Gratingless integrated tunneling multiplexer for terahertz waves: supplement

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Supplementary Information—Gratingless Integrated Tunneling Multiplexer for Terahertz Waves

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S1. Leaky-mode analysis

The following analysis of the optical tunneling process is adapted from the procedure described in [29], as applied to the scenario that is illustrated in Fig. 1(a), albeit approximated with two-dimensional (2D) geometry that is invariant in the z-direction. This approach is an alternative to the full three-dimensional (3D) treatment of the problem, which significantly increases its complexity.

We define \( \tilde{\beta} \) as the complex wavevector of the guided leaky mode, for which the real component is \( \beta_{\text{waveguide}} \), which is associated with phase change during propagation, and the imaginary component is \( -\alpha_{\text{leak}} \), which describes power loss due to leakage from the waveguide into the silicon slab. We also define

\[
\alpha_s = \sqrt{\tilde{\beta}^2 - k_0^2},
\]

and,

\[
k_s = \sqrt{(k_0 n)^2 + \tilde{\beta}^2},
\]

which are associated with exponential decay of evanescent field strength in air, and the propagation of confined waves in the dielectric in the x-direction, respectively. The x-positions \( a = W/2 \) and \( b = a + s \) are designated, and hence there is dielectric of refractive index \( n \) enclosed between \( x = -a \) and \( x = a \), as well as for \( x \geq b \).

The procedure that we follow is to ensure that tangential field components are continuous across the dielectric-air boundaries, by deriving the value of \( \tilde{\beta} \) that satisfies this constraint. For the mode that is employed in the present work, the electric-field vector is coplanar with the \( xy \)-plane, and the magnetic-field vector is parallel to the z-axis. Following the example of [29], we are searching for a modal field distribution phasor of the following form,

\[
H_z(x) = \begin{cases} 
H_i \exp(\alpha_s x) & \text{if } x \leq -a \\
H_{ii} \cos(k_s x + \varphi_1) & \text{if } |x| \leq a \\
H_{ii} \cosh(\alpha_s x + \varphi_2) & \text{if } a \leq x \leq b \\
H_{iv} \exp(j k_s (b - x)) & \text{if } x \geq b,
\end{cases}
\]

where \( H_i \) to \( H_{iv} \) are complex amplitudes, and \( \varphi_1 \) and \( \varphi_2 \) are associated with displacement of the center point of the trigonometric and hyperbolic expressions in \( x \). The value \( \varphi_1 \) is non-zero due to the asymmetry of the structure. The complex amplitude terms in Eq. S3 are selected to ensure that the tangential magnetic field is continuous across all dielectric boundaries,

\[
H_i = \frac{\cos(-k_s a + \varphi_1)}{\exp(-\alpha_s a)},
\]

\[
H_{ii} = 1,
\]
Fig. S1. Results of leaky mode analysis, in comparison to 2D full-wave simulations, showing (a),(b) distributions of field components tangential to the dielectric-air boundary, where a dielectric of index $n$ is shown in gray, (c),(d) the dependence of dispersion and rate of leakage upon separation, at 350 GHz, and (e),(f) dependence of dispersion and scan angle upon frequency, where separation $s = 50 \, \mu m$.

\[
H_{iii} = \frac{\cos(k_x a + \varphi_1)}{\cosh(\alpha_x a + \varphi_2)},
\]  

(S6)
\[ H_{iv} = \frac{\cos(k_x a + \varphi_1)}{\cosh(\alpha_x a + \varphi_2)} \cosh(\alpha_x b + \varphi_2). \]  \hfill (S7)

The Ampere-Maxwell equation can be expressed as \( \nabla \times \vec{H} = j \omega \epsilon_0 \mu_0 \vec{E} \) in linear, isotropic, non-conducting media of relative permittivity \( \epsilon_r \). As the magnetic field is entirely parallel to the \( z \)-axis, this reduces to,

\[ j \omega \epsilon_0 \vec{E} = \left( \frac{1}{\epsilon_r} \frac{\partial H_z}{\partial y} - \frac{\partial H_x}{\partial x} \right). \]  \hfill (S8)

The electric field component that is parallel to dielectric-air boundaries can therefore be expressed,

\[ -j \omega \epsilon_0 E_x(x) = \begin{cases} \alpha_x H_1 \exp(\alpha_x x) & \text{if } x \leq -a, \\ -\frac{k_x}{n} H_{ii} \sin(k_x x + \varphi_1) & \text{if } |x| \leq a, \\ \alpha_x H_{ii} \sinh(\alpha_x x + \varphi_2) & \text{if } a \leq x \leq b, \\ -\frac{j k_x}{n} H_{iv} \exp(j k_x (b-x)) & \text{if } x \geq b. \end{cases} \]  \hfill (S9)

Now we employ this expression to determine the \( \varphi \) terms. We obtain the following expression for \( \varphi_1 \) when enforcing the electric field’s boundary condition at \( x = -a \),

\[ \varphi_1 = \arctan\left(-\frac{\alpha_x n^2}{k_x}\right) + k_x a, \]  \hfill (S10)

and we obtain a second expression at \( x = a \),

\[ \varphi_1 = \arctan\left(-\frac{\alpha_x n^2}{k_x} \tanh(\alpha_x a + \varphi_2)\right) - k_x a. \]  \hfill (S11)

Equating Eqs. S10 and S11 yields an expression for \( \varphi_2 \),

\[ \varphi_2 = \arctanh\left(-\frac{k_x}{\alpha_x n^2} \tan\left(2k_x a + \arctan\left(-\frac{\alpha_x n^2}{k_x}\right)\right)\right) - \alpha_x a. \]  \hfill (S12)

A second expression for \( \varphi_2 \) is obtained by enforcing the tangential electric field boundary condition at \( x = b \),

\[ \varphi_2 = \arctanh\left(-\frac{j k_x}{\alpha_x n^2}\right) - \alpha_x b. \]  \hfill (S13)

Equating these two expressions for \( \varphi_2 \) yields a dispersion relation that is solely dependent upon \( \beta \), which is related to \( \alpha_x \) and \( k_x \), as defined in Eqs. S1 and S2. Numerical optimization techniques are employed to search for the value of \( \beta \) that renders Eqs. S12 and S13 equal.

The results of leaky-mode analysis are presented in Fig. S1, along with the simulated results, for comparison. In contrast to that which is presented in Fig. 1, the simulations employed here make use of 2D geometry that is invariant in the \( z \)-dimension, for closer correspondence to the analytical leaky mode analysis model. It can be seen that both methods to analyze the leaky mode are in close agreement, and hence the analytical model is validated.

**S2. Design procedure**

The design procedure is performed at a single frequency, for which 350 GHz is chosen based on the range of frequencies that are accessible to the available experimental equipment. The uncurled length of the tunneling waveguide must be discretized into finite steps in order to support iteration along its length. In order to closely approximate a continuous curvature, we select \( \Delta l = 20 \mu m \), which is less than 1/40 of a free-space wavelength.
S2.1. Tailored separation for aperture magnitude distribution

The desired beam shape is given in Eq. 2. Input parameters to the design procedure are initial separation, \(s(0)\), Gaussian beam width, \(w_{\text{Aperture}}\), and truncation magnitude level, which is 50% for this work. The total uncurled length of the tunneling waveguide, \(l_{\text{Max}}\), is determined by the latter two parameters. For a given \(\Delta l = 20 \mu\text{m}\)-step, the amount of power that is leaked from the waveguide to the slab is,

\[
P_{\text{leak}}(l) = (1 - \exp \left[-2\alpha_{\text{leak}}(s(l))\Delta l\right]) P_{\text{waveguide}}(l).
\]  

(S14)

The excitation of the tunneling waveguide is arbitrarily set to \(P_{\text{waveguide}}(l = 0) = 1\) W, and hence the leaked power at the initial step is determined from this value using Eq. S14. The desired outgoing truncated-Gaussian leaked beam shape, which we will call \(P_{\text{Desired}}(l) = A(l)^2\), is scaled to this initial value. For any given subsequent step, the power that arrives from the prior step is given by,

\[
P_{\text{waveguide}}(l) = P_{\text{waveguide}}(l - \Delta l) - P_{\text{leak}}(l - \Delta l).
\]  

(S15)

This is a single step in the iterative procedure, and hence it is necessary to compute the leaked power at a given point before proceeding to the next point. This information is combined with Eq. S14 to determine the required rate of leakage,

\[
\alpha_{\text{leak}}(s(l)) = \frac{-1}{2\Delta l} \ln \left[1 - \frac{P_{\text{Desired}}(l)}{P_{\text{waveguide}}(l)}\right].
\]  

(S16)

The relation between the separation and leakage, which is shown in Fig. 1(f), is employed to determine the separation that corresponds to the desired rate of leakage. In this work, \(s(l)\) decreases monotonically with respect to increase in \(l\). If the required value of \(s(l)\) is lower than it is practically feasible due to fabrication constraints, then the minimum viable value of separation is selected.

The initial separation, \(s(0)\), is a free parameter that is selected at the outset of the design procedure, and hence a suitable value for this parameter must be sought. If \(s(0)\) is too large, then there will be significant power in the tunneling waveguide at its point of termination. On the other hand, a value that is too small will result in the premature encounter of the minimum viable value of \(s\). In both cases, the resultant field distribution of the slab-confined beam will be skewed to one side, and hence will be a poor approximation of the desired truncated-Gaussian shape. As such, the value of \(s(0)\) must be manually optimized for any given selection of \(w_{\text{Aperture}}\) using trial-and-error.

S2.2. Curvature for in-slab focusing

The target focus is situated along a line that makes angle \(\theta(l = 0)\) with the \(x\)-axis. This will induce the initial orientation of the tunneling waveguide to be parallel to the \(y\)-axis, for alignment with the associated port. The distance from the start point at \(l = 0\) to the target focus is a free parameter, \(d(0)\). The total phase change that a wave undergoes in this distance is equal to

\[
\phi_{\text{Total}}(0) = -d(0)\beta_{\text{Slab}}.
\]  

(S17)

The phase-change that the leaky-mode undergoes in a \(\Delta l\)-step at position \(l\) is \(-\beta_{\text{waveguide}}(s(l))\Delta l\). At this stage in the design, the value of separation, \(s(l)\), is pre-determined along the tunneling waveguide length. As a consequence, the value of \(\beta_{\text{waveguide}}(s(l))\) is also fixed, in accordance with the information given in Fig. 1(c). Thus, the accumulated phase at each point in the tunneling waveguide may be computed as,

\[
\phi_{\text{waveguide}}(l) = -\int_{0}^{l} \beta_{\text{waveguide}}(s(\ell))d\ell.
\]  

(S18)
In order to achieve focusing at the intended position, this acquired phase must be compensated by shortening the distance to the focus in the slab medium. This will induce constructive interference to occur at the target focus. This requirement may be expressed,

$$\phi_{\text{Total}}(0) = \phi_{\text{Waveguide}}(l) - d(l)\beta_{\text{Slab}}.$$  \hspace{1cm} (S19)

which yields the required distance to the target focus, for all $l$,

$$d(l) = d(0) - \frac{1}{\beta_{\text{Slab}}} \int_{0}^{l} \beta_{\text{Waveguide}}(s(\ell))d\ell.$$  \hspace{1cm} (S20)

These integrals are computed as a Riemann-sum, due to the discretization of the tunneling waveguide structure.

An iterative curving procedure is employed to ensure that all points along the length of the tunneling waveguide are at the appropriate distance from the target focus. For a given step along the waveguide, we seek a point that is both $\Delta l$ away from the current position, and $d(l)$ away from the target focus, to be the subsequent step. If no such point can be found, then this is an indication that the chosen value of $d(0)$ is too small. However, it is desirable to have as short a focal length as possible, as this engenders compact device size, and reduces the waist of the focused beam [39]. Thus, in order to obtain the smallest possible ratio of $d(0)$ to $l_{\text{Max}}$, we manually search for the minimum $d(0)$ for which the curving procedure completes successfully.

S3. Thermal stability

The thermal stability of the multiplexer device is of key interest to its intended application of high-volume communications. This is investigated in simulation, where thermal expansion is represented by physically scaling the model in all spatial dimensions. It is understood that intrinsic silicon undergoes linear expansion, and that a physical increase in side-length of $\Delta L/L = 0.1\%$ corresponds to an approximate increase of 200°C with respect to room temperature [37]. For

Fig. S2. Simulated impact of thermal expansion upon the 4×1 integrated multiplexer that is detailed in the main text, over (a) the operation bandwidth of the multiplexer, and (b) a small, 5-GHz subset of the operation bandwidth, in order to show the frequency shift in detail. The bounds of the plot in (b) are indicated with a rectangular box in (a).
this reason, the simulation model is scaled by a factor of 1.001, as a reasonable upper-bound for thermal expansion that the device may undergo during deployment. The results of this simulation are given in Fig. S2, where it can be seen that the heated device undergoes a minor downward frequency shift. The precise value of this downward frequency shift is quantified using cross-correlation, and is found to be 0.34 GHz, 0.34 GHz, 0.31 GHz, and 0.29 GHz for channels 1–4, respectively. We reason that this is unlikely to negatively impact performance, and hence we infer that this integrated multiplexer exhibits robust thermal stability.

S4. Calibration and coupling efficiency

The multiplexer device is interfaced with the measurement system via a 2 mm-long linear-tapered spike with an 80 µm-wide snub tip. This taper is inserted directly into the internal air-filled volume of a hollow rectangular metallic waveguide, as shown in Fig. S3(a). In experiment, precise alignment is achieved using micrometer stages. In order to calibrate out this coupling loss, a reference device containing only the coupling sections is fabricated and is measured along with that of the multiplexer, and subsequently used for normalization. This transmission magnitude cannot be measured directly, owing to a lack of a suitable means to normalize the response of the measurement system itself, and hence simulation must serve to show coupling efficiency. The associated simulation model is given as an inset to the plot. For this structure, the port on the left side bears an effective medium buttress that is identical to that of the multiplexer that is shown in Fig. 3(a). The right-hand side is clad with an effective medium, following the example of [17] and [18]. Put simply, the left- and right-hand ports of the displayed reference device correspond to the combined port and channel ports of the multiplexer, respectively. Hollow waveguides are attached to both of these ports, and are excited with their fundamental modes. The resulting simulated transmission magnitude through the reference device is shown in Fig. S3(b). It can be seen that coupling loss is increased at lower frequencies, owing to reduced field confinement that commutes a greater proportion of modal fields into the surrounding space, where they are not admitted into the micro-scale aperture of the hollow rectangular waveguides. The transmission magnitude encounters −6 dB at ~300 GHz. Assuming that roughly half the coupling loss is
contributed by each port, this means that the 3 dB bandwidth of a given coupler is limited to frequencies above 300 GHz. Thus, the operation bandwidth of the multiplexer that is the main subject of this work is wholly within this range. Overall coupling loss is decreased to ~0.5 dB at higher frequencies, corresponding to ~94% peak efficiency for a single coupler.

S5. Over-etching

Microscope images of the tunneling waveguide are given in Fig. S4, showing reduction of waveguide width due to over-etching. This alters the dispersion of the waveguide’s leaky mode, and hence it alters the beamforming behavior of the slab-confined beam, causing an upward shift in the center frequency of each channel.

S6. Dispersion performance

For a given profile of complex transmission magnitude, the dispersion-limited bandwidth of a channel at a specific carrier frequency may be estimated by considering of a subset of the operation bandwidth that is centered upon that carrier frequency. The impulse response of this subset is approximated as a Gaussian pulse, which is described by its full-width at half-maximum duration in the time-domain, $\Delta \tau$. In order to relate this pulse to the transmission response, group delay is computed therefrom, and peak deviation (i.e. from highest to lowest value of time delay) is evaluated. This is considered to represent the time-window within which the Gaussian pulse exists, and hence it is also given by $\Delta \tau$. As a consequence, the full-width at half-maximum bandwidth of the Gaussian pulse is given by [15],

$$\text{Bandwidth} = \frac{\sqrt{2 \ln 2}}{\pi \Delta \tau} \quad [\text{Hz}] \quad (S21)$$

Only deviations in group delay within the subset of operation bandwidth are considered for this calculation. Thus, the calculation of dispersion-limited bandwidth is iterative; the range of frequencies under consideration is progressively expanded until Eq. S21 holds, and the final frequency span that meets this condition is reported as the dispersion-limited bandwidth. It is noted that, as dispersion pertains exclusively to phase response, the magnitude response is essentially neglected for the purposes of this calculation.

For all channels of the frequency-division multiplexer that is detailed in Section 3.A, group delay is extracted from the simulated phase response, and results are given in Fig. S5(a)–(d). It can be seen that, within the frequency allocation of each channel, there is little variation in group delay. The dispersion-limited bandwidth all channels is estimated using Eq. S21, and the results given in Fig. S5(e) show peak bandwidth in the vicinity of 8–11 GHz for all four channels. This is reasonably consistent with the achievable error-free data rates that are observed in Fig. 5(b). It is noted that the calculated dispersion-limited bandwidth is lower than the measured and simulated spectral bandwidth, and hence we may conclude that dispersion is the primary limiting factor upon the achievable data rate.
S7. Crosstalk

We aim to quantify the impact of crosstalk on the performance of the multiplexer device that is the main subject of this work. The configuration that is under consideration is displayed in Fig. S6(a), showing two multiplexers that are connected at the common port. One multiplexer serves to join the signals supplied to each input channel port, and the other (i.e. the demultiplexer) separates them. The transmission magnitude through a single multiplexer is given by $|T_m(f)|$ [W/W] for the $m$th channel. The input signal at each channel port of the multiplexer is represented abstractly as a Gaussian-distributed power-spectral density that is shifted to the center frequency of the relevant channel. The bandwidth of this input signal is identical for all channels, and is variable,

$$G_m(f) = \exp\left[-a(f - f_{\text{center},m})^2\right], \quad [\text{W/Hz}]$$

(S22)
Fig. S6. Impact of cross-talk, showing (a) the configuration that is considered in analysis, with two back-to-back frequency-division (de)multiplexers, showing the transit of the desired signal at channel 1, and cross-talk between channels 2 and 3, and (b) calculated signal-to-interference ratio for each channel.

\[ a = \frac{\ln \sqrt{2}}{(\text{Signal Bandwidth}/2)^2} \]  \hspace{1cm} (S23)

The signal that is received at the nth output channel due to stimulus at the mth input channel is given by,

\[ |X_{n,m}(f)| = |G_{m}(f)||T_{m}(f)||T_{n}(f)|, \quad [\text{W/Hz}] \]  \hspace{1cm} (S24)

as it incorporates transit from input port m to the common port of the multiplexer, and subsequently from the common port of the demultiplexer to output port n. It is noted that this neglects coupling loss in order to isolate the impact of crosstalk upon the multiplexer itself. The total output power of the desired signal that is observed at output port n may be computed by integration over the frequency domain,

\[ S = \int_{-\infty}^{\infty} |X_{n,n}(f)|\,df, \quad [\text{W}] \]  \hspace{1cm} (S25)

likewise, the power of the interference signal is computed by integration, and the overall power is summed over all interfering input ports,

\[ I = \sum_{m\neq n} \int_{-\infty}^{\infty} |X_{n,m}(f)|\,df, \quad [\text{W}] \]  \hspace{1cm} (S26)

These integrals imply that the detector bandwidth is infinite, which is an approximation that serves as a crude representation of the broadband SBD that is deployed in the experimental demonstration of terahertz-range communications that is detailed in the main text. Finally, the impact of crosstalk is quantified by computation of the ratio of desired signal power to interfering power. This procedure is applied to the measured transmission that is observed in Fig. 3(g) of the main text, and results are given in Fig. S6(b), showing crosstalk power ranging from 10 dB to 20 dB.