1. SINGLE PHOTON PURITY AND INDISTINGUISHABILITY

The single photon purity of the source is deduced from the $g^{(2)}(\Delta t)$ histograms shown in Figs. S1(a,b) for the normal repetition rate of the laser, 81 MHz, and for the ×4 increased repetition rate used for the experiment. In this second case, the limited time resolution of our detectors (≈700 ps) results in some temporal overlap of the neighbouring peaks. We deduce the values of $g^{(2)}(0)$ by integrating the coincidence counts over a 2 ns time bin. The values deduced from the histograms presented in Figs. S1(a,b) are respectively $g^{(2)}(0)=0.035\pm0.003$ and $g^{(2)}(0)=0.071\pm0.003$, respectively.

For the three photon experiments, we use photons delayed by respectively ≈48 ns and ≈98 ns. We thus study the photon indis-
tistinguishability for various delays. This is achieved by interfering in an unbalanced Mach-Zehnder interferometer two photons delayed by 12, or 98 ns, rendering the correlation counts presented in Figs. S1 (c,d), respectively (see Ref. 11 of the main text for further details). The raw Hong-Ou-Mandel visibility \( V_{\text{HOM}} \) and corrected mean-wavepacket overlap deduced from the histograms are respectively \( V_{\text{HOM}}^{\text{12ns}} = 0.850 \pm 0.007 \), \( M_{12\text{ns}} = 0.920 \pm 0.007 \) and \( V_{\text{HOM}}^{\text{98ns}} = 0.810 \pm 0.009 \), \( M_{98\text{ns}} = 0.880 \pm 0.009 \).

2. EXPERIMENTAL CHARACTERIZATION OF THE PHOTONIC TRITTER

The experimental unitary matrix of the tritter \( U_{\text{EXP}} \) is determined following the method described in Ref. [1]. In this procedure, the absolute value of the matrix element \( |U_{jk}^{\text{EXP}}| \) is determined by inserting a continuous-wave laser (set at the same wavelength as the QD emission) in the tritter input \( j \) and measuring the laser intensity in each tritter output \( k \).

The four phases \( \theta_{jk} \) of the matrix elements \( |U_{jk}^{\text{EXP}}|e^{i\theta_{jk}} \), with \( jk = \{22, 23\} \) \( jk = \{32, 33\} \), are measured by splitting the laser beam in two paths and connecting them to the in inputs \( j = 1, 2 \) \( j = (1, 2, 3) \); the variation of the laser phase between the two inputs (for example, slowly displacing the position of one of the input fiber couplers) produces oscillations in the intensity outputs \( k = 1, 2, 3 \). Setting the oscillation of output 1 as a reference, the phase shift with respect to outputs 2 and 3 will determine the experimental phase values \( \theta_{22}, \theta_{23} \) \( \theta_{32}, \theta_{33} \). For the sake of clarity and as an example, we show in Fig. S2 the measure of \( \theta_{22}, \theta_{23} \); the laser light has been injected in inputs 1 and 2, the relative phase shift between the two inputs produce constant oscillations in the three output modes intensities, from where the matrix phases are directly extracted.

We deduce the following experimental matrix:

\[
U_{\text{EXP}} = \frac{1}{\sqrt{3}} \begin{pmatrix}
0.971 & 1.011 & 0.979 \\
1.023 & 0.950 \times e^{2\pi i 0.322} & 1.028 \times e^{2\pi i 0.656} \\
1.005 & 1.036 \times e^{2\pi i 0.670} & 0.993 \times e^{2\pi i 1.331}
\end{pmatrix}
\]

(S1)

The average errors of \( |U_{jk}^{\text{EXP}}| \) and \( \theta_{jk} \) are less that 0.1% and 1%, respectively.

Following the procedure described in Ref. [2], the fidelity of the experimental tritter to the ideal matrix \( U_{\text{th}} \) is calculated as

\[
F = 1 - \sum_{i<j} \sum_{k<l} |\langle V_{ijkl} | U_{ijkl}^{\text{th}} | V_{ijkl} \rangle|^2 / 18,
\]

where the visibility \( V_{ijkl} \) is given by the ratio \( V_{ijkl} = (P_{ijkl}^{C} - P_{ijkl}^{Q}) / P_{ijkl}^{C} \) and each of these probabilities are \( P_{ijkl}^{C} = |U_{ijkl}^{C}|^2 + |U_{ijkl}^{L}|^2 \) and \( P_{ijkl}^{Q} = |U_{ijkl}^{Q}|^2 + |U_{ijkl}^{Q}|^2 \).

3. MODEL OF IMPERFECT SINGLE PHOTON PURITY

In this section we derive the fraction of residual laser light in the source. As discussed in the main text, the imperfect single photon purity reveals the presence of scattered laser photons mixed with the single photon emission arising from the quantum dot (QD). We make the assumption that the presence of this two photon distributions in the collection fiber (and in the subsequent parts of the tritter setup) are independent and statistically mixed. Under such conditions, the photon-number probability distribution of the laser and QD mixture can be analytically reconstructed by the probability generating function formalism [3].

The probability generating function is defined as

\[
G(s) = \sum_{n} s^n p_n,
\]

where \( p_n \) are the different photon number Fock states probabilities. For an statistical mixture of two independent photon distributions (laser and QD), the probability generating function of the mixed ensemble, \( G_{\text{mix}}(s) \), is calculated as the product of the corresponding laser and QD components, \( G_{\text{L}}(s) \) and \( G_{\text{Q}}(s) \), respectively, i. e., \( G_{\text{mix}}(s) = G_{\text{L}}(s) G_{\text{Q}}(s) \). The QD single photon emission is

\[
G_{\text{Q}}(s) = p_0^{\text{Q}} + p_1^{\text{Q}} s = (1 - \mu_{\text{QD}}) + p_{0Q} s
\]

where \( \mu_{\text{QD}} = p_{1Q} \) is the average photon number of the QD emission, and the laser...
is \( G_{L}(s) = \sum_{n=0}^{\infty} s^{n} p_{n}^{L} = \sum_{n=0}^{\infty} s^{n} \frac{\mu_{L}^{n} e^{-\mu_{L}}}{n!} = e^{-\mu_{L}(1-s)} \) (where \( \mu_{L} \) is the average photon number of the scattered laser).

The analytical expression of the second order correlation of the ensemble, \( g_{tot}^{(2)}(0) \) is given by
\[ g_{tot}^{(2)}(0) = \frac{\mu_{tot}}{2(\mu_{tot})^{2}} \frac{d}{ds} G_{tot}(s) \bigg|_{s=1} = \left( \frac{\mu_{L} + \mu_{QD}}{2} \right)^{2} \] (where the total average photon number is \( \mu_{tot} = \mu_{L} + \mu_{QD} \)). The resulting second order correlation function is
\[ g_{tot}^{(2)}(0) = \chi(2 + \chi)/(1 + \chi)^{2}, \] where \( \chi = \mu_{L}/\mu_{QD} \), see dark blue trace in Fig. S3.

**Fig. S3.** Second order correlation function \( g_{tot}^{(2)}(0) \) versus the ration of average photon number of laser and QD \( \mu_{L}/\mu_{QD} \); in a light blue full trace, the approximation of the \( g_{tot}^{(2)}(0) \simeq 2\mu_{L}/\mu_{QD} \). The dashed grey lines mark the value of the experimental \( g_{tot}^{(2)}(0) \) and the corresponding ratio of \( \mu_{L}/\mu_{QD} \).

As expected, the value of \( g_{tot}^{(2)}(0) \) saturates to unity (poissonian statistics), when \( \mu_{L} \gg \mu_{QD} \). For low \( g_{tot}^{(2)}(0) \) values, it can be approximated by \( g_{tot}^{(2)}(0) \simeq 2\mu_{L}/\mu_{QD} \), as shown by the light-blue line in Fig. S3 for \( \mu_{L} \ll \mu_{QD} \). Considering the experimental value of single photon purity, \( g^{(2)}(0) = 0.071 \pm 0.003 \), we deduce the fraction of laser photon normalized to the fraction of photons emitted by the QD to \( \frac{\mu_{L}}{\mu_{QD}} \simeq \frac{g^{(2)}(0)/2}{0.038 \pm 0.003} \), where we have assumed that higher order Fock terms from the laser \( p_{n}^{L} \) are completely negligible since \( \mu_{L} \ll \mu_{QD} \).

**REFERENCES**