Setting mean transcription rate as constant does not affect rhythmic pattern

Eqs. (1) and (2) from the Methods section are copied below.

Long-tailed mRNA: \[ \frac{dL}{dt} = \kappa_{\text{trsc}}(t) - \kappa_{\text{deA}}(t)L + \kappa_{\text{polyA}}(t)S \] (S1)

Short-tailed mRNA: \[ \frac{dS}{dt} = \kappa_{\text{deA}}(t)L - \kappa_{\text{polyA}}(t)S - \kappa_{\text{dgrd}}(t)S \] (S2)

Note that \( \kappa_{\text{trsc}}(t) = k_{\text{trsc}} (1 + A_{\text{trsc}} \cos(\omega(t - \varphi_{\text{trsc}}))) \). We can divide both sides of Eqs. (S1) and (S2) by the mean transcription rate constant, \( k_{\text{trsc}} \) and obtain the following equations:

\[ \frac{d(L/k_{\text{trsc}})}{dt} = (1 + A_{\text{trsc}} \cos(\omega t - \varphi_{\text{trsc}})) - \kappa_{\text{deA}}(t) \frac{L}{k_{\text{trsc}}} + \kappa_{\text{polyA}}(t) \frac{S}{k_{\text{trsc}}} \] (S3)

\[ \frac{d(S/k_{\text{trsc}})}{dt} = \kappa_{\text{deA}}(t) \frac{L}{k_{\text{trsc}}} - \kappa_{\text{polyA}}(t) \frac{S}{k_{\text{trsc}}} - \kappa_{\text{dgrd}}(t) \frac{S}{k_{\text{trsc}}} \] (S4)

Eqs. (S3) and (S4) show that changing \( k_{\text{trsc}} \) only causes a strictly proportional change of \( L(t) \) and \( S(t) \). The rhythmicity patterns, including the peak phases and relative amplitudes, of all output quantities, will not be affected at all. When \( k_{\text{trsc}} \) changes, the means of \( L(t) \) and \( S(t) \) change proportionally, but mean L/S ratio remains the same.