

# Time-multiplexed heralded single-photon source: supplementary material

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## 1. EXPERIMENTAL DETAILS

A schematic diagram of our experimental setup is shown in Fig. 2 in the main text. We used a frequency-tripled mode-locked Nd:YAG laser ( $\lambda = 355$  nm,  $\Delta t \sim 5$  ps,  $\tau = 8.33$  ns) to pump a bismuth barium borate (BiBO) crystal that generates non-collinear photon pairs centered at 710 nm via Type-I phase-matching. Each photon produced by the SPDC was passed through an interference filter (IF,  $\Delta\lambda = 20$  nm), and then coupled into a single-mode fiber (SMF), with a coupling efficiency above 70% [1]. We used a cascade of four SPDs, each a silicon avalanche photodiode (Si-APD), with detection efficiency of  $\sim 70\%$ . The detector cascade allows us to reduce the effect of each SPD’s saturation and to resolve the approximate number of photon pairs produced in a time slot. By measuring the ratio of single- and coincidence-count rates from the SPDC source, we estimate that the total system detection efficiency of the signal photons  $\eta_{Ds} = 41.8 \pm 0.4\%$ . A field-programmable gate array (FPGA) module processes input signals from the SPDs, and triggers PCs at heralded and output time slots.

A Herriott-cell-type optical delay line [2] is made of two concave spherical mirrors separated by  $\sim 1$  m. The idler photon enters the cell through a hole in one mirror, bounces 54 times between the two mirrors, and escapes from the cell through a hole in the other mirror. We used one way (round trip) of the Herriott cell to implement  $\Delta t_{DL} \sim 200$  ns ( $\sim 400$  ns), with transmission of 89.3% (84.6%).

For an adjustable photon storage cavity, we used a custom Brewster-angled polarizing beamsplitter (PBS1) and PC comprising a pair of rubidium titanyl phosphate (RTP) crystals. The

single-pass cavity transmission ( $T_c = 97.0\%$ ) is limited by imperfect reflection of mirrors and PBS1, and the PC’s imperfect transmission and polarization switching. A PC and PBS2 after the cavity are used as an optical shutter, which opens in an output time window of the multiplexing only if at least one heralding signal is detected within  $N$  multiplexed time slots, and drops unheralded photons. The total transmission of other optics for the idler mode  $T_i$  is 92.4%.

In order to investigate the multiplexed single-photon probability using Eq. (1) in the main text, we estimated the system detection efficiency for the multiplexed photon  $\eta_{Di} = 56.8\%$ , accounting for the measured transmission of the non-polarizing 50:50 beam splitter (NPBS, 89.9%), fiber coupling efficiency and transmission (90.2%), and SPD detection efficiency (70%, as listed in manufacturer specification sheet).

## 2. THEORY OF A TIME-MULTIPLEXED HERALDED SINGLE-PHOTON SOURCE

Here we show theoretical details of a time-multiplexed single-photon source with inefficient optical components. As discussed in the main text, for the case that multiple time slots within  $N$  are heralded, it is more efficient to use the photon that is created last. The probability of emitting an  $m$ -photon state by multiplexing  $N$  time slots and storing the last-born photons is

given by

$$P_M(m) = \sum_{j=1}^N \left(1 - \sum_{k_1=1}^{\infty} P_c(k_1) P_d(1|k_1)\right)^{N-j} \times \sum_{k_2=1}^{\infty} P_c(k_2) P_d(1|k_2) P_e(m|j, k_2), \quad (\text{S1})$$

where  $P_c(k)$ ,  $P_d(1|k)$ , and  $P_e(m|j, k)$  are, respectively, the probability of generating  $k$  photon pairs in a time slot, detecting a signal photon conditioned by  $k$  photon pairs created, and emitting  $m$  idler photons conditioned by  $k$  photon pair generations in the  $j$ -th time slot. The first part in Eq. (S1) is the probability that a single signal photon detection has not occurred in the “last”  $N - j$  time slots. The second part is the probability that a single signal photon is detected at the  $j$ -th time slot, and  $m$  photons are released at the output time window. For pure heralded photons via SPDC generating separable photon pairs,  $P_c(k)$  follows a thermal distribution [3]:  $P_c(k) = p^k / (1 + p)^{k+1}$ , where  $p$  is the mean number of created photon pairs per time slot. However, the observed purity of our heralded idler photons is  $\text{Tr}\rho^2 \sim 0.05$ , indicating that our SPDC source generates photon pairs into  $s \sim 1/\text{Tr}\rho^2 \sim 20$  Schmidt modes. Assuming that each of  $s$  Schmidt modes has the identical mean photon numbers per time slot  $p/s$ ,  $P_c(k)$  follows the distribution:

$$P_c(k) = \frac{(p/s)^k}{(1 + p/s)^{k+s}} \binom{k+s-1}{k}. \quad (\text{S2})$$

For  $s \gg 1$ , and thus for our SPDC source, Eq. (S2) closely follows a Poissonian distribution:

$$P_c(k) \simeq \frac{e^{-p} p^k}{k!}. \quad (\text{S3})$$

The detection probability of a signal photon  $P_d(1|k)$  is given by

$$P_d(1|k) = \sum_{l=1}^k \eta_{Ds}^l (1 - \eta_{Ds})^{k-l} \binom{k}{l} \left(\frac{1}{D}\right)^{l-1}, \quad (\text{S4})$$

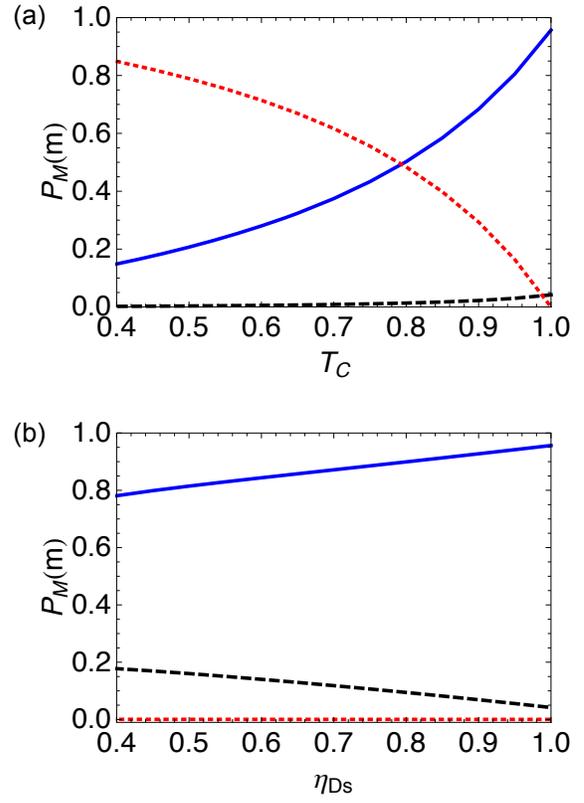
where  $\eta_{Ds}$  is the total transmission of the signal photons from the SPDC crystal to SPDs, and  $D$  is the number of SPDs used for a trigger-detector cascade to herald idler photons. Here, we assume that the SPDs are “bucket” detectors that discriminate between zero and one-or-more photons, and the detector cascade distributes the signal photons to  $D$  detectors with an equal probability ( $1/D$ ). By substituting  $D \rightarrow \infty$ , Eq. (S4) can also describe a photon-number-resolving detector:  $P_d(1|k) = \eta_{Ds} (1 - \eta_{Ds})^{k-1}$ . The idler photon’s transmission to an output time slot  $P_e(m|j, k)$  is given by

$$P_e(m|j, k) = (T_i T_c^{N-j})^m (1 - T_i T_c^{N-j})^{k-m} \binom{k}{m}. \quad (\text{S5})$$

The  $T_c$  and  $T_i$  denote the idler photon’s storage efficiency in the switchable storage cavity and other optics transmission (including an initial delay line, fiber coupling efficiency, etc.), respectively.

For the case without an extra optical delay line to determine the last heralded idler photon,  $P_M(m)$  for storing the earliest-born photons is given by [4]:

$$P_M(m) = \sum_{j=1}^N \left(1 - \sum_{k_1=1}^{\infty} P_c(k_1) P_d(1|k_1)\right)^{j-1} \times \sum_{k_2=1}^{\infty} P_c(k_2) P_d(1|k_2) P_e(m|j, k_2). \quad (\text{S6})$$



**Fig. S1.** Theoretical multiplexed  $m$ -photon probability  $P_M(m)$  versus (a) transmission of a storage cavity  $T_c$  and (b) system detection efficiency of signal mode  $\eta_{Ds}$ . We used Eq. (S1), substituting fixed values of  $N = 30$ ,  $T_i = 1$ ,  $D = 4$ ,  $p = 0.35$ , and  $\eta_{Ds} = 1$  for (a) ( $T_i = 1$  for (b)). Blue solid curve:  $P_M(1)$ . Black dashed curve:  $P_M(2)$ . Red dotted curve:  $P_M(0)$ .

The above equation is similar to Eq. (S1) except that the first part in Eq. (S6) describes the probability that the single signal photon is not detected in the “first”  $j - 1$  time slots. As shown in Fig. 3 (c) in the main text, Eq. (S6) has an optimal number of time slots  $N_{\max}$  to maximize  $P_M(m)$ , because increasing  $N$  effectively results in larger photon storage times, and associated larger loss in the storage cavity.  $N_{\max}$  is obtained by solving  $\partial P_M(m)/\partial N = 0$  in  $N$ :

$$N_{\max} \simeq \frac{\log(\log(1 - \alpha)) - \log(\log T_c)}{\log T_c - \log(1 - \alpha)}, \quad (\text{S7})$$

$$\alpha = \sum_{k_1=1}^{\infty} P_c(k_1) P_d(1|k_1). \quad (\text{S8})$$

For our experiment,  $N_{\max} \sim 14$  and  $\sim 29$  for  $p = 0.35$  and  $0.07$ , respectively.

Figure S1 (a) shows theoretical calculations of  $P_M(m)$  versus  $T_c$  with fixed experimental parameters  $N = 30$ ,  $T_i = 1$ ,  $D = 4$ ,  $\eta_{Ds} = 1$ , and  $p = 0.35$ . In order to obtain a large enhancement in  $P_M(1)$  (see blue solid line in Fig. S1 (a)), one needs very high ( $> 95\%$ ) cavity transmission; even a time-multiplexed source with a moderately high (70-80%)  $T_c$  performs only up to  $\sim 50\%$  of the case for  $T_c = 1$ . In contrast, the system detection efficiency  $\eta_{Ds}$  is a much less critical parameter for  $P_M(1)$  (see blue solid line in Fig. S1 (b)), though a higher detection efficiency for signal photons can better suppress the undesirable two-photon

emission probability  $P_M(2)$  (black dashed line in Fig. S1 (b)), and thus the second-order correlation function  $g^{(2)}(\tau = 0)$ , another important factor for single-photon sources. We find that optimizing the value of  $p$  for each  $T_c$  and  $\eta_{Ds}$  slightly improves the  $P_M(1)$  probability.

## REFERENCES

1. B. G. Christensen, K. T. McCusker, J. Altepeter, B. Calkins, T. Gerrits, A. E. Lita, A. Miller, L. K. Shalm, Y. Zhang, S. W. Nam, N. Brunner, C. C. W. Lim, N. Gisin, and P. G. Kwiat, "Detection-loophole-free test of quantum nonlocality, and applications," *Phys. Rev. Lett.* **111**, 130406 (2013).
2. C. Robert, "Simple, stable, and compact multiple-reflection optical cell for very long optical paths," *Appl. Opt.* **46**, 5408–5418 (2007).
3. A. Christ, K. Laiho, A. Eckstein, K. N. Cassemiro, and C. Silberhorn, "Probing multimode squeezing with correlation functions," *New J. Phys.* **13**, 033027 (2011).
4. E. Jeffrey, N. A. Peters, and P. G. Kwiat, "Towards a periodic deterministic source of arbitrary single-photon states," *New J. Phys.* **6**, 100 (2004).