Optical analog computing of two-dimensional spatial differentiation based on the Brewster effect: supplement

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SI. Calculation of the reflection matrix

Fig.S1 Scheme of the reflection of the central wave and the local wave vector at the air-glass interface.

In order to convert the frame \((x_i, y_i, z_i)\) to the frame \((X_i, Y_i, Z_i)\), these three steps should be performed (Fig.S1). Firstly, we transform the coordinate system \((x_i, y_i, z_i)\) around the \(y\) axis at the incident angle \(\theta\) to the frame \((x, y, z)\). The first changing matrix is given by:
Secondly, we need to transform the frame \((x, y, z)\) around the \(z\) axis by an angle \(\theta\) to the frame \(XYZ\), and the second matrix can be written as the follow:

\[
M_{xyz \rightarrow XYZ} = \begin{bmatrix}
1 & \frac{k_y}{k_0 \sin \theta} & 0 \\
-\frac{k_y}{k_0 \sin \theta} & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]  \(\text{(S2)}\)

At the end step, we transform the frame \(XYZ\) around the \(y\) axis by an angle \(-\theta\) to the frame \(X_iY_iZ_i\), and the third matrix is given by:

\[
M_{XYZ \rightarrow X_iY_iZ_i} = \begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}.
\]

\(\text{(S3)}\)

In summary, the matrix of the whole transformation can be written as the following:

\[
M_{x_i,y_i,z_i \rightarrow xyz} = M_{XYZ \rightarrow X_iY_iZ_i} M_{xyz \rightarrow XYZ} M_{y_i,z_i \rightarrow xyz}
\]

\[
= \begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix} \begin{bmatrix}
1 & \frac{k_y}{k_0 \sin \theta} & 0 \\
-\frac{k_y}{k_0 \sin \theta} & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\cos(-\theta) & 0 & \sin(-\theta) \\
0 & 1 & 0 \\
-\sin(-\theta) & 0 & \cos(-\theta)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
1 & \frac{k_y \cot \theta}{k_0} & 0 \\
-\frac{k_y \cot \theta}{k_0} & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]  \(\text{(S4)}\)

The two-dimensional form of this matrix can be written as:
Following the procedure above, we can transform the frame \( x, y, z \) to the frame \( X, Y, Z \):

\[
M_{x,y,z \rightarrow X,Y,Z} = \begin{bmatrix}
1 & \frac{k_y \cot \theta_t}{k_0} \\
-\frac{k_y \cot \theta_t}{k_0} & 1
\end{bmatrix}.
\]  

(S5)

As we know, for any arbitrary wave vector, the reflected field is determined by

\[
E_{x,y,z}^{p,s} = r_p E_{x,y,z_0}^{p,s} \quad r_p \quad \text{and} \quad r_s \quad \text{refer to the Fresnel reflection coefficient of the p and s wave, respectively. Therefore, the reflection matrix is as the follows:}
\]

\[
M_R = M_{x,y,z \rightarrow x,y,z} \begin{bmatrix} r_p & 0 \\ 0 & r_s \end{bmatrix} M_{x,y,z \rightarrow x,y,z}^{-1}
\]

\[
= \begin{bmatrix}
0 & \frac{k_y \cot \theta_t}{k_0} \\
-\frac{k_y \cot \theta_t}{k_0} & 0
\end{bmatrix} \begin{bmatrix}
r_p & 0 \\
0 & r_s
\end{bmatrix} \begin{bmatrix}
0 & \frac{k_y \cot \theta_t}{k_0} \\
-\frac{k_y \cot \theta_t}{k_0} & 0
\end{bmatrix}.
\]  

(S7)

SII. Calculation of spatial spectral transfer function

When the input field \( \vec{E}_{in} \) passes through the GLP1, we can get the field with \( \alpha \) linear polarization with \( x \) axis:

\[
\vec{E}_{in} = \vec{E}_{in} \begin{bmatrix} \cos \alpha \\ \sin \alpha \end{bmatrix}.
\]  

(S8)

After the light beam passing through the QWP, whose optical axis is 0° with \( x \) direction, and the Jones matrix of the QWP can be written as:
\[
\langle QWP \rangle = \begin{bmatrix}
\exp \left(-\frac{\pi i}{4}\right) & 0 \\
0 & \exp \left(\frac{\pi i}{4}\right)
\end{bmatrix}
\]  
(S9)

Then the light beam reflected at the glass-air interface, the angular spectrum can be obtained by the relation \( \tilde{E}_r = R_M \langle QWP \rangle \tilde{E}_in \). The reflection matrix can be written as:

\[
R_M = \begin{bmatrix}
r_p & k \Delta \\
-k \Delta & r_s
\end{bmatrix},
\]  
(S10)

where \( r_p \) and \( r_s \) are the Fresnel reflection coefficients of p and s waves, \( \Delta = (r_p + r_s) \cot \theta / k_0 \), \( k_0 = 2\pi / \lambda \) is the wave vector in the vacuum, \( \theta \) is incident angle. Based on the boundary condition, the angular spectrum of reflected light field can be written as

\[
\tilde{E}_r = \tilde{E}_in \left\{ \begin{array}{c}
\exp \left(-\frac{\pi i}{4}\right) r_p \cos \alpha + \exp \left(\frac{\pi i}{4}\right) \Delta \sin \alpha k_y e_s \\
+ \exp \left(\frac{\pi i}{4}\right) r_s \sin \alpha - \exp \left(-\frac{\pi i}{4}\right) \Delta \cos \alpha k_y e_y
\end{array} \right\}.
\]  
(S11)

Here, \( e_x = \frac{1}{\sqrt{2}}(e_+ + e_\uparrow) \), \( e_y = \frac{i}{\sqrt{2}}(e_- - e_\uparrow) \). After substituting them into the (S4), the angular spectrum of reflected light field can also be written as

\[
\tilde{E}_r = \frac{\tilde{E}_in}{\sqrt{2}} \left\{ \exp \left(-\frac{\pi i}{4}\right) \cos \alpha (r_p - i \Delta k_y) e_+ + \exp \left(-\frac{\pi i}{4}\right) \cos \alpha (r_p + i \Delta k_y) e_s \\
+ i \exp \left(-\frac{\pi i}{4}\right) \sin \alpha (r_s - i \Delta k_y) e_- - i \exp \left(-\frac{\pi i}{4}\right) \sin \alpha (r_s + i \Delta k_y) e_y \right\},
\]  
(S12)

according to the Taylor series expansion, \( r_p \) and \( r_s \) can be expanded as follows

\[
r_p(k_x) = r_p(k_x = 0) + k_x \left[ \frac{\partial r_p(k_x)}{\partial k_x} \right]_{k_x=0},
\]  
(S13)

\[
r_s(k_x) = r_s(k_x = 0) + k_x \left[ \frac{\partial r_s(k_x)}{\partial k_x} \right]_{k_x=0}.
\]  
(S13)

Then the angular spectrum of reflected light field can be rewritten as
\[ \mathbf{E}_r = \exp \left( -\frac{\pi}{4} i \right) \frac{\sin \alpha \mathbf{E}_m r_s}{\sqrt{2}} \left[ \left( 1 + \left( \frac{r_p}{r_s \tan \alpha} \Delta_H + \Delta_V \right) k_x + i \left( \delta_H + \delta_V \right) k_y \right) e_x ight. \\
\left. - \left( 1 + \left( -\frac{r_p}{r_s \tan \alpha} \Delta_H + \Delta_V \right) k_x + i \left( \delta_H - \delta_V \right) k_y \right) e_y \right] \], \quad (S14)

where \[ \Delta_H = \frac{\partial \ln r}{k_0 \partial \theta}, \quad \Delta_V = \frac{\partial \ln r}{k_0 \partial \theta}, \quad \delta_H = \frac{\Delta}{r_s \tan \alpha}, \quad \delta_V = \frac{\Delta}{r_s}. \] Then we set \[ \Delta = \frac{r_p}{r_s \tan \alpha} \Delta H, \quad \Delta y = \delta_V, \] and the reflected field can be written as

\[ \mathbf{E}_r \approx \exp \left( -\frac{\pi}{4} i \right) \frac{\sin \alpha \mathbf{E}_m r_s}{\sqrt{2}} \left[ \exp \left( \Delta x k_x + i \Delta y k_y \right) e_x - \exp \left( -\Delta x k_x - i \Delta y k_y \right) e_x \right]. \quad (S14) \]

Here, we have introduced the approximation: \[ 1 + \Delta x k_x + \Delta y k_y \approx \exp \left( \Delta x k_x + \Delta y k_y \right). \]

Then the reflected field passes through second polarizer whose polarization axis is chosen as \[ 0^\circ. \] Therefore, the output field in the whole differentiator system can be acquired as

\[ \mathbf{E}_{\text{out}} = \exp \left( -\frac{\pi}{4} i \right) \frac{\sin \alpha \mathbf{E}_m r_s}{\sqrt{2}} \left[ \exp \left( \Delta x k_x + i \Delta y k_y \right) - \exp \left( -\Delta x k_x - i \Delta y k_y \right) \right]. \quad (S15) \]

In order to get the homogeneous two-dimensional isotropic edges in the subsequent experiments, we need to ensure that \[ \Delta x = \Delta y, \] and then we can obtain the \[ \alpha = 67.1^\circ. \]

According to the expression of the spatial transfer function

\[ H(k_x, k_y) = \frac{\mathbf{E}_{\text{out}}(k_x, k_y)}{\mathbf{E}_m(k_x, k_y)}. \quad (S16) \]

After substituting Eq. (S9) into Eq. (S10), we obtain

\[ H(k_x, k_y) \propto i\mathbf{E}_m r_s \sin \left( -i \Delta x k_x + i \Delta y k_y \right). \]

\[ \approx \Delta x k_x + i \Delta y k_y. \quad (S17) \]

**SIII. Calculation the theoretical bandwidth of the differentiator**

In the Fig.S2, we chose three different beam widths in the x and y axis, respectively. In our experimental system, the beam width is 0.02mm, which is red lines shown in the Fig.S2. We find that the gain of the system become larger when the beam width is smaller.
The first-order derivative of the input field in the y direction. The ordinate value is normalized.

**SIV. Calculation the theoretical of the output field**

The output field in position space can be obtained by the Fourier transform:

\[
E_{\text{out}}(x, y) = \int \int \tilde{E}_{\text{out}}(k_x, k_y) \exp[i(k_x x + k_y y)] dk_x dk_y. \tag{S18}
\]

After substituting Eq. (S8) into Eq. (S11), the output field in position space can be written as

\[
E_{\text{out}}(x, y) \approx r_x \left[ E_{\text{in}}(x + \Delta x, y + i \Delta y) - E_{\text{in}}(x - \Delta x, y - i \Delta y) \right] \\
= \Delta x \frac{\partial E_{\text{in}}(x, y)}{\partial x} + i \Delta y \frac{\partial E_{\text{in}}(x, y)}{\partial y}. \tag{S19}
\]

Therefore, the output field is approximately proportional to the two-dimensional spatial differentiation of the input field.

**Supplementary References**