} \)

In this section we demonstrate how to obtain the polynomial forms presented in Eq. (15) of the primary manuscript, used for the construction of functions \( p_{\min}(x,y) \) and \( q_{\min}(x,y) \) (and, from these, of \( p_{\max}(x,y) \) and \( q_{\max}(x,y) \)). The starting point is the first equation in Eqs. (12) of the primary manuscript, which we report here:

\[
q_{\min}(x,y) = \int_{x_{i}}^{y} \left( \frac{\partial p_{\min}(t,y)}{\partial y} \right) \, dt + g_{1}(y) \quad (S1)
\]

This equation expresses the fundamental condition \( \partial q / \partial x = \partial p / \partial y \) applied to \( p_{\min} \) and \( q_{\min} \). We assume that \( p_{\min}(x,y) \) can be written in the polynomial shape

\[
p_{\min}(x,y) = \sum_{n=0}^{\infty} a_{mn} x^{m} y^{n} = \sum_{n=0}^{\infty} a_{mn} x^{m} y^{n} + \sum_{m=0}^{\infty} b_{m} x^{m}
\]

(S2)

Application of Eq. (S1) to Eq. (S2) yields the following expression for \( q_{\min}(x,y) \)

\[
q_{\min}(x,y) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_{mn} \frac{n}{m+1} x^{m+1} y^{n-1} + \sum_{n=0}^{\infty} c_{n} y^{n}
\]

assuming, for the sake of consistency, that \( g_{1}(x) \) (the fraction of \( q_{\max}(x,y) \) depending only on \( x \)) can be expressed as a univariate polynomial \( \sum_{m=0}^{\infty} c_{n} x^{n} \). Such definitions for \( p_{\min} \) and \( q_{\min} \) ensure that they satisfy condition Eq. (S1); in other words, together they define one wavefront.

The symmetries of the design system (with respect to planes \( x = \pm y \), \( y = 0 \) and \( x = 0 \)) further constrain Eq. (S2) and Eq. (S3) as follows. Symmetry with respect to \( x = \pm y \) implies that \( p(x, y) = q(y, x) \). This is achieved for the above polynomials by making \( m = n + 1 \) and \( c_{n} = b_{m} \) for every \( m \). Then, symmetry with respect to planes \( x = 0 \) and \( y = 0 \) implies that \( p(x, y) = p(-x, y) \) and \( q(x, y) = q(-y, x) \). This is obtained on \( p_{\max}(x,y) \) and \( q_{\max}(x,y) \) by having even powers on \( y \) and \( x \), respectively. Altogether, these conditions lead to the final form for the two polynomials

\[
p_{\min}(x,y) = \sum_{n=0}^{\infty} a_{mn} x^{2n+1} y^{2n+2} + \sum_{m=0}^{\infty} b_{m} x^{m}
\]

\[
q_{\min}(x,y) = \sum_{n=0}^{\infty} a_{mn} x^{2n+1} y^{2n+1} + \sum_{m=0}^{\infty} b_{m} y^{m}
\]

(S4)

In conclusion, use of polynomials like Eq. (S4) for the components \( p \) and \( q \) ensures that the resulting optical momentum functions define a proper wavefront. Plus, the geometrical symmetries of the target emission pattern are automatically taken into account.

2. SMS CALCULATION OF THE 2-SURFACE LENS

The SMS method in 3D (SMS3D, [1]) is used for design of the lens. The chosen refractive index of the material is 1.5848 (Polycarbonate at 587 nm wavelength). The freeform wavefronts \( W F_{k} \), \( k = 1, 2, 3 \) and 4, generated from functions \( p_{\max}, p_{\max}, q_{\max}, q_{\min} \) serve as the output wavefronts for the calculation. They are defined over the \( 10 \times 10 \text{mm}^{2} \) reference exit aperture placed at \( z = 0 \) (see Fig. S1(a)). The square 1-mm-side source is placed at \( z = -4.2 \text{ mm} \). These initial parameters are chosen to obtain a highly compact configuration. The input wavefronts are instead four spherical wavefronts \( W F_{k} \), \( k = 1, 2, 3 \) and 4, stemming from the four source corners (as indicated in Fig. S1(a)). The SMS3D design starts with the calculation of a starting curve (“seed rib”, [1]). This is achieved by imposing the couplings of the wavefront \( W F_{1} \) with
WF3, and of WF3, with WF1. The resulting seed rib is visible in Fig. S1(a). From this initial curve, the full 3D SMS process is performed by requiring the couplings of wavefront WF2 with WF4i, and of WF4 with WF2o. Several curves, called "SMS chains" [1], are calculated in this phase (the first ones are shown in Fig. S1(a)). They are subsequently fitted with NURBS surfaces to obtain the final SMS surfaces forming the lens (Fig. S1(b)). During the SMS calculation, symmetry with respect to planes $x = 0$ and $y = 0$ is forced onto the points forming the SMS curves, to comply with the symmetry of the target pattern and of the system geometry.

A comment on the application of the SMS method to this design case is in order. Rigorously, with two surfaces the SMS3D method allows to perfectly couple only two pairs of input-output wavefronts. Here, we have calculated a 2-surface lens by requiring the system to simultaneously couple four pairs of wavefronts. It has indeed been shown that, in some circumstances, the SMS method achieves partial control over wavefronts used only in the preliminary calculation of the seed rib (and not entering the full 3D calculation), see for instance [2, 3]. In the present case, because of the symmetry of the problem, the output wavefronts WFko can be transformed into each other by a 90° rotation about the $z$ axis. The same applies to the input wavefronts WFki. By forcing this symmetry on the SMS surfaces, we obtain the same degree of control over wavefront pairs WF2, WF4, and WF4, WF2o (used in the full 3D calculation) and over wavefront pairs WF1i, WF3i, and WF3, WF1o (used for the calculation of the starting seed rib). Having just two surfaces, the four wavefront couplings cannot be perfect. In the case under consideration, though, we can accept a slight reduction in wavefront control: in exchange, we get an extremely compact lens configuration with just two optical surfaces. Ray tracing simulations, reported in the primary manuscript, demonstrate that the designed lens still performs very close to the required illumination functionality.

Fig. S1. (a) Geometry of the SMS calculation: position of the source, pictorial representation of the input and output wavefronts, first SMS chains calculated from the seed rib. The size of the reference exit aperture and of the wavefronts is not to scale. (b) The final SMS lens.

References