This document provides supplementary information to “Photonic hyperinterfaces for light manipulations,” https://doi.org/10.1364/OPTICA.392356. The permittivities of silver and silicon are shown in Section I, the permittivity tensors of MA and MB are shown in Section II, the simulated optical absorption of MA with a flat surface or a zigzag surface is shown in Section III, the calculated optical absorption of MA with a flat surface is shown in Section IV, the rotated iso-frequency contours of MA and MB are shown in Section V, the Energy flow patterns of a point source embedded in hyperinterfaces IFBA and IFAB are shown in Section VI, the time-averaged energy flow patterns inside the SMM for $\lambda=450, 524, 680, 793$ nm are shown in Section VII, and the measured absorption spectra of the SMMs and the HMMs at different angles are shown in Section VIII.

### Section I. Permittivities of silver and silicon

Figures S1(a) and S1(b) show the experimental and interpolated complex refractive index of silver [1] and silicon [2] as a function of wavelength, respectively, where the circles represent the experimental data from [1, 2], and the solid lines are derived from the cubic spline interpolation. In Fig. S1(a)/S1(b), the blue and green circles represent the experimental refractive index ($n$) and the extinction coefficient ($k$) of silver/silicon, respectively, while the red and black lines represent the interpolated $n$ and $k$ of silver/silicon, respectively. Figures S1(c) and S1(d) show the calculated and interpolated permittivity of silver and silicon as a function of wavelength. In Fig. S1(c)/S1(d), the blue and green circles represent the calculated real and imaginary parts of permittivity of silver/silicon, respectively, while the red and black lines represent the interpolated real and imaginary parts of permittivity of silver/silicon, respectively.
Section II. Permittivity tensors of MA and MB

When the layers in Fig. S2(a) are sufficiently thin, the optical properties of the alternating multilayer structure can be represented by an anisotropic effective medium with an effective permittivity of the form
\[
\varepsilon_{\text{normal}} = \begin{bmatrix}
\varepsilon_x & 0 \\
0 & \varepsilon_y \\
\end{bmatrix}
\]

\[
\varepsilon_{\text{normal}} = \begin{bmatrix}
\varepsilon_x & 0 \\
0 & \varepsilon_y \\
\end{bmatrix} = \left( (f \varepsilon_x^1 + (1-f)\varepsilon_y^2)^{-1} \right)
\]

where \( f = d_i / (d_i + d_o) \) is the filling ratio of the isotropic material whose permittivity is \( \varepsilon_i \).

Fig. S2. Schematics of the alternating multilayer structures with different tilt angles. (a) Alternating multilayer structure composed of two kinds of isotropic materials with permittivities \( \varepsilon_i \) and \( \varepsilon_e \). The normal direction of the structure is along the \( u \) direction, and the \( v \) direction is perpendicular to the \( u \) direction. (b) Alternating multilayer structure with normal direction along the \( u \) direction at a fixed angle \( -\alpha \) from the \( x \) direction. (c) Alternating multilayer structure with normal direction along the \( u \) direction at a fixed angle \( \alpha \) from the \( x \) direction.

Fig. S3. Simulated absorption spectra of the initial HMMs at an incident angle \( \theta = 0^\circ \), where the red curve indicates the absorption of the initial HMM based on effective permittivity tensor, while the blue curve indicates that of the initial HMM based on the multilayer structure.

Figure S3 shows the simulated absorption spectra of the initial HMMs based on the effective permittivity tensor and the multilayer structure, respectively. Then we can find that although the above absorption curves are slightly different, they exhibit similar absorption characteristics, which means that the effective permittivity tensor can effectively describe the optical absorption characteristics of the hyperbolic metamaterial.

Rotate the alternating multilayer structure in Fig. S2(a) by \( \alpha \) degrees clockwise (counterclockwise) about the origin, we can get the alternating multilayer structure with normal direction along the \( u \) direction at a fixed angle \( -\alpha \) (+\( \alpha \)) from the \( x \) direction, as shown in Figs. S2(b) and S2(c). Such alternating multilayer structures can be interpreted using the concept of anisotropic media [3]. The effective permittivity tensors of the alternating multilayer structures in Figs. S2(b) and S2(c) can be written as
\[
\varepsilon_-\alpha = \begin{bmatrix}
\varepsilon_x & 0 \\
0 & \varepsilon_y \\
\end{bmatrix} = \left( (f \varepsilon_x^1 + (1-f)\varepsilon_y^2)^{-1} \right)
\]

and
\[
\varepsilon_+\alpha = \begin{bmatrix}
\varepsilon_x & 0 \\
0 & \varepsilon_y \\
\end{bmatrix} = \left( (1-f)\varepsilon_x^1 + f\varepsilon_y^2 \right)
\]

respectively, where \( \varepsilon_-\alpha = \varepsilon_+\alpha = \varepsilon_x \cos^2 \alpha + \varepsilon_y \sin^2 \alpha \), \( \varepsilon_-\alpha = \varepsilon_+\alpha = \varepsilon_x \cos^2 \alpha + \varepsilon_y \sin^2 \alpha \), and \( \varepsilon_-\alpha = \varepsilon_+\alpha = \varepsilon_x \sin \alpha \cos \alpha (\varepsilon_x - \varepsilon_y) \).

Section III. MA (MB) with a flat surface vs a zigzag surface

Figures S4(a)-S4(c) show the 2D effective medium models of MA, MB and the SMM with a zigzag or a flat surface, respectively. Based on the FEM simulation results from COMSOL Multiphysics, we find that the optical absorption curves for two different angles of incidence (AOI) at different wavelengths of MA, MB and the SMM with a flat surface are almost the same with those of a zigzag surface, as shown in Figs. S4(d)-S4(f), where the short dashed lines correspond to the absorption curves of MA, MB and the SMM with a flat surface, while the solid lines correspond to those with a zigzag surface.

Fig. S4. (a)-(c) 2D effective medium models of MA, MB and the SMM with a zigzag or a flat surface, respectively. (d)-(f) Simulated absorption curves for two AOI at different wavelengths of MA, MB and the SMM with a zigzag or a flat surface.

Section IV. Optical absorption of MA (MB)

Let us consider a TM plane wave with the angle \( \theta \) incident from the air to MA (with a flat surface) along the \( x \)-axis, as shown in Fig. S5, the \( y \)-axis is the boundary between the air and MA, and the green, orange and red arrows indicate the incident, reflected and refracted (transmitted) waves, respectively. Besides, MA is supposed to be thick enough to have a zero transmittance, then the optical absorption of MA can be written as \( A = 1 - R \), where \( A \) represents the absorptivity and \( R \) represents the reflectivity of the incident light by MA. For an incident TM plane wave
\[
\vec{H}_{\text{in}} = H_0 e^{i(\omega t + \phi_{\text{in}})} e^{i\theta} \hat{z} , \quad x < 0,
\]

the reflected wave is written as
\[
\vec{H}_{\text{ref}} = r \cdot H_0 e^{i(\omega t + \phi_{\text{ref}})} e^{i\theta} \hat{z} , \quad x < 0,
\]

while the transmitted wave is expressed as
\[
\vec{H}_{\text{tr}} = t \cdot H_0 e^{i(\omega t + \phi_{\text{tr}})} e^{i\theta} \hat{z} , \quad x > 0,
\]
Fig. S5. The schematic diagram of a TM plane wave with an angle $\theta$ incident from the air to MA. The green solid arrow indicates the incident wave, the orange solid arrow indicates the reflection wave and the red solid arrow indicates the refraction wave.

Fig. S6. (a)-(b) Calculated optical absorption maps of the initial HMM and MA(MB) based on Eqs. (S13) and (S12) vs the angle and wavelength of the incident TM plane waves. (c) Simulated and calculated absorption curves for two AOI at different wavelengths.

where $\beta' = \beta - k_y \sin \theta$ is the wave vector along the y-axis, $k_x = \omega/c_0 \cos \theta$ and $k_y'$ are the wave vectors normal to the y-axis in the air and MA, respectively, $\theta$ is the incident angle, and $r$ and $t$ are the coefficients of reflection and transmission, respectively. Besides, the corresponding electric-field distributions can be obtained by one of the Maxwell's equations $\mathbf{E} = -\nabla \times \mathbf{H} / i\omega \varepsilon \mathbf{E}$.

After matching the boundary conditions at the interface $x=0$ ($\mathbf{H}_{\text{int}} + \mathbf{H}_{\text{ref}} = \mathbf{H}_{\text{int}}$ & $\mathbf{E}_{\text{int,y}} + \mathbf{E}_{\text{ref,y}} = \mathbf{E}_{\text{int,y}}$), we have

$$r = t^{-1},$$

where $\epsilon_{\text{xx}}^{(\alpha - \omega)}$ and $\epsilon_{\text{yy}}^{(\alpha - \omega)}$ are the components of the permittivity tensor of MA (see Eq. S2), and $\epsilon_x$ and $\epsilon_y$ are the components of the permittivity tensor of the initial HMM (see Eq. S1). Then we can obtain

$$t = 2[1 + \frac{1}{k_x \epsilon_y \epsilon_y} (\epsilon_{\text{yy}}^{(\alpha - \omega)} \beta + \epsilon_{\text{xx}}^{(\alpha - \omega)} k_y')]^{-1},$$

by combining Eqs. (S7)-(S9), we can rewrite the coefficients of reflection $r$ as

$$r = 2 / (1 + \sqrt{\epsilon_x \epsilon_x \epsilon_x \epsilon_x - \sin^2 \theta - 1}),$$

In this case, the optical absorption of MA can be written as

$$A_{\text{MA}} = 1 - \left| t \right|^2 = 1 - 2 / (1 + \sqrt{\epsilon_x \epsilon_x \epsilon_x \epsilon_x - \sin^2 \theta - 1}),$$

Since $\epsilon_{\text{xx}}^{(\alpha - \omega)} = \epsilon_y^{(\alpha - \omega)} + \epsilon_y^{(\alpha - \omega)}$, MA and MB should have the same optical absorption expression, that is

$$A_{\text{MA}} = A_{\text{MB}} = 1 - \left| t \right|^2 = 1 - 2 / (1 + \sqrt{\epsilon_x \epsilon_x \epsilon_x \epsilon_x - \sin^2 \theta - 1}),$$

where $\epsilon_{\text{xx}}^{(\alpha - \omega)} = \epsilon_{\text{xx}}^{(\alpha - \omega)}$. In addition, set the rotation angle $\alpha$ to be 0, we can obtain the optical absorption of the initial HMM of the form

$$A_{\text{HMM}} = 1 - \left| t \right|^2 = 1 - 2 / (1 + \sqrt{\epsilon_x \epsilon_x \epsilon_x \epsilon_x - \sin^2 \theta - 1}).$$

Based on the permittivities of silver and silicon in Section 1, the optical absorption maps of the initial HMM and MA (MB) versus the angle and the wavelength of incidence can be calculated using Eqs. (S12) and (S13), as plotted in Figs. S6(a) and S6(b), respectively. To show the details of the absorption characteristics and to confirm our calculations, we plot some calculated (solid lines) and simulated (dashed lines) absorption curves for two different angles of incidence in Figs. S6(c) and S6(d). By comparing the absorption maps of the initial HMM and MA (MB) in Figs. S6(a) and S6(b) and the selected absorption curves for two AOI at different wavelengths in Figs. S6(c) and S6(d), we find that the initial HMM and MA (MB) have almost the same absorption characteristics and our calculation results are consistent with the simulation results.

**Section V. Rotated iso-frequency contour**

We consider two-dimensional systems in the x-y plane and the TM incident waves. Then the dielectric properties of an anisotropic metamaterial can be described by a tensor

$$\mathbf{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{bmatrix}.$$  

(S14)

In general, $\varepsilon_{yy}$ is set to be 0, and based on Maxwell's equations, the corresponding dispersion relation can be written as

$$\frac{k_y^2}{\varepsilon_{yy}} + \frac{k_x^2}{\varepsilon_{xx} - \alpha^2} = \frac{\omega^2}{c^2},$$

(S15)

where $k_x$ ($k_y$) is the wave vector along the x (y) direction, $\omega$ is the angular frequency and $c$ is the velocity of the EM wave in vacuum. If we rotate the anisotropic metamaterial by an angle of alpha, $\varepsilon_{xy} = -\varepsilon_{yx}$ or $\varepsilon_{xx} = \varepsilon_{yy}$ and $\varepsilon_{yy} = \varepsilon_{yy}$, then there will be a cross term in the dispersion relation of the rotated metamaterial, written as

$$\frac{k_x^2}{\text{det}(\varepsilon)/\varepsilon_{xx}} + \frac{k_y^2}{\text{det}(\varepsilon)/\varepsilon_{yy}} + \frac{k_y^2}{\text{det}(\varepsilon)/\varepsilon_{yy}} = \frac{\omega^2}{c^2},$$

(S16)
where \( \text{det}(\mathbf{ϵ}) = \mathbf{ϵ}_{xx}\mathbf{ϵ}_{yy} - \mathbf{ϵ}_{xy}^2 \). Solve the above equation, we can find that the iso-frequency contour will be tilted by an angle \( \alpha \). For example, Fig. 3(d) shows the tilted IFCs of the initial HMMs [shown in Fig. 3(b)] by the geometric angle \( \alpha \) (the purple curve) and \( -\alpha \) (the blue curve) at \( \lambda = 765 \) nm.

Section VI. Energy flow patterns of a point source embedded in hyperinterfaces IFBA and IFAB

Figures S7(a) and S7(b) show the simulated time-averaged energy flow patterns of a directly embedded point source at the hyperinterfaces IFBA and IFAB when \( \lambda = 765 \) nm, respectively. Figures S7(c)-S7(d) show the tailored and spliced time-averaged energy flow patterns of an embedded point source at the hyperinterfaces IFBA and IFAB when \( \lambda = 612 \) nm, where the upper part pattern of MA and the lower part pattern of MB make up the pattern of IFBA, while the upper part pattern of MB and the lower part pattern of MA make up the pattern of IFAB. By comparing the energy flow patterns in Figs. S7(a) and S7(c) or in Figs. S7(b) and S7(d), we find the time-averaged energy flow patterns induced by the directly embedded point sources are almost the same with those tailored and spliced.

Figures S8(a) and S8(b) show the simulated time-averaged energy flow patterns of a directly embedded point source at hyperinterfaces IFBA and IFAB when \( \lambda = 612 \) nm, respectively. Figures S8(c)-S8(d) show the tailored and spliced time-averaged energy flow patterns of an embedded point source at hyperinterfaces IFBA and IFAB when \( \lambda = 612 \) nm, where the upper part pattern of MB and the lower part pattern of MA make up the pattern of IFBA, while the upper part pattern of MB and the lower part pattern of MA make up the pattern of IFAB.

Section VII. Time-averaged energy flow patterns inside the SMM for \( \lambda = 450, 524, 680, 793 \) nm

Fig. S9. (a) The time-averaged energy flow pattern of an embedded point source (with a fixed magnetic field at a tiny circle) inside the initial HMM when \( \lambda = 450 \) nm. (b)-(c) The time-averaged energy flow patterns of an embedded point source at the hyperinterfaces IFBA and IFAB when \( \lambda = 450 \) nm. (d) The 2D FEM simulation results of the time-averaged energy flow pattern in the SMM for a normal incident TM wave when \( \lambda = 450 \) nm. (e) IFCs of MA and MB when \( \lambda = 450 \) nm, where the purple curves denote the IFC of MA, the deep blue curves denote the IFC of MB, the orange circle is the IFC of the air, the red dashed arrow denotes the corresponding Poynting vector of the incident wave and the black dashed arrow denotes the corresponding Poynting vector of the refraction wave.

For \( \lambda < 623 \) nm, the intrinsic absorption of MA (MB) is extremely strong and the absorption of the SMM will be mainly determined by the refraction characteristics of the hyperinterfaces IFBA and IFAB. Considering a point source (excited by a fixed magnetic field at a tiny circle) embedded in the initial HMM, through FEM simulations, we can obtain the time-averaged energy flow pattern of the point source for \( \lambda = 450 \) nm, as shown in Fig. S9(a). By rotating, cutting and splicing the energy flow pattern in Fig. S9(a) (similar operations from the main text), we can obtain the time-averaged energy flow patterns of an embedded point source at the hyperinterfaces IFBA and IFAB when \( \lambda = 450 \) nm, as shown in Figs. S9(b) and S9(c), respectively. Only the right part of the above energy flow patterns should be considered as the TM wave is incident from the left along the positive direction of the x-axis. The simulated time-averaged energy flow inside the SMM for a normal incident TM plane wave from the air to the SMM with a wavelength of 450 nm is shown in Fig. S9(d). By comparison, we can find the rightward energy flow pattern at IFBA in Fig. S9(b) matches the simulated energy flow pattern at IFBA in Fig. S9(d) and the rightward energy flow pattern at IFAB in Fig. S9(c) matches the
simulated energy flow pattern at IF_{AB} in Fig. S9(d). In this case, high-k modes can be excited around the hyperinterfaces, which will enhance the absorption of the SMM.

On the other hand, for $\lambda = 450$ nm, both of MA and MB have very strong optical absorption [see Figs. 2(a) and 2(c) in the main text] and $\beta = 35^\circ > \alpha$. If we focus on the IFCs of MA and MB in Fig. S9(e), where the red dashed arrow (labeled as $S_1$) denotes the corresponding Poynting vector of the incident wave while the black dashed arrow (labeled as $S_1$) denotes that of the refraction wave, we can find that the incident energy flow in MB will be refracted in MA (vice versa) and the refracted energy flow will propagate inward the SMM. Then the SMM should have similar absorption to MA (MB). Besides, due to the extra absorption introduced by the excited high-k modes, the absorption of the SMM should be a little larger than that of MA(MB), as shown in Fig. 2(c) in the main text.

**Fig. S10.** (a) The time-averaged energy flow pattern of an embedded point source (with a fixed magnetic field at a tiny circle) inside the initial HMM when $\lambda = 524$ nm. (b)-(c) The time-averaged energy flow patterns of an embedded point source at the hyperinterfaces IF_{BA} and IF_{AB} when $\lambda = 524$ nm. (d) The 2D FEM simulation results of the time-averaged energy flow pattern in the SMM for a normal incident TM wave when $\lambda = 524$ nm. (e) IFCs of MA and MB when $\lambda = 524$ nm, where the purple curves denote the IFC of MA, the deep blue curves denote the IFC of MB, the orange circle is the IFC of the air, the red dashed arrow denotes the corresponding Poynting vector of the incident wave.

Figure S10(a) shows the simulated time-averaged energy flow pattern of a point source embedded in the initial HMM for $\lambda = 524$ nm. Figures S10(b) and S10(c) show the tailored and spliced time-averaged energy flow patterns of an embedded point source at the hyperinterfaces IF_{BA} and IF_{AB} when $\lambda = 524$ nm, respectively. The rightward energy flow pattern only appears around IF_{AB} in Fig. S10(c) since $\beta < \alpha$ when $\lambda = 524$ nm. The simulated time-averaged energy flow inside the SMM for a normal incident TM plane wave from the air to the SMM with a wavelength of 524 nm is shown in Fig. S10(d). By comparison, we can find the rightward energy flow pattern around IF_{AB} in Fig. S10(c) matches the simulated energy flow pattern at IF_{AB} in Fig. S10(d). In this case, high-k modes can be excited around IF_{AB} only, which will also enhance the absorption of the SMM.

On the other hand, for $\lambda = 524$ nm, both of MA and MB have very strong optical absorption [see Figs. 2(a) and 2(c) in the main text] and $\beta = \alpha$. If we focus on the IFCs of MA and MB in Fig. S10(e), we can find that there is no transmission between MA and MB since one of the asymptotes of the purple hyperbola in Fig. S10(e) coincides with the y-axis. For example, the corresponding Poynting vector indicated by red dashed arrow and labeled as $S_1$ in Fig. S10(e) for MB do not have a solution to match the momentum for MA. Then the incident energy flow (from the air to the SMM) will directly propagate inward the SMM and the SMM should have similar absorption to MA (MB). Besides, due to the extra absorption introduced by the excited high-k modes, the absorption of the SMM should be a little larger than that of MA(MB), as shown in Fig. 2(c) in the main text.

**Fig. S11.** (a) The time-averaged energy flow pattern of an embedded point source (with a fixed magnetic field at a tiny circle) inside the initial HMM when $\lambda = 680$ nm. (b)-(c) The time-averaged energy flow pattern of an embedded point source at the hyperinterfaces IF_{BA} and IF_{AB} when $\lambda = 680$ nm. (d) The 2D FEM simulation results of the time-averaged energy flow pattern in the SMM for a normal incident TM wave when $\lambda = 680$ nm. (e) IFCs of MA and MB when $\lambda = 680$ nm, where the purple curves denote the IFC of MA, the deep blue curves denote the IFC of MB, the orange circle is the IFC of the air, the red dashed arrow denotes the corresponding Poynting vector of the incident wave.

For $\lambda > 623$ nm, the intrinsic absorption of MA (MB) is relatively weak and the absorption of the SMM will be mainly determined by the excited high-k modes. Figure S11(a) shows the simulated time-averaged energy flow pattern of a point source embedded in the initial HMM for $\lambda = 680$ nm. Figures S11(b) and S11(c) show the tailored and spliced time-averaged energy flow patterns of an embedded point source at the hyperinterfaces IF_{BA} and IF_{AB} when $\lambda = 680$ nm, respectively. The rightward energy flow pattern only appears around IF_{AB} in Fig. S11(c) since $\beta < \alpha$ when $\lambda = 680$ nm. The simulated time-averaged energy flow inside the SMM for a normal incident TM plane wave from the air to the SMM with a wavelength of 680 nm is shown in Fig. S11(d). By comparison, we can find the rightward energy flow pattern around IF_{AB} in Fig. S11(c) matches the simulated energy flow pattern at IF_{AB} in Fig. S11(d). In this case, the excited high-k modes will be converged around IF_{AB} only, resulting in smaller absorption (compared to those converged at both hyperinterfaces IF_{BA} and IF_{BA}).

On the other hand, if we focus on the IFCs of MA and MB in Fig. S11(e), where the red dashed arrow (labeled as $S_1$) denotes the corresponding Poynting vector of the incident wave while the black dashed arrow (labeled as $S_1$) denotes that of the refraction wave, we can find that the incident energy flow in MB will be negatively refracted in MA (vice versa) and the refracted energy flow will propagate outward the SMM [Fig. S11(d)]. Then part of the incident energy flow will propagate outward the SMM, resulting in smaller absorption.

Figure S12(a) shows the simulated time-averaged energy flow pattern of a point source embedded in the initial HMM for $\lambda = 793$ nm. Figures S12(b) and S12(c) show the tailored and
spliced time-averaged energy flow patterns of an embedded point source at the hyperinterfaces IFBA and IFAB when $\lambda = 793$ nm, respectively. The rightward energy flow pattern only appears around IFBA in Fig. S12(c) since $\beta > \alpha$ when $\lambda = 793$ nm. The simulated time-averaged energy flow inside the SMM for a normal incident TM plane wave from the air to the SMM with a wavelength of 793 nm is shown in Fig. S12(d). By comparison, we can find the rightward energy flow pattern around IFAB in Fig. S12(c) matches the simulated energy flow pattern at IFBA in Fig. S12(d). In this case, the excited high-k modes will be converged around IFBA only, resulting in smaller absorption (compared to those converged at both interfaces IFAB and IFBA).

On the other hand, if we focus on the IFCs of MA and MB in Fig. S12(e), where the red dashed arrow (labeled as $S_i$) denotes the corresponding Poynting vector of the incident wave while the black dashed arrow (labeled as $S_r$) denotes that of the refraction wave, we can find that the incident energy flow in MB will be refracted in MA (vice versa) and the refracted energy flow will propagate inward the SMM [Fig. S12(d)]. Then part of the incident energy flow will propagate inward the SMM.

Fig. S12. (a) The time-averaged energy flow pattern of an embedded point source (with a fixed magnetic field at a tiny circle) inside the initial HMM when $\lambda = 793$ nm. (b)-(c) The time-averaged energy flow patterns of an embedded point source at the hyperinterfaces IFBA and IFAB when $\lambda = 793$ nm. (d) The 2D FEM simulation results of the time-averaged energy flow pattern in the SMM for a normal incident TM wave when $\lambda = 793$ nm. (e) IFCs of MA and MB when $\lambda = 793$ nm, where the purple curves denote the IFC of MA, the deep blue curves denote the IFC of MB, the orange circle is the IFC of the air, the red dashed arrow denotes the corresponding Poynting vector of the incident wave.

Section VII. Measured absorption spectra of the SMMs and the initial HMMs at different angles

Fig. S13. (a)-(d) Measured absorption spectra of the fabricated SMMs and the initial HMMs at an incident angle $\theta = 15^\circ / 30^\circ / 45^\circ / 60^\circ$, respectively. In those figures, 1# indicates sample 1, while 2# indicates sample 2.

References