MONETARY POLICY AND HETEROGENEOUS LABOR MARKETS

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ABSTRACT

Chaudhuri, Pritha PhD, Purdue University, August 2019. Monetary Policy and Heterogeneous Labor Markets. Major Professor: Trevor S. Gallen.

Labor market indicators such as unemployment and labor force participation show a significant amount of heterogeneity across demographic groups, which is often not incorporated in monetary policy analysis. This dissertation is composed of three essays that explore the effect of labor market heterogeneity on the design and conduct of monetary policy. The first chapter, **Effect of Monetary Policy Shocks on Labor Market Outcomes**, studies this question empirically by looking at dynamics of macroeconomic outcomes to a monetary policy shock. I construct a measure of monetary policy shock using narrative methods that represent the unanticipatory changes in policy. Impulse response of unemployment rates for high and low-skill workers show low-skill workers bear a greater burden of contractionary monetary policy shock. Their unemployment rates increase by almost four times that of the high-skill group. Even though we see differences in dynamic response of unemployment rates, the empirical analysis shows some puzzling results where effects of contractionary shock are expansionary in nature. Moreover, these results are plagued by the recursiveness assumption that the shock does not affect current output and prices, which is at odds with theoretical models in the New Keynesian literature. In the second chapter, **Skill Heterogeneity in an Estimated DSGE Model**, I use a structural model to better identify these shocks and study dynamic responses of outcomes to economic shocks. I build a dynamic stochastic general equilibrium model, which captures skill heterogeneity in the U.S. labor market. I use Bayesian estimation techniques with data on unemployment and wages to obtain distribution of key parameters of the model. Low-skilled workers have a higher elasticity of labor

supply and labor demand, contributing to the flatness of the wage Phillips curve estimated using aggregate data. A contractionary monetary policy shock has immediate effects on output and prices, lowering both output and inflation. Moreover, it increases unemployment rates for both high and low-skill groups, the magnitude being larger for the latter group. The presence of labor market heterogeneity will have new implications for the design of monetary policy, that I study in the third chapter, **Optimal Monetary Policy with Skill Heterogeneity**. I design an optimal policy for the central bank where policymakers respond to the different inflation-unemployment trade-off between high and low-skill workers. The monetary authority must strike a balance between stabilization of inflation, GDP and outcomes of high and low-skill workers separately. This optimal policy can be implemented by a simple interest rate rule with unemployment rates for high and low-skill workers and this policy is welfare improving.

1. EFFECT OF MONETARY POLICY SHOCKS ON LABOR MARKET OUTCOMES

1.1 Introduction

Demographic groups have diverse experiences over the business cycle. Several studies have shown that recovery since the Great Recession has been unequal for for different sections of the labor market.[1] Outcomes of low-skill workers, that is those people with low education, typically fare worse during recessions. The recovery in employment levels since the Great Recession of 2008 has been varied for different skill levels, as illustrated in Figure 1.1 (a). People with high-skill, or higher education levels such as Bachelors degree, have seen a steady increase in employment levels since the end of the recession. However, low-skill or lower educated groups have not had the same experience. Individuals with some college education or Associates degree experience a slight improvement in their outcomes that does not begin until late 2011. As for individuals with high school degrees or less, their outcomes have shown almost no improvement since the end of the recession. Figure 1.1 (b) depicts a similar story for recovery in labor force participation rates.

In this paper, I show a contractionary monetary policy shock causes different responses of high and low-skill unemployment rates thus implying that a policy change would have different implications for the two groups. The monetary policy shocks that I use follow the identification method introduced in Romer and Romer (2004). This strategy identifies the unanticipatory changes in monetary policy actions taken by the Federal Reserve after controlling for the information the policymakers have regarding the state of the economy from Greenbook forecasts. I employ three different empirical methods, following three canonical papers in the literature, and look at dynamic

[1] Zago (2018); Elsby et al. (2010)

responses of macroeconomic outcomes to the Romer and Romer (2004) shocks over the period 1979 to 2008. The particular variables of interest are a measure of output, for which I use the industrial production index, and price indices, for which I use the consumer price index and a measure of global price index. To see how monetary policy shocks affect different skill groups in the labor market I use high and low-skill unemployment rates and the wage premium, the difference between high and low-skill wages.

The results from the empirical analysis are somewhat puzzling. In general contractionary monetary policy shocks have an expansionary effect on the economy during this time period, a result that has also been shown by Ramey (2016) and Barakchian and Crowe (2013). Industrial production is seen to rise and unemployment levels show a decrease when there is an increase in the policy rate. However, the responses of unemployment rates for the two skill groups behave differently. The magnitude of impact for the low-skill unemployment is much larger than the high-skill and more persistent. In a standard vector autoreression framework the low-skill unemployment reaches a peak of 5% increase in unemployment in less than a year, whereas that increase is only about 1% for the high-skill and takes longer.

The empirical analysis conducted in this paper reinforces the idea that monetary policy has different implications on different sections of the economy. Moreover, the results in this paper are puzzling and raises questions on how well true monetary policy shocks are being identified given the superior nature of how policy is being conducted by the Federal Reserve in recent years. Over the past couple of decades the Federal Open Markets Committee (FOMC) has been communicating its policy intentions to the markets and the public so as to mitigate sudden impacts of a policy change. In that setting trying to analyze the impact of monetary policy on the labor market and its heterogeneous participants is better done through a structural model where the true shocks are well identified by combining model with data. These models also provide a framework to study the design of policy in such a setting and quantify its welfare implications. This paper provides an empirical motivation to that idea.

1.1.1 Related Literature

Over the years several papers have studied monetary policy shocks and their effects on the macroeconomy. There have been various different methods that have been used to identify monetary policy shocks and estimating impulse responses of endogenous variables to these shocks. Early works of Sims (1972) and Barro (1977) developed time series methods to investigate the effects of monetary policy. The first paper to introduce a narrative method of identifying a policy shock was Romer and Romer (1989). Several papers in the 1990s used vector autoregression (VAR) models and structural VAR models with a particular form of Cholesky decomposition to identify the monetary policy shock.

One of the most common papers in the literature that uses the VAR framework is Christiano et al. (1999) (CEE henceforth). Here the authors introduce the "recursiveness assumption" according to which the variables introduced in the first block of the VAR, usually output, inflation and commodity prices, does not respond to the shock within that period. Romer and Romer (2004) (RR henceforth) develop a narrative method to construct a new measure of monetary policy shocks by using Greenbook forecasts and other historical documents. The authors estimate impulse responses of industrial production and prices to these shocks through single equation regressions and a variant of the VAR specification in CEE.

One problem that might arise with VAR and SVAR methods is that if the model is misspecified the estimated impulse responses will be distorted. Jordà (2005) find an alternative local projection method to estimate impulse responses and impose no restriction on the data generating process. Directly regressing the variable of interest on the shock gives us an estimate of the response of the variable to the shock and is used to compute the impulse responses. Another common method of shock identification is the high frequency approach that uses high frequency data such as news announcements around FOMC meetings and changes in fed funds futures. Gertler and Karadi (2015) uses this approach and investigates the effect of a monetary policy shock on financial variables through a proxy SVAR model. Lastly, some alternative to VAR models are regime switching models of monetary policy as in Owyang and Ramey (2004) and factor augmented VAR approach developed in Bernanke et al. (2005).

Several papers have documented the disparate impact of monetary policy on labor market outcomes for different demographic groups in the economy. Carpenter and Rodgers (2004) show a contractionary monetary policy shock lowers the employment-to-population ratio of less skilled workers, African Americans and out-of-school teenagers by increasing their unemployment rates. Seguino and Heintz (2012) find monetary policy to be neither race nor gender neutral as disinflationary policy increases unemployment for black women and men more than white women and men. Thorbecke (2001) finds greater burden of a contractionary monetary policy on minorities due to wage gap that exists between minorities and whites. These papers however employ a standard policy shock which is an innovation in the policy rate or the Federal Funds Rate. This paper contributes to the literature on disparate impacts of monetary policy by using a different measure of monetary policy shocks and considering skill differentials as the source of heterogeneity in the economy.

1.2 Identification of Monetary Policy Shocks

Identification of the monetary policy shock is key before looking at the effect of the shock on macroeconomic outcomes. As Ramey (2016) notes, the shock must be exogenous to current and lagged endogenous outcomes and to other exogenous shocks in the system. It should also represent unanticipated movements in the exogenous variable. In terms of monetary policy these shocks usually represent changes in the preferences of policymakers at central banks. The most common challenge to identifying such shocks is the problem of foresight on the part of the policymaker or news about future policy actions. An example of the latter is the Federal Reserve adopting forward guidance since the 1990s which would result in changes in the policy rate being anticipated in advance by the economy. Several papers have used the high frequency approach to identify "news" shocks (Gürkaynak et al. (2005); Barakchian and Crowe (2013); Gertler and Karadi (2015), to name a few) and study it's impact on the economy.

To tackle the foresight problem of the policymaker and obtain a shock series that is free of the anticipated part of policy, Romer and Romer (2004) (RR henceforth) use narrative methods to identify innovations to monetary policy that are free of anticipatory changes in the policy rate by the Federal Reserve. In this paper I use this new measure constructed by RR. The authors first derive a measure of Federal Reserve intentions for the Federal Funds Rate around FOMC meetings between the years 1969 until 1996. This is the narrative part of the approach where the authors use historical documents such as the Record of Policy actions, Minutes and Transcripts of Federal Open Market Committee and Monetary Policy alternatives document also known as the Bluebook. This represents the set of information available to the Fed. Using this the authors construct a measure of changes in the target Federal Funds Rate at each FOMC meeting that represents the Federal Reserve's intentions.

In the second step the authors wish to remove information that is available to the policymaker about the state of the economy from intended policy actions. To do this RR use the Greenbook forecasts prepared by the staff to get a series of intentions that are free of anticipatory movements. The particular macroeconomic indicators they use include growth rate of GDP, GDP deflator and unemployment rate. Using FOMC meetings as the unit of observation, regressing the change in the target rate on Greenbook forecasts gives us estimated residuals that represent the unanticipated movements and are defined as monetary policy shocks by RR.

The specification used is as follows

$$\Delta ff_m = \alpha + \beta ffb_m + \sum_{i=-1}^{2} \gamma_i \Delta y^F_{mi} + \sum_{i=-1}^{2} \lambda_i \big(\Delta y^F_{mi} - \Delta y^F_{m-1i}\big) + \sum_{i=-1}^{2} \phi_i \pi^F_{mi} + \sum_{i=-1}^{2} \theta_i \big(\pi^F_{mi} - \pi^F_{m-1i}\big) + \rho u^F_{m0} + \epsilon_m$$

where m denotes the FOMC meeting, Δff_m is the change in the intended funds rate around the FOMC meeting and ffb_m is the target funds rate going into the meeting. Δy_m^F, π_m^F and u_m^F are Greenbook forecasts from meeting m for real output growth, inflation and unemployment in months around meeting m ($i = -1$ is the previous month, 0 is current, 1 and 2 are the following two months). Only the current month's unemployment forecast is used in the specification due to the Okun's law relationship between output and unemployment.

The fitted residuals $\hat{\epsilon}_m$ is the monetary policy shock as defined by RR. This shock series is then converted to a monthly series by allocating each shock to the month in which the FOMC meeting occurred. If there were two meetings in a month the sum of the shocks for that month was used and if there was no meetings in a month the shock was assigned a 0 value. Following Coibion et al. (2017), I extend the RR data set to December 2008 to include more recent changes in the target policy rate.

1.2.1 Discussion: History of U.S. Monetary Policy

The shock series obtained shows a large amount of variation that can be used to identify the effects of monetary policy on macroeconomic outcomes. Before moving on to the dynamic effects, I first discuss some observations from the shock series that is consistent with the history of U.S. monetary policy. Figure 1.2 shows a graph of the monetary policy shock measure (presented quarterly) with the shaded regions representing U.S. recessions. The 1970s saw Arthur Burns as the chairman of the Federal Reserve and the "stop-go" policy where inflationary expectations were crucial. During this time the view of the central bank, though not explicit, was highly anchored on the Keynesian idea of the Phillips curve trade-off that lower inflation required high unemployment rates. This period saw considerable movement in the Federal Funds Rate with expansionary policies in the early 1970s resulting in negative values of the shock series. This view changed in 1973-74 when the Fed said the current level of inflation was too high and interest rates spiked in 1974.

This period was followed by the Volker-Greenspan era of monetary policy that was accompanied by disinflation and moderate unemployment. Paul Volker rejected the Keynesian approach and followed a monetarist regime through which the Fed provided a nominal anchor through a rule that would stabilize inflationary expectations in the market. This led to the inflation scare after the 1982-83 recession resulting in rising rates where the shock series shows positive values. The "leaning against the wind with credibility" policy of the Federal Reserve continued when Alan Greenspan took over from Volker as chairman in 1987. We can see similar rises in interest rates during 1994 in response to inflation scares. The Volker-Greenspan era of monetary policy regime created a long period of economic stability and reduced volatility in economic fluctuations and is commonly known as the Great Moderation.

However, the stagflation of the 1970s and the disinflation and moderate unemployment rates of the 1980s and 1990s paved the way for New Keynesian models for monetary policy that combined the price stability and full employment goal for the Federal Reserve. During the Volker-Greenspan era the Fed changed it's manner of conducting policy by communicating its strategies to the markets through changes in the funds rate and in turn anchoring inflation expectations. The New Keynesian approach built on this idea and included the Phillips curve relationship between inflation and unemployment where the price system would be allowed to determine real variables. The policy maintained nominal expectational stability and avoided exploiting the Phillips curve trade-off through a policy rule for the funds rate.

The Great Recession that began in 2008 raised questions about the non-activist policy of the Volker-Greenspan era. The inflation targeting nature of monetary policy and disruption to financial intermediation contributed to the recession, as argued by the critics of the Federal Reserve. The early 2000s saw expansionary policies with interest rates being lower than what the Fed staff forecasts would have called for. Taylor (2009) argues that these low rates led to the housing boom and bust, one of the causes of the Great Recession. Beginning in 2003 the Fed continued to raise rates till 2007. As the recession started to set in the Fed started to lower rates and then we see a sharp fall in the shock series starting in 2008.[2]

The continuous change in how monetary policy is conducted by the central bank, and how policymaker's views and perceptions influence it, is captured by this shock series. The fact that monetary policy was influenced by strong Keynesian views in the 1970s to monetarist views in the 1980s can be understood through fluctuations in the series and this variation will help identify the effect of monetary policy on macroeconomic outcomes. Moreover, changes in the conduct of policy in the 1990s and 2000s, when the Federal Reserve started communicating it's intentions more clearly, has been included while developing this shock series. The shock series represents unanticipatory changes in the policy rate given the information available to policymakers at the time of making policy decisions. Hence, this series is a good representation of a monetary policy "shock" measure that can be used to study how policy effects movements in macroeconomic outcomes.

1.3 Effects of Monetary Policy Shocks

In this section I explore the dynamics of labor market outcomes in response to a monetary policy shock. To provide a basis for comparison I use empirical specifications from three canonical papers. I start with the specification in Christiano et al. (1999) that uses the standard VAR approach with innovations to the federal funds rate as the monetary policy shock. Secondly, I discuss results from the single dynamic equation and hybrid VAR approach specified in Romer and Romer (2004) using the new RR shock measure. Finally, to relax the restrictions imposed in the VAR specifications I also compute impulse responses using Jordà (2005)'s local projections method with the RR shocks.

[2]For more information on the history and evolution of U.S. monetary policy see Hetzel (2018).

1.3.1 Data

Before moving on to the empirical strategy let me describe the macroeconomic outcomes that are used and their sources. Since the unit of time considered in the analysis is a month, I use industrial production index as the measure of output. This data is released by the Board of Governors of the Federal System and maintained by the FRED database. The outcome used in the analysis is the log of this index. The price indices used in the analysis include consumer price index and a global price index. The consumer price index is a measure of average change in monthly prices for goods and services available to urban consumers. This data is available at the U.S. Bureau of Labor Statistics and retrieved by the FRED. The global price index is a representative of the global markets and the data is available from the International Monetary Fund. The log values of these price indices are used in the analysis. The Federal funds rate data is available from the Board of Governors.

Labor market outcomes are taken from the Current Population Survey (CPS), a monthly survey of employment and labor markets of approximately 60,000 households. The public use micro data files are maintained by the Bureau of Labor Statistics and the NBER has prepared extracts of these files since 1979. An individual (representing a household) is interviewed for the first 4 months, ignored for the next 8 and interviewed again for the last 4 months. The Outgoing Rotation Group consists of households who have been asked to report weekly hours and earnings in months 4 and 8. These interviews form the Merged Outgoing Rotation Group files for a month in a year. I use these files to calculate the unemployment rate and wages. First I divide the population into high and low-skilled households/individuals using their education levels. An individual is high-skilled if he has a Bachelors degree or higher. Using the employment/labor force status recode I can calculate the population of employed and unemployed for both high and low-skill groups and hence the unemployment rates. Wages in this data set is calculated using the weekly earnings of a household and the hours worked per week. The wage premium is the difference in high and low-skill wages.

1.3.2 Christiano, Eichenbaum and Evans (1999) model

A key assumption made by Christiano et al. (1999) is the "recursiveness assumption" for identifying the monetary policy shocks. Let us consider a standard VAR framework

$$Z_t = B_1 Z_{t-1} + ... + B_q Z_{t-q} + \epsilon_t$$

where $Z_t = (X_{1t}, S_t, X_{2t})$ consists of 3 blocks. Here S_t is the instrument of the monetary authority such as the Federal Funds Rate. The monetary authority sets policy according to a rule given by

$$S_t = f(\Omega_t) + \sigma_s \epsilon_t^s$$

where Ω_t is the information set available to the policymaker and $\sigma_s \epsilon_t^s$ is the monetary policy shock. The recursiveness assumption states that ϵ_t^s is orthogonal to Ω_t which means the monetary policy shock does not affect contemporaneous values of the variables in the information set.

I can re-write the above system in a simplified form with 3 blocks as follows

$$X_{1t} = C_1 S_t + D_1 X_{2t} + \epsilon_{1t}$$

$$S_t = C_2 X_{1t} + D_2 X_{2t} + \epsilon_{2t}$$

$$X_{2t} = C_3 X_{1t} + D_3 S_t + \epsilon_{3t}$$

The contemporaneous values of the variables in X_{1t} appear in Ω_t and hence $C_2 \neq 0$. Also, the policymaker does not observe the variables in X_{2t} when setting the policy rate and hence $D_2 = 0$. The main assumption that the authors make is that the elements of X_{1t} are unaffected by the monetary policy shock or the variables of X_{2t}. This means $C_1 = 0$ and $D_1 = 0$. In Christiano et al. (1999) X_{1t} includes output and price indices and X_{2t} includes monetary aggregates like M1, M2 or non-borrowed reserves. The recursiveness assumption implies changes in Federal funds rate does not affect output, price indices and monetary aggregates within that period.[3] Since the monetary policy shock ϵ_{2t} is the only focus here, no more assumptions are required for identification.

I use a specification similar to Christiano et al. (1999) but do not include the third block of variables that contain money stock measures. The first block of variables include monthly data for log of industrial production, log consumer price index, log world commodity price index, unemployment rates for high and low-skill workers and skill premium. The second block consists of the Federal Funds Rate, ordered last such that an innovation to the variable is orthogonal to contemporaneous values of variables in the first block and all its lags. This innovation is the monetary policy shock.

Figure 1.3 shows the impulse response functions and 95% bootstrap confidence bands for the estimated system over the sample period 1979:1 to 2008:12. The Federal funds rate shows a temporary increase and then falls back to zero in 2 years as would be the case for a contractionary monetary policy shock. However, this contractionary shock does not result in the classical effects and instead show some initial expansionary effects.[4] Industrial production production shows an initial increase for the first 5 months before the contractionary effect of the shock starts to take hold. Once it begins to fall it reaches its trough 15 months later at about 0.15%. Similarly, unemployment rates for high and low-skilled workers shows and initial decrease before increasing, which is the standard result for a contractionary shock. The thing to note here is that once the unemployment rate jumps up, the magnitude of increase is much larger for the low-skill group, reaching a peak of 5% after a year. On the other hand high-skill unemployment only increases by about 1% due to the shock after 1.5 years. Both unemployment rates return to normal in 40 months but the contractionary effects are

[3]Ramey (2016) notes that this assumption is at odds with estimated New Keynesian DSGE models like Smets and Wouters (2007) where monetary policy shocks have immediate effects on output and prices. I find similar results in the model I propose in chapter 2.

[4]These expansionary effects are also found by Ramey (2016) and Barakchian and Crowe (2013).

much greater in magnitude for the low-skill group than the high-skill. The effect of skill premium is very small, showing an initial increase of about 0.1% before decreasing slightly and returning to zero 20 months after the shock.

1.3.3 Romer and Romer (2004) model

In this paper the authors construct a new measure of monetary policy shocks. The authors argue that by regressing the targeted Fed funds rate on Greenbook forecasts and using the residuals as monetary shocks, they are able to control for the foresight of the policymaker or any superior information he might possess. Thw resulting shock series thus only represents the unanticipatory changes in monetary policy. Following the authors I first estimate the effect of the shock on outcomes using a single dynamic regression with lagged values of the dependent variable and the identified shock. The specification is as follows

$$x_t = \alpha_0 + \sum_{k=1}^{11} \alpha_k D_{kt} + \sum_{i=1}^{24} \beta_i x_{t-i} + \sum_{j=1}^{36} \gamma_j S_{t-j} + e_t$$

which includes monthly dummies D_{kt} and lagged values of the dependent variable x_{t-i}. Lagged values of the identified shocks are also included S_{t-j}. In line with the recursiveness assumption in CEE, the monetary policy shock does not affect contemporaneous values of the dependent variable. I consider three different dependent variables: change in log industrial production, high-skill unemployment and low-skill unemployment. Like the authors, for industrial production I include 24 lags of the dependent variable and 36 lags of the monetary policy shock. Following Coibion (2012) I include the same number of lags for the unemployment rate equations. Using the estimates β_i's and γ_j's the impulse response to the monetary policy shocks can be computed. The single equation regressions illustrated in Figure 1.4 show expansionary effects of the monetary policy shock. Industrial production increases and unemployment rates for high and low-skill decrease. The decrease in unemployment rates is persistent for the high-skill but for the low-skill there seems to be a contractionary effect of the shock after 30 months and unemployment rate starts to increase.

Once again we see a greater burden of the contractionary effects on low-skill workers than high-skill.

Romer and Romer (2004) also estimate a VAR system, similar to Christiano et al. (1999), but use their measure of the shock. I use a similar specification where the first block of the VAR contains log industrial production, log CPI, log commodity price index, unemployment rates for high and low-skill workers and the skill wage premium. The second block contains the cumulative RR shock to provide a similar measure as the Fed funds rate. This is ordered last to maintain the recursiveness assumption that the shock does not affect macroeconomic outcomes within a month.

In Figure 1.5 we see the impulse responses of industrial production, unemployment rates, wage premium and the monetary policy shock measure. The response of the targeted Federal funds rate is more persistent in this case and does not return to zero soon. Here the expansionary effects of the contractionary monetary policy shock are more prominent with a constant increase in industrial production that reaches a peak of approximately 0.6% in 4 years. Unemployment rates show a decrease that takes a long time to revert back to normal. However we see the unemployment rate for high and low-skill groups behave differently. The high-skill unemployment decreases and reaches its trough of about 6% in 10 months and starts gradually moving towards zero over time. On the other hand, low-skill unemployment has a more gradual decrease and reaches a trough of about 10% unemployment rate in 1.5 years. So the magnitude of impact on the low-skill group is larger and their unemployment rate also takes a longer time to revert back to zero. In this case I find that a contractionary shock helpd the low-skill workers more than the high-skill. The effect on wage premium is again small and increases to 0.1% and remains at that level. So with a decrease in unemployment levels, the gap in wages for these workers increases persistently.

1.3.4 Jorda (2005) model

The VAR specifications provide some unconventional results where contractionary monetary policy shocks have expansionary effects with a decrease in output and increase in unemployment rates. It is possible that misspecification of the VAR model is leading to biased estimates of the impulse responses. The local projection method introduced by Jordà (2005) is a way to overcome this issue. This method imposes fewer restrictions and is robust to misspecification of the data generating process and also allows for non-linearity in the specifications. Specifically it is free of the recursiveness assumption however the impulse response estimates may be noisy. I estimate the following specification

$$x_{t+h} = \alpha_h + \gamma_h \epsilon_t + \beta_h Y_t + e_{t+h}$$

The dependent variable x at horizon $t + h$ is regressed on the shock ϵ at t. The coefficient γ_h is the impulse response of the variable to the shock at horizon h. Y_t is a list of control variables that include current and two lagged values of log industrial production, log CPI, log price index, unemployment rates for high and low-skill, wage premium and Fed funds rate. I also include lagged values of the shock as in Ramey (2016).

Figure 1.6 shows the impulse responses for industrial production, Federal funds rate, unemployment rates for high and low-skill and the wage premium. I find an initial increase in the funds rate that is standard for a contractionary policy shock, that eventually returns to zero in about 2 years. This result is similar to the results obtained in section 1.3.2 and 1.3.3. The impulse response estimates for the remaining variables are quite noisy and the expansionary effect of the shock still remains.[5] Industrial production response is extremely erratic, but the pattern shows

[5]The volatility of these impulse responses increases significantly depending on the sample period. If the same estimation was carried out using data starting at 1969 and ending in 1996, as in Ramey (2016), the responses would be a lot smoother. This leads me to believe the behavior of macroeconomic variables during the sample period I use might cause the large volatility in the impulse response. Moreover, splitting the unemployment rate by skill types also results in these fluctuations as the aggregate unemployment response to a monetary policy shock for 1969-1996 is smooth. An

an initial increase that remains for almost 1.5 years before returning to zero. The unemployment rates again show the opposite effect where they decrease initially before increasing. The effect of the shock is larger for the low-skill group than the high-skill. Low-skill unemployment decreases by about 0.2% in initial 10 months before starting to increase and reaches a peak of about 0.5% in 2 years. In contrast the initial decrease in high-skill unemployment is 0.1% and reverts back to zero soon after without any substantial increase. Once again we see a greater burden on low-skill unemployment as a result of a monetary policy shock than on high-skill.

To summarize, all three estimation methodologies show a similar theme where contractionary monetary policy shocks produce expansionary effects in output and unemployment rates. The explanation provided by Ramey (2016) says the FOMC responds to more information now other than Greenbook forecasts, which results in the recursiveness assumption where the shock is orthogonal to output and prices in the current period. This assumption is at odds with New Keynesian DSGE models that find immediate effects on prices and output. Moreover, as discussed in section 1.2.1, monetary policy has been conducted differently in the past two decades with the FOMC adopting forward guidance and informing the markets and the public regarding their intentions. With less erratic monetary policy and policymakers now having more information about the state of the economy, information that goes beyond the scope of the Greenbook forecasts, it is harder to identify true shocks to policy. In that case, to truly understand the effect of monetary policy on labor market outcomes it is essential to turn to DSGE models where these shocks are well identified.

1.4 Conclusion

In this paper, I provide empirical motivation on how monetary policy affects aggregate dynamics of unemployment rates for groups with different skill levels. When

alternative explanation to the volatility is the nature of impulse responses produced by the Jorda method. Since this method does not impose any underlying dynamics on the variables, unlike a VAR, the impulse response estimates may end up being noisy.

the economy is divided into high and low-skill workers I find that a monetary policy shock results in different dynamics of unemployment for the two groups. The magnitude of impact is much larger for the low-skill group. The contractionary effects of the shock results in low-skill unemployment increasing by almost 4 times the high skill unemployment. If monetary policy shocks do effect the heterogeneous agents in the economy differently, the monetary authority would want to incorporate that knowledge in its policymaking decisions.

The identification of monetary policy shocks, as shown in this paper and various others before this[6], is slightly problematic especially in the last decade. Even with newer methods of shock identification, such as narrative methods using Greenbook forecasts as in this paper or high frequency identification using high frequency data[7], true shocks are hard to identify. This is mainly because central banks are doing a much better job of communicating their decisions to the public thereby reducing uncertainty in economic activity. Since the 1990s the Federal Reserve has pursued forward guidance and communicated it's decisions well, reducing market uncertainty and the economy has seen stable activity. Thus, true shocks to policy are becoming harder to identify which can be seen in the results of this paper where contractionary shocks produce expansionary effects on output and unemployment.

Thus arises the need for a structural model of the economy with a specified rule for setting monetary policy that includes an idiosyncratic shock for policy. New Keynesian DSGE models, such as in Erceg et al. (2000), Galí (2011) and Smets and Wouters (2007) specify a Taylor (1993)-type rule for monetary policy. According to this rule, the policy rate of the central bank responds to macroeconomic outcomes such as real GDP and inflation and hence the policymaker is responding to the current state of the economy while setting monetary policy. Using these models the literature shows monetary policy has a current effect on output and prices that is at odds with the recursiveness assumption of the empirical literature. These structural models are

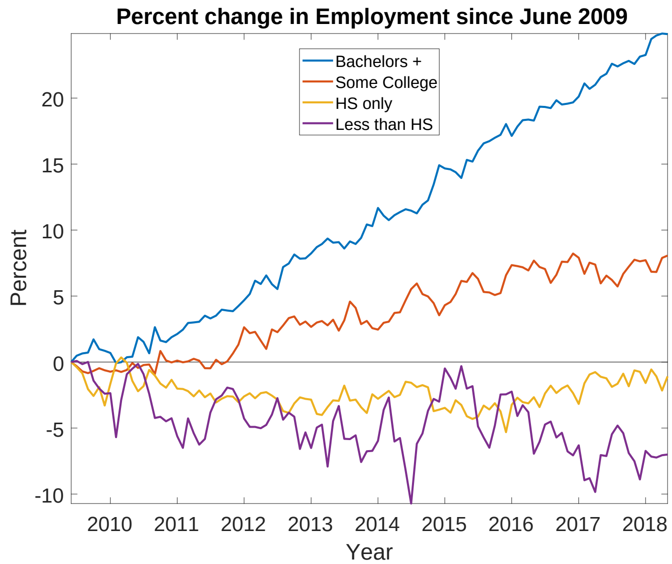
[6]See for example Ramey (2016), Barakchian and Crowe (2013), etc.

[7]As in Gertler and Karadi (2015)

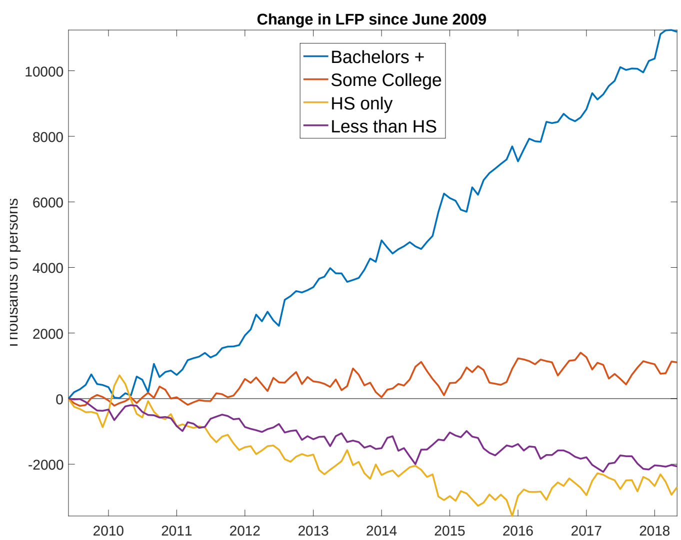
also used to obtain closed form solutions for the loss function of a central bank, as in Erceg et al. (2000) and Bilbiie (2008). A change in the weights on inflation and output in this loss function represents a shift in policy and contributes to a shock.

DSGE models used for monetary policy analysis often ignore the heterogeneity present in the labor market, thus ignoring how a change in policy might have different affects on labor market participants. This paper has shown empirically how monetary policy shocks, even with existing identification issues, result in different dynamic responses of outcomes for agents with different skill levels. In the following two chapters of this dissertation I explore this question through a structural model where I reformulate the labor market to have high and low-skill workers and explore the dynamic responses of their labor market outcomes to a monetary policy shock. I also explore what monetary policy would be in a setting with labor market heterogeneity and provide some insights for policymakers.

Figures



(a) Monthly change in employment levels



(b) Monthly change in labor force participation

Figure 1.1. Labor Market Recovery since the Great Recession

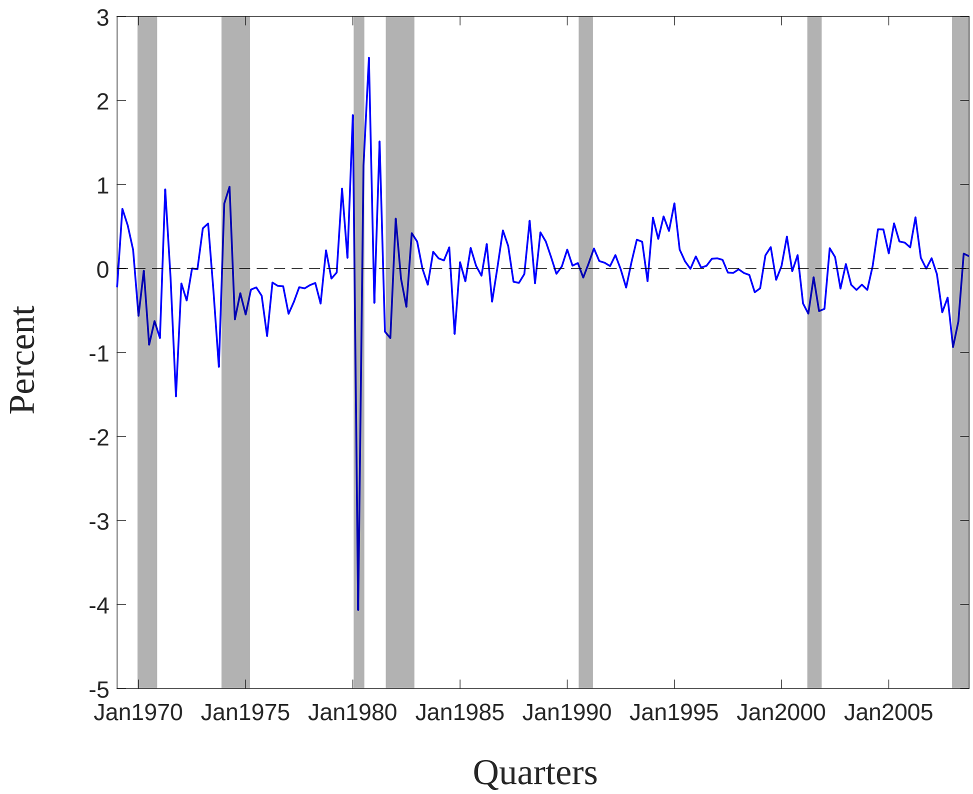
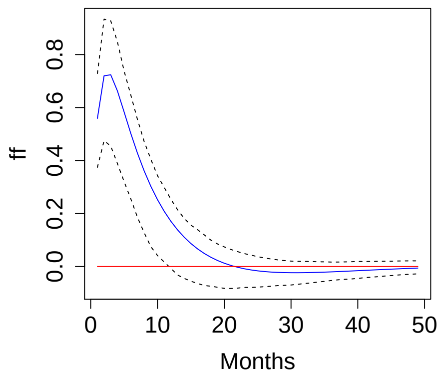
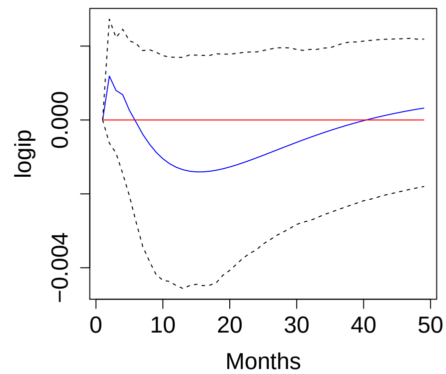


Figure 1.2. Romer and Romer (2004) Measure of Monetary Policy Shocks

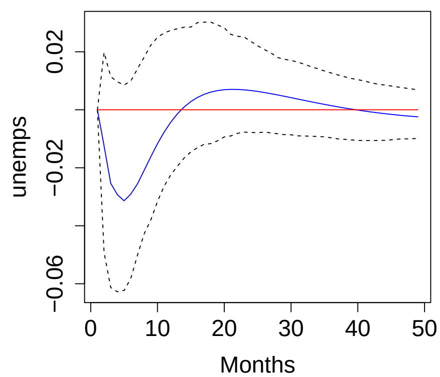
Note: The sample starts at 1969 and ends in 2008 before the Federal funds rate reached the zero lower bound. The monthly series is aggregated to quarterly. The shaded regions represent U.S. recessions.



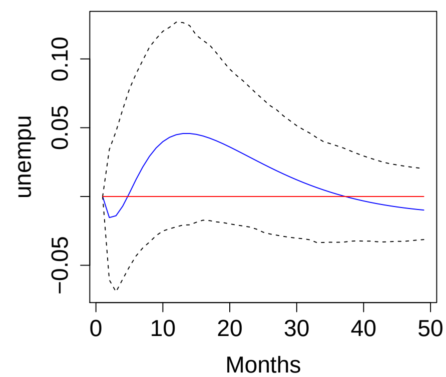
(a) Federal funds rate



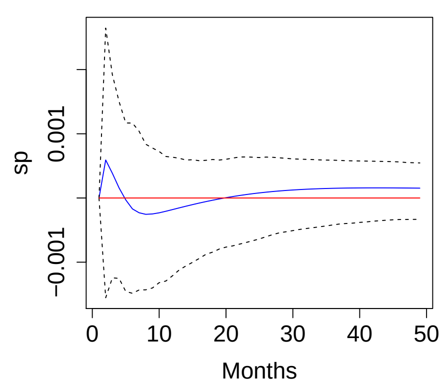
(b) Industrial Production



(c) High-skill unemployment

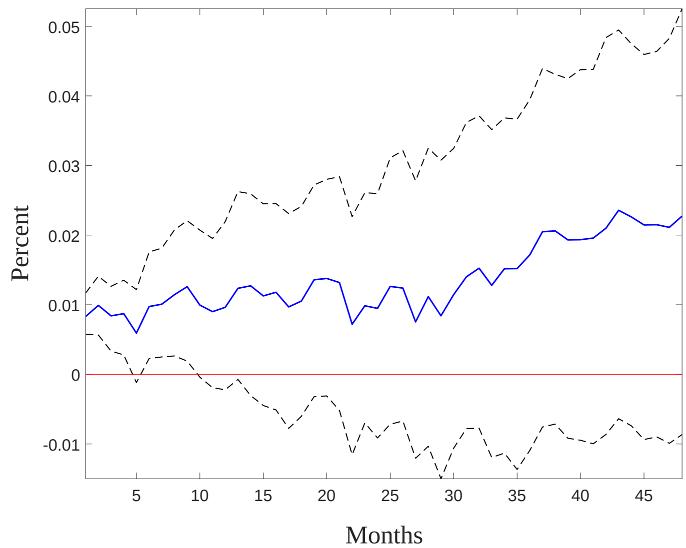


(d) Low-skill unemployment

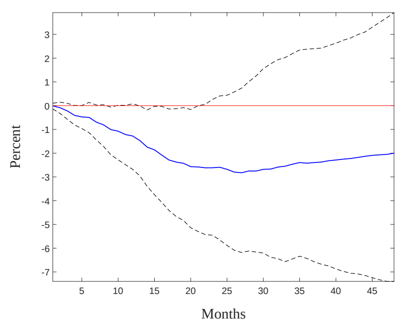


(e) Skill wage premium

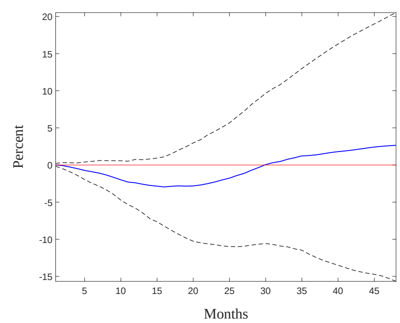
Figure 1.3. Impulse Response Functions: CEE model



(a) Industrial Production

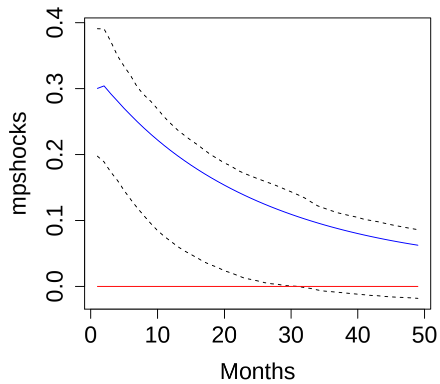


(b) High-skill unemployment

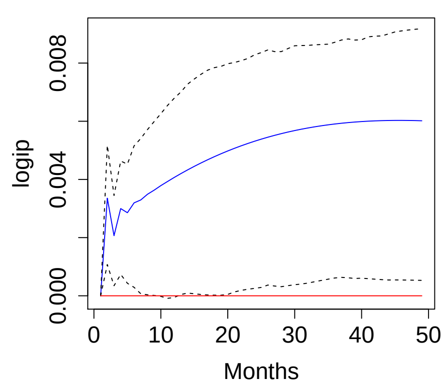


(c) Low-skill unemployment

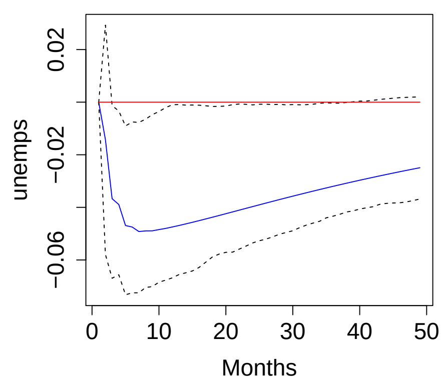
Figure 1.4. Impulse Response Functions: RR model, single dynamic equation



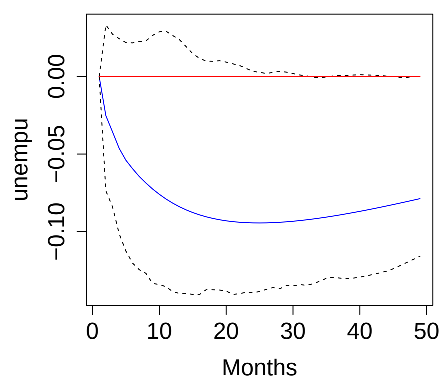
(a) Federal funds rate



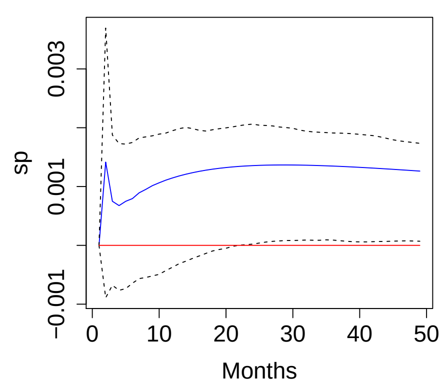
(b) Industrial Production



(c) High-skill unemployment

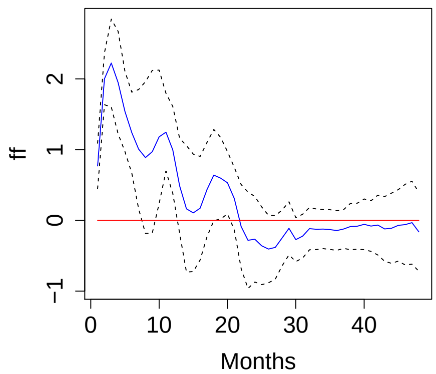


(d) Low-skill unemployment

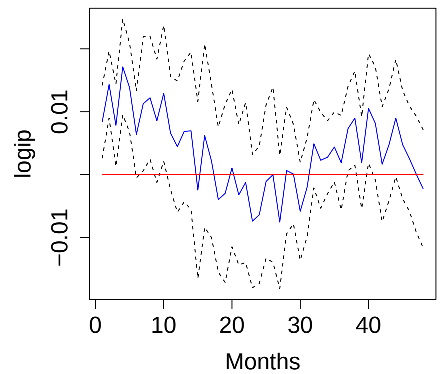


(e) Skill wage premium

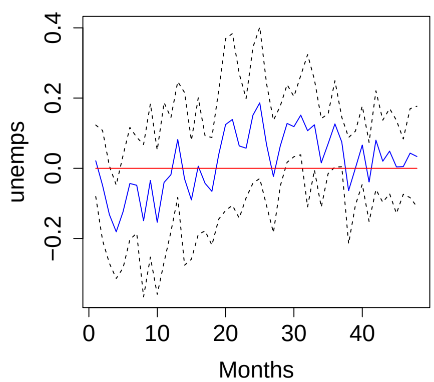
Figure 1.5. Impulse Response Functions: RR model, VAR specification



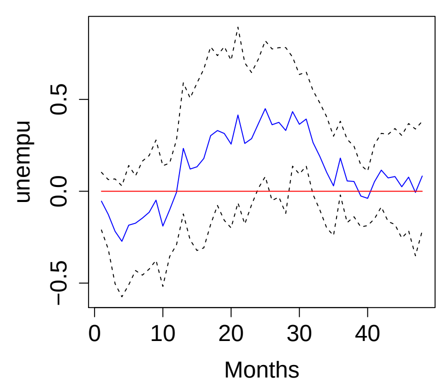
(a) Federal funds rate



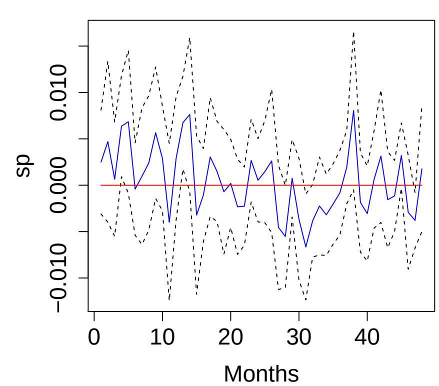
(b) Industrial Production



(c) High-skill unemployment



(d) Low-skill unemployment



(e) Skill wage premium

Figure 1.6. Impulse Response Functions: Jorda (2005) local projections

2. SKILL HETEROGENEITY IN AN ESTIMATED DSGE MODEL

2.1 Introduction

Lael Brainard, a member of the Board of Governors of the Federal Reserve System, said "A deeper understanding of labor market disparities is central to the mission of the Federal Reserve ..." to better assess full employment and improve overall economic activity. Standard models used for monetary policy analysis typically do not incorporate heterogeneity in the labor market, such as disparities in employment, labor force participation and wages among agents. In the previous chapter I show empirically that changes in monetary policy have a disproportionate impact on workers of various skill levels. Before moving on to policy recommendations, it is essential to understand the varied responses of labor market participants to monetary policy in order to make predictions about aggregate dynamics of the economy. This paper incorporates skill heterogeneity in the labor market to analyze the effect of various macroeconomic shocks on outcomes of the agents.

I build a New Keynesian model with price and wage rigidity to study aggregate dynamics of an economy with skill heterogeneity among households in the labor market. I estimate this framework using data from Bureau of Economic Analysis, Board of Governors of the Federal Reserve System and Current Population Survey. The estimated framework accurately represents the behavior of unemployment and wages in the U.S. economy. The two channels through which high and low-skill workers differ in the labor market are their elasticity of labor supply and wage elasticity of demand. High and low-skill workers operate in separate markets and earn different wages, generating different wage Phillips curves. I also incorporate different unemployment rates for the two skill types, which arises through different market power in the labor market as in Galí (2011). I assume low-skill households are financially constrained and exhibit hand-to-mouth behaviour. This assumption is made to generate different consumption levels among the two skill types and isn't completely unrealistic as low-skill households would have lower income and hence almost negligible savings and would be unable to participate in financial markets.[1] Using data on unemployment and wages for high and low-skill workers in the U.S. economy obtained from the Current Population Survey, I estimate key structural parameters of the model through Bayesian estimation techniques as in Smets and Wouters (2007); Herbst and Schorfheide (2015). I use these estimates to show the existence of labor market heterogeneity and study the response of high and low-skill outcomes to a monetary policy shock, technology shock and a demand shock.

Estimation results verify that high and low-skill workers differ in the labor demand and supply parameters. Low-skill workers have a higher Frisch elasticity as their labor hours are less responsive to wage changes. This results in a lower marginal rate of substitution for low-skill workers, that is, they are willing to forgo less consumption for leisure time. The estimate for wage elasticity indicates if wages increase for both groups, firms demand fewer low-skill workers and are willing to lay them off faster than high-skill workers. Several studies show shift in demand towards college educated or high-skill workers by firms and the existence of job polarization where routine "middle-skill occupations have seen a decline in employment. In this paper, these jobs are categorized as low-skill and the wage elasticity estimates point to decline in firm demand for low-skill workers. The estimates for Frisch elasticity and wage elasticity result in a steeper wage Phillips curve for the high-skill compared to the low-skill, as seen in the data. This means low-skill wage inflation responds less to fluctuations in unemployment than high-skill and has greater implications on monetary policy for which the Phillips curve is a key tool. The estimate for elasticity of substitution

[1] I show in the appendix that relaxing this assumption does not significantly change the results. I however continue to work with this assumption so as to generate different consumption levels for high and low-skill workers.

between the skill types indicate high and low-skill workers are imperfect substitutes in production.

I consider a descriptive Taylor (1993)-type rule where the interest rate responds to wage inflation for high and low-skill workers along with price inflation and output gap. While a standard Taylor rule does not respond to wage inflation or consider heterogeneity among economic agents, I find the policy rate responds less than one-for-one to wage inflation for high and low-skill. Moreover, there is significant heterogeneity in these responses, 0.46 for high-skill and 0.29 for low-skill workers, which indicates the policy rate responds more to high-skill wage inflation. This result that monetary policy responds differently to heterogeneous agents in the economy serves as a motivation for the next chapter of this dissertation, where I find a closed form solution for optimal monetary policy with skill heterogeneity. In a scenario where policy responds to labor market heterogeneity, it is essential to look at how macroeconomic aggregates respond to aggregate shocks before formulating optimal policy. In response to a contractionary monetary policy shock I find a fall in high and low-skill consumption as a result of decrease in aggregate demand. Unemployment rates for both skill types experience a decrease, thus giving me the standard contrctionary results for the shock. Larger wealth effects and decreasing wage income results in a greater increase in low-skill unemployment. For a positive technology shock, high-skill consumption increases and low-skill consumption decreases due to the latter's inability to plan for the future. Low-skill unemployment rate increases more than high-skill as a result of the shock. A positive demand shock has the same effects as that of an expansionary monetary policy, like other New Keynesian models.

Past and present Federal Reserve chairs have discussed the puzzle of stable wage inflation even though labor market indicators suggest the U.S. economy is at full employment level since the Great Recession[2]. This has led economists to question the reliability of the wage Phillips curve trade-off and its importance for monetary policy. In this paper, using data from the CPS, I show that the unemployment-

[2]Yellen (2017); Powell et al. (2018)

inflation trade-off is stronger for high-skill workers. I also build a framework which accurately captures the empirical result that the wage Phillips curve relationship is different for high and low-skill workers. Using the framework built in this chapter and the key parameter values obtained, in the next chapter I show the welfare implications of optimal monetary policy when skill differentials exist among workers in the labor market.

2.1.1 Related Literature

Several studies have documented the importance of skill differences over the business cycle. Prasad (1996) and Riley and Young (2007) note importance of skill heterogeneity in the business cycle framework, discussing variation in cyclicality of employment and wages with skill levels. Krusell et al. (2000) use capital-skill complementarity to explain the rising skill premium over the past 30 years. Lindquist (2004) reports the same hypothesis can account for cyclicality and volatility of skill premium with a higher volatility of low-skill hours and lower volatility of low-skill wages.

A wide range of research has been carried out on heterogeneous agents in New Keynesian models, addressing a variety of questions. Ravenna and Walsh (2012) introduce worker heterogeneity through efficiency levels in a New Keynesian model with labor market frictions to study the design of optimal policy. Kaplan et al. (2018) build a new framework with incomplete markets (known as Heterogeneous Agent New Keynesian, or HANK, models) and analyze the direct and indirect effects of monetary policy shock on household consumption. Lester (2014) formulates a standard New Keynesian framework with worker heterogeneity thorugh different wage stickiness parameters and looks at welfare effects of monetary policy.

The framework used in this paper is closely related to Erceg et al. (2000) where the authors use nominal price and wage rigidity in a New Keynesian model to study the design of optimal policy. Galí et al. (2007) and Furlanetto (2011) incorporates rule-of-thumb behavior in the above set-up to study its implications for fiscal policy. However, these studies are unable to perfectly calibrate structural parameters of the model and experiment with various parameter ranges to analyze their results. I introduce labor market heterogeneity through differences in Frisch elasticity and wage demand elasticity between high and low-skill workers. I also incorporate unemployment in this framework, similar to Galí (2011), arising from the labor unions market power. This allows me to use data on unemployment and wages for the separate skill groups to estimate the parameters and show the existence of heterogeneity in the labor market. Moreover, the high and low-skill workers operate in separate labor markets thus giving rise to a wage differential, unlike other papers in the literature where all agents earn the same wage. With unemployment and wages I am able to replicate the empirical finding of a flatter low-skill wage Phillips curve.

2.2 A New Keynesian Model with Heterogeneous Labor Markets

I modify a standard New Keynesian model with nominal rigidity in price and wages as in Erceg et al. (2000) and introduce skill heterogeneity among workers in the households. The high and low-skill operate in separate labor markets and firms hire both types of workers for output production paying them different wages. The channels through which high and low-skill workers differ are their labor supply and demand elasticity. Moreover, following Galí et al. (2007), I assume high-skill workers are forward-looking, consume their permanent income and save for the future. Low-skill workers follow a "rule-of-thumb" of consuming their current income every period. Segmented labor markets with differences in demand and supply elasticity is a new feature introduced in this paper.

2.2.1 Households

There is a continuum of households of unit mass indexed by $(j, h) \in [0, 1] \times [0, 1]$. j represents the occupation or labor service in which a worker in the household specializes and h is the disutility from working. I assume labor services are heterogeneous and can be of two types, high-skill (H) or low-skill (L). λ is the fraction of workers in the economy who are high-skill, such that $j \in [0, \lambda]$ are high-skill workers who are forward-looking and save to smooth consumption over time. $j \in [\lambda, 1]$ are the low-skill workers who are hand-to-mouth. Unlike a standard New Keynesian framework, this paper assumes two representative households, one providing high-skill labor and the other low-skill labor.[3]

The period utility for a household of type $s \in \{H, L\}$ is given by

$$U(C_t^s, N_t^s(j); Z_t) = \Big(\frac{C_t^{s\,1-\sigma}}{1-\sigma} - \chi \frac{N_t^s(j)^{1+\gamma^s}}{1+\gamma^s}\Big) Z_t \tag{2.1}$$

where $C_t^s = \Big(\int_0^1 C_t^s(i)^{\frac{\epsilon_t^p - 1}{\epsilon_t^p}} di\Big)^{\frac{\epsilon_t^p}{\epsilon_t^p - 1}}$ is the consumption index for each skill type and $C_t^s(i)$ is the amount of good i consumed, $i \in [0, 1]$. The time-varying elasticity of substitution for product varieties is given by ϵ_t^p. $z_t = \ln(Z_t)$ is a discount rate shock (or demand shock) common to all households which follows an AR(1) process given by $z_t = \rho_z z_{t-1} + \varepsilon_t^z$ where $\rho_z \in [0, 1]$ and ε_t^z is a white noise term with mean zero and standard deviation σ_z.

γ^s determines the Frisch elasticity of labor supply for skill type s. The Frisch elasticity denotes how responsive a worker's labor supply is to changes in wages. Juhn et al. (1993) find low-skill workers have a steeper labor supply curve, hence a higher Frisch elasticity, that suggests a small change in their wages will generate a larger response in their labor hours. Frisch elasticity is the first channel through which high and low-skill workers differ in the labor market. χ is an exogenous parameter for labor disutility.

[3] The existence of rule-of-thumb consumers was first introduced in Campbell and Mankiw (1989). This assumption is not a key focus of this paper and has been introduced to ensure different consumption levels for high and low-skill workers. With two representative households and access to nominal bonds for both skill types, the consumption-savings problem generates two Euler equations. Thus, a pricing problem arises for nominal bonds. In appendix A.2 I report the results of the posterior estimates of parameters when the assumption is relaxed. Relaxing the assumption does not significantly change the estimates.

Each type of household seeks to maximize the net present value of discounted utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t^s, N_t^s(j); Z_t)$$

subject to a budget constraint. Workers in the high-skill household have access to a risk-free nominal bond that allows them to perfectly smooth consumption and their budget constraint is given by

$$\int_0^1 P_t(i) C_t^H(i) di + \frac{B_t^H}{1+i_t} \leq B_{t-1}^H + \int_0^1 W_t^H(j) N_t^H(j) di + D_t^H$$

where $P_t(i)$ is the price of good i, $W_t^H(j)$ is the nominal wage for a high-skill worker with occupation j, i_t is the nominal interest rate in the economy, B_t^H is the bond holding in period t and D_t^H is the dividend received from ownership of firms. The household discounts future at the rate $\beta \in [0,1]$. Low-skill workers in the household do not have access to nominal bonds and consume labor income every period. Their household budget constraint is given by

$$\int_0^1 P_t(i) C_t^L(i) di = \int_0^1 W_t^L(j) N_t^L(j) di$$

Households must choose the optimal amount of consumption expenditure among different goods. The solution to this problem yields a set of demand equations (log-linearized) for each type of good i for high and low-skill households[4]

$$c_t^H(i) = -\epsilon_t^p (p_t(i) - p_t) + c_t^H \tag{2.2}$$

$$c_t^L(i) = -\epsilon_t^p (p_t(i) - p_t) + c_t^L$$

where the price index for goods is $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon_t^p} di \right)^{\frac{1}{1-\epsilon_t^p}}, \quad p_t \equiv \ln(P_t)$.

The high-skill workers are forward-looking and save for the future that allows them to smooth consumption. Their consumption-savings decision leads to an Euler equation given by

$$c_t^H = \mathbb{E}_t\{c_{t+1}^H\} - \frac{1}{\sigma}\Big(i_t - \rho - E_t\{\pi_{t+1}^p\} - (1-\rho_z) z_t \Big) \tag{2.3}$$

[4]Details of derivation in appendix section A.1.1. Lower case letters denote logs.

where $\pi_t^p \equiv p_t - p_{t-1}$ denotes price inflation and $\rho \equiv -\ln(\beta)$. The low-skill workers are hand-to-mouth and consume all their labor income every period. They do not have a consumption-savings decision to make.

Since labor markets are monopolistically competitive, workers in households form labor unions who have a market power and set wages in each market. A standard model with only price rigidity has a labor supply condition from the household's problem. When wage rigidity is included in the framework, households supply labor that meets the firm labor demand and the former condition is replaced by a wage inflation equation discussed in the next subsection.

Wage Setting

Wages are set by labor unions in a monopolistically competitive market. Each household with occupation j is a labor union that pools across all labor disutility $h \in [0,1]$. There is a continuum of high-skill labor unions $j \in [0,\lambda]$ and a continuum of low-skill labor unions $j \in [\lambda, 1]$. In the presence of nominal wage rigidities, the labor union of type $s \in \{H, L\}$ can update wage in period t with probability $1-\theta_w$ to choose optimal wage rate $W_t^{s*}(j)$. If wage is not updated, the past period wage prevails. Thus, labor unions face a dynamic utility maximization problem, and set wages such that they meet firm labor demand for each skill type.[5]

High and low-skill labor unions operate in separate labor markets, set different wages and face distinct optimization problems. The resulting optimality condition provides separate equations for high-skill wage inflation $\pi_t^{Hw} \equiv w_t^H - w_{t-1}^H$ and low-skill wage inflation $\pi_t^{Lw} \equiv w_t^L - w_{t-1}^L$ given by[6]

$$\pi_t^{Hw} = \beta \mathbb{E}_t\{\pi_{t+1}^{Hw}\} - \Theta_w^H(\mu_t^{Hw} - \mu^{Hw}) \tag{2.4}$$

and

$$\pi_t^{Lw} = \beta \mathbb{E}_t\{\pi_{t+1}^{Lw}\} - \Theta_w^L(\mu_t^{Lw} - \mu^{Lw}) \tag{2.5}$$

[5]The firm labor demand schedules (2.10) are discussed in section 3.2

[6]Details in appendix A.1.1.

where the average (log) wage mark-up for type s is $\mu_t^{sw} = (w_t^s - p_t) - mrs_t^s$ and $mrs_t^s = \xi + \sigma c_t^s + \gamma^s n_t^s$, $\xi \equiv \ln(\chi)$. Due to market power of labor unions, the average mark-up is the difference between real wage and the households marginal rate of substitution. If nominal rigidity in wages are absent and wage is flexible, the desired log wage mark-up is $\mu^{sw} = \ln\left(\frac{\epsilon_w^s}{\epsilon_w^s - 1}\right)$ where ϵ_w^s denotes the wage elasticity of demand. This demand elasticity is the second channel through which high and low-skill workers differ in the labor market.

Unemployment

Following Galí (2011), I introduce unemployment in this model, which acts as a driving force of wage inflation in the U.S. economy. Unemployment arises in this framework as a result of labor unions' market power. In the absence of monopolistic competition and sticky wages, real wage equals the marginal rate of substitution for a household of type $s \in \{H, L\}$ and this level of employment is a "shadow" labor force denoted by $\bar{N}_t$. If market frictions were non-existent, that is labor markets were perfectly competitive, a marginal household of type s with occupation j has the following labor supply condition

$$\frac{W_t^s(j)}{P_t} = \chi (C_t^s)^\sigma \bar{N}_t^s(j)^{\gamma^s}$$

Log-linearization of above equation and integrating over j yields a participation equation for skill-type s as

$$w_t^s - p_t = \xi + \sigma c_t^s + \gamma^s \bar{n}_t^s$$

where $\bar{N}_t^H \equiv \int_0^\lambda \bar{N}_t^H(j)dj$ and $\bar{N}_t^L \equiv \int_\lambda^1 \bar{N}_t^L(j)dj$ and $\bar{n}_t^s \equiv \ln(\bar{N}_t^s)$.

Labor unions set wages as a mark-up over the competitive wage in frictional labor markets and maintain employment below the "shadow" labor force level. This gives rise to unemployment, which is the difference between the labor force and employment provided by the unions, and would not exist in the absence of monopolistic competition and nominal wage rigidity. The unemployment rate can be defined as log difference between the "shadow" labor force and average employment with market frictions

$$u_t^s \equiv \bar{n}_t^s - n_t^s$$

Combining this definition with the average wage mark-up $\mu_t^{sw} = (w_t^s - p_t) - (\xi + \sigma c_t^s + \gamma^s n_t^s)$ and the labor participation equation, the unemployment rate can be written as

$$\gamma^s u_t^s = \mu_t^{sw} \tag{2.6}$$

The above equation clearly shows that unemployment in this model arises from the labor unions' market power given by the mark-up of real wage over marginal rate of substitution due to monopolistic competition and sticky wages. Low-skill workers have a lower marginal rate of substitution and hence their wage inflation is less responsive to unemployment fluctuations. Fluctuations in unemployment arise from variations in the wage mark-up. Following (2.6), the natural rate of unemployment is given as $u^{sn} = \frac{\mu^{sw}}{\gamma^s}$, which prevails when wages are flexible. Thus, unemployment can be generated even when sticky wages are absent and only market imperfection exists. The natural rate of unemployment is increasing in the Frisch elasticity (determined by γ^s) and decreasing in the wage elasticity of demand (or increasing in the mark-up given by μ^{sw}).

Using the definition of unemployment, the wage inflation equation can be re-written as

$$\pi_t^{sw} = \beta E_t\{\pi_{t+1}^{sw}\} - \Theta_w^s \gamma^s (u_t^s - u^{sn}) \qquad s \in \{H, L\} \tag{2.7}$$

The above formulation of the wage inflation equation is often referred to as the New Keynesian Wage Phillips curve. It is a relationship between wage inflation and unemployment in the economy. The ratio between demand and supply elasticity is key in understanding unemployment rate differences among high and low-skilled workers. If the high-skilled earn a higher wage mark-up than low-skilled, thus having a lower wage elasticity of demand ($\epsilon_w^H < \epsilon_w^L$), the above equations imply that the former have a lower natural unemployment rate than the latter. Again, if high-skilled workers have a flatter labor supply curve, which means a lower Frisch elasticity ($\gamma^H > \gamma^L$), their flexible price unemployment rates are lower than the low-skilled workers. Slopes of the New Keynesian Wage Phillips curve depends on the demand and supply elasticity for the workers $\Theta_w^H = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\gamma^H\epsilon_w^H)}$ and $\Theta_w^L = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\gamma^L\epsilon_w^L)}$. If the above assumptions about elasticity are true, the high-skilled workers face a steeper wage Phillips curve.[7]

2.2.2 Firms

There is a continuum of monopolistically competitive firms of unit mass indexed by $i \in [0,1]$, each producing a differentiated product. Each firm hires both high and low-skill labor, which is aggregated into a labor input index using CES technology as in Goldin and Katz (2007). The production function of the firm is given by

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{2.8}$$

where

$$N_t(i) = \Big[\lambda\big(A_t^H N_t^H(i)\big)^{\frac{\eta-1}{\eta}} + (1-\lambda)\big(A_t^L N_t^L(i)\big)^{\frac{\eta-1}{\eta}}\Big]^{\frac{\eta}{\eta-1}} \tag{2.9}$$

$$N_t^H(i) = \Big(\frac{1}{\lambda}\int_0^\lambda N_t^H(i,j)^{\frac{\epsilon_w^H-1}{\epsilon_w^H}}\,dj\Big)^{\frac{\epsilon_w^H}{\epsilon_w^H-1}}$$

$$N_t^L(i) = \Big(\frac{1}{1-\lambda}\int_\lambda^1 N_t^L(i,j)^{\frac{\epsilon_w^L-1}{\epsilon_w^L}}\,dj\Big)^{\frac{\epsilon_w^L}{\epsilon_w^L-1}}$$

A_t is an aggregate technology shock which follows an AR(1) process $a_t = \rho_a a_{t-1} + \varepsilon_t^a$, $a_t \equiv \ln(A_t)$ and ε_t^a is a white noise process with mean zero and standard deviation σ_a. The decreasing returns to scale parameter in the production function is $\alpha \in [0,1]$. A_t^H and A_t^L are high-skill and low-skill labor augmenting technology shocks.[8]

[7] I estimate the parameter values and report these results in section 2.4.1. The results show high-skill workers have a steeper wage Phillips curve compared to low-skilled.

[8] Given the structure of the production function the shocks A_t, A_t^H and A_t^L aren't separately identified as scaling up A_t^H and A_t^L and scaling down A_t by the same proportion would not change the average or marginal products. However, the nature of identification does not matter for the conclusions of this paper as all the results shown henceforth will be conditional on one particular shock ceteris paribus. The shock series follow univariate AR(1) processes and during estimation I assume that the variance-covariance matrix of the shock series is a diagonal matrix. Thus, the shocks have zero covariance and the system is identified.

The elasticity of substitution between high-skill and low-skill labor in firm production is η. The wage elasticity of demand for high-skill and low-skill occupations are ϵ_w^H and ϵ_w^L, respectively. This elasticity represents substitution between the j occupations for each skill type. Juhn et al. (1993) find high-skill worker have a lower wage elasticity of demand, thus earning a higher wage mark-up, which leads to higher wages.

The solution to a firms' cost minimization problem leads to a set of demand schedules for high and low-skill labor[9]

$$n_t^H(i,j) = -\epsilon_w^H(w_t^H(j) - w_t^H) + n_t^H(i) \text{ and } n_t^H(i) = (\eta-1)a_t^H - \eta(w_t^H - w_t) + n_t(i)$$

$$n_t^L(i,j) = -\epsilon_w^L(w_t^L(j) - w_t^L) + n_t^L(i) \text{ and } n_t^L(i) = (\eta-1)a_t^L - \eta(w_t^L - w_t) + n_t(i) \quad (2.10)$$

The average nominal wages for the high and low-skilled households are aggregated as follows

$$W_t^H = \Big(\frac{1}{\lambda}\int_0^\lambda W_t^H(j)^{1-\epsilon_w^H} dj\Big)^{\frac{1}{1-\epsilon_w^H}}$$

$$W_t^L = \Big(\frac{1}{1-\lambda}\int_\lambda^1 W_t^L(j)^{1-\epsilon_w^L} dj\Big)^{\frac{1}{1-\epsilon_w^L}}$$

The overall wage index for the economy is

$$W_t = \Big[\lambda\Big(\frac{W_t^H}{A_t^H}\Big)^{1-\eta} + (1-\lambda)\Big(\frac{W_t^L}{A_t^L}\Big)^{1-\eta}\Big]^{\frac{1}{1-\eta}}$$

where $w_t \equiv \ln(W_t)$, $w_t^H \equiv \ln(W_t^H)$ and $w_t^L \equiv \ln(W_t^L)$.

Price Setting

The goods market is characterized by monopolistic competition. Each firm sets a price at which to sell it's product in the presence of nominal rigidities. A firm can update it's price in period t with probability $1 - \theta_p$ and choose the optimal price $P^*(t)$. If a firm cannot update, the price is same as last period. Thus a firm faces a dynamic profit maximization problem, and chooses price to meet household demand

[9]Details of the derivation are included in appendix section A.1.2.

for products given by equation (2.2). The resulting optimality condition is an inflation equation for prices[10]

$$\pi_t^p = \beta E_t \left\{ \pi_{t+1}^p \right\} - \Theta_p(\mu_t^p - x_t^p) \tag{2.11}$$

where

$$\mu_t^p = \ln(1-\alpha) - rw_t - \frac{\alpha}{1-\alpha} y_t + \frac{1}{1-\alpha} a_t \tag{2.12}$$

is the average price mark-up in the economy and $\Theta_p = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\epsilon^p}$. In the absence of sticky prices the desired mark-up $x_t^p = \ln\left(\frac{\epsilon_t^p}{\epsilon_t^p - 1}\right)$ is time-varying and follows an AR(1) process with mean $\mu^p = \ln(\frac{\epsilon^p}{\epsilon^p - 1})$, autoregressive coefficient ρ_p and an innovation term ε_t^p.

The above equation is the New Keynesian Phillips curve and is similar to one obtained in a standard New Keynesian model with homogeneous labor. If average mark-up falls short of the desired level, firms update prices such that the mark-up adjusts and drives up inflation.

2.2.3 Market Clearing and Equilibrium

Goods Market

Goods market clearing requires quantity of each good produced to be equal to quantity demanded. This means total amount of a good produced by firm i must equal the amount of consumption for that good by the households. So in equilibrium we have

$$Y_t(i) = C_t(i)$$

Aggregate output is defined as $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon_t^p - 1}{\epsilon_t^p}} di \right)^{\frac{\epsilon_t^p}{\epsilon_t^p - 1}}$ and consumption of good i by high and low-skilled household is $C_t(i) = \lambda C_t^s(i) + (1-\lambda) C_t^u(i)$.

Making use of household demand for product varieties, the output market clearing can be written as

$$Y_t = C_t(1 + \Delta_t)$$

[10]Details of the derivation are included in appendix section A.1.2.

where $C_t = \lambda C_t^s + (1-\lambda)C_t^u$ and $\Delta_t = \left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon_t^p} di\right)^{\frac{\epsilon_t^p}{\epsilon_t^p - 1}}$.[11]

Labor Market

The average employment across all high-skill and low-skill households is

$$N_t^H = \frac{1}{\lambda}\int_0^1 \int_0^\lambda N_t^H(i,j)djdi \quad \text{and} \quad N_t^L = \frac{1}{1-\lambda}\int_0^1 \int_\lambda^1 N_t^L(i,j)djdi$$

Given firm demand, the labor market clearing condition is[12]

$$N_t^H = \frac{1}{\lambda}(A_t^H)^{\eta-1}\Big(\frac{W_t^H}{W_t}\Big)^{-\eta}\Big(\frac{Y_t}{A_t}\Big)^{\frac{1}{1-\alpha}}\Delta_t^H \Delta_t^p$$

$$N_t^L = \frac{1}{1-\lambda}(A_t^L)^{\eta-1}\Big(\frac{W_t^L}{W_t}\Big)^{-\eta}\Big(\frac{Y_t}{A_t}\Big)^{\frac{1}{1-\alpha}}\Delta_t^L \Delta_t^p$$

where $\Delta_t^H = \int_0^\lambda \left(\frac{W_t^H(j)}{W_t^H}\right)^{-\epsilon_w^H} dj$ is the high-skill wage dispersion, $\Delta_t^L = \int_\lambda^1 \left(\frac{W_t^L(j)}{W_t^L}\right)^{-\epsilon_w^L} dj$ is low-skill wage dispersion and $\Delta_t^p = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon_t^p}{1-\alpha}}$ is price dispersion.

Output and Wage Gap

Output gap $\tilde{y}_t$ is defined as the (log) deviation between output and its natural or flexible price counterpart y_t^n, such that $\tilde{y}_t \equiv y_t - y_t^n$. Real wage for skill type s is $\omega_t^s \equiv w_t^s - p_t$, which allows me to define the real wage gap as $\tilde{\omega}_t^s \equiv \omega_t^s - \omega_t^{sn}$. Similar to natural level of output, ω_t^{sn} is the natural or flexible price level of real wage for skill type s. The deviation of price mark-up from it's natural counterpart can now be expressed in terms of output and wage gap as

$$\mu_t^p - x_t = -\tilde{\omega}_t - \Big(\frac{\alpha}{1-\alpha}\Big)\tilde{y}_t$$

Substituting the above expression into the price inflation equation (2.11) yields

$$\pi_t^p = \beta E_t\left\{\pi_{t+1}^p\right\} + \Theta_p\tilde{\omega}_t + \kappa_p\tilde{y}_t \tag{2.13}$$

[11]In the neighborhood of zero inflation steady-state, Δ_t is zero up to a first-order approximation Galí (2015). This means $Y_t = C_t$.

[12]Details of derivation in appendix section A.1.3.

where $\kappa_p = \frac{\alpha}{1-\alpha}\Theta_p$.

Similarly, the deviation of average wage mark-up from its natural counterpart can be expressed in terms of wage and output gap as

$$\mu_t^{sw} - \mu^{sn} = (1-\sigma)\tilde{\omega}_t^s + \eta(\sigma+\gamma^s)\Big(\tilde{\omega}_t^s - \tilde{\omega}_t\Big) - \frac{\sigma+\alpha}{1-\alpha}\tilde{y}_t$$

allowing me to write the wage inflation inflation for skill type s as

$$\pi_t^{sw} = \beta E_t\left\{\pi_{t+1}^{sw}\right\} + \kappa_1^s\tilde{\omega}_t^s + \kappa_2^s\Big(\tilde{\omega}_t^s - \tilde{\omega}_t\Big) + \kappa_3^s\tilde{y}_t \tag{2.14}$$

where $\kappa_1^s = (1-\sigma)\Theta^s, \quad \kappa_2^s = \eta(\sigma+\gamma^s)\Theta^s, \quad \kappa_3^s = \left(\frac{\sigma+\alpha}{1-\alpha}\right)\Theta^s$.

2.2.4 Equilibrium Dynamics

The optimality conditions (2.3), (2.4), (2.5), (2.6), (2.8), (2.13) and (2.14) along with wage gap identity

$$\tilde{\omega}_t^s \equiv \tilde{\omega}_{t-1}^s + \pi_t^{sw} - \pi_t^p - \Delta\omega_t^{sn}$$

form the non-policy block of the model. The above equilibrium conditions together with a monetary policy rule closes the model. For equilibrium dynamics I use a descriptive Taylor-type rule, where interest rate responds to wage inflation of high and low-skill workers separately

$$i_t = \rho_R i_{t-1} + (1-\rho_R)(\rho + \phi_p\pi_t^p + \phi_y\tilde{y}_t + \phi_H\pi_t^{Hw} + \phi_L\pi_t^{Lw}) \tag{2.15}$$

ρ_R is the interest rate smoothing parameter that determines the degree to which current interest rate depends on it's past period value. The policy rate also responds to price inflation. Ascari et al. (2017) find that price inflation targeting has negative welfare costs and wage inflation targeting is necessary when wages are sticky. Following their finding I include separate wage inflation terms for the high and low-skill workers. By estimating the response coefficients in the Taylor-type rule I let the data indicate how interest rates respond to heterogeneity in the labor market.

2.3 Bayesian Estimation

In this section, I describe the Bayesian estimation strategy used to estimate the key parameters of the model presented above. A Bayesian DSGE model consists of a joint distribution of data and model parameters. This joint distribution can be factored into two components. The first component, which is the likelihood function, is the distribution of data given model parameters. The data consists of time series for GDP growth, inflation and the Federal Funds Rate. I also use unemployment rates and wages for high and low-skill workers. The second component of the joint distribution is the prior distribution of the parameters. The likelihood function is used to update a priori beliefs about model parameters using prior distributions and sample information. After updating beliefs, information regarding model parameters is summarized by the posterior distribution. Bayes Theorem provides a link between the likelihood, prior and posterior distribution.

2.3.1 Likelihood Function

The first step in the estimation process is forming the likelihood function, which is the probability density of the data given model parameters. To obtain the likelihood function I present the model in a linear rational expectations form proposed by Sims (2002)[13]

$$\Gamma_0 \mathbb{S}_t = \Gamma_1 \mathbb{S}_{t-1} + \Psi \epsilon_t + \Pi \zeta_t$$

where Γ_0, Γ_1, Ψ and Π are matrices made-up of structural parameters of the model Θ. $\mathbb{S}_t$ is the vector of endogenous variables, ϵ_t is the vector of innovations and ζ_t are expectation errors.

[13]There exists other solution methods to linear rational expectations models such as Blanchard and Kahn (1980) and Christiano (2002).

Solution to the above system of equations can be written as a state-transition equation

$$\mathbb{S}_t = \Phi(\Theta)\mathbb{S}_{t-1} + \mathcal{R}(\Theta)\epsilon_t \qquad \epsilon_t \sim \mathcal{N}(0, \Sigma)$$

The innovations to the system ϵ_t are independently and identically distributed with a diagonal variance-covariance matrix Σ.[14] Endogenous variables of the model are related to time series data through a measurement equation

$$\mathbb{Y}_t = \mathcal{A}(\Theta) + \mathcal{B}(\Theta)\mathbb{S}_t + u_t \qquad u_t \sim \mathcal{N}(0, \mathcal{H})$$

The above two equations make up the state-space representation of the model required for the estimation process. Coefficients Φ, $\mathcal{R}$, $\mathcal{A}$, $\mathcal{B}$ are functions of structural parameters of the model Θ. The likelihood function of the model is

$$\mathcal{L}(\mathcal{Y}^{\mathcal{T}}|\Theta) = \prod_{t=1}^{T} p(\mathbb{Y}_t|\mathcal{Y}^{t-1}, \Theta)$$

where $\mathcal{Y}^t = \{\mathbb{Y}_1, ..., \mathbb{Y}_T\}$ are observables from the data.

Kalman Filter

The likelihood function for a log-linearized DSGE model is evaluated using a Kalman filter. Filtering removes the noise u_t from the data and generates optimal forecasts of the state variables $\mathbb{S}_t$. Evaluation of the Kalman filter includes the following steps

1. Initialize the state variable in period 0 with prior $\mathbb{S}_0 \sim \mathcal{N}(\hat{\mathbb{S}}_{0|0}, \mathbb{P}_{0|0})$

2. At period t, belief about the state vector is $\mathbb{S}_t|\mathcal{Y}^{t-1} \sim \mathcal{N}(\hat{\mathbb{S}}_{t|t-1}, \mathbb{P}_{t|t-1})$ where $\hat{\mathbb{S}}_{t|t-1} = \Phi\hat{\mathbb{S}}_{t-1|t-1}$ and $\mathbb{P}_{t|t-1} = \Phi\mathbb{P}_{t-1|t-1}\Phi' + \mathcal{R}\mathcal{R}'$

3. The marginal distribution of $\mathbb{Y}_t$ conditional on the data $\mathcal{Y}^{t-1}$ is $\mathbb{Y}_t|\mathcal{Y}^{t-1} \sim \mathcal{N}(\hat{\mathbb{Y}}_{t|t-1}, \mathbb{F}_{t|t-1})$ where $\hat{\mathbb{Y}}_{t|t-1} = \mathcal{A}+\mathcal{B}\hat{\mathbb{S}}_{t|t-1}$ and $\mathbb{F}_{t|t-1} = \mathcal{B}\mathbb{P}_{t|t-1}\mathcal{B}'+\mathcal{H}$

[14] Σ is a diagonal matrix, where the diagonal elements are the standard errors for the innovations in ϵ_t and the off-diagonal elements are zero.

4. The joint predictive distribution $p(\mathbb{S}_t, \mathbb{Y}_t|\mathcal{Y}^{t-1})$ is

$$\mathbb{S}_t, \mathbb{Y}_t|\mathcal{Y}^{t-1} \sim \mathcal{N}\left(\begin{bmatrix} \hat{\mathbb{S}}_{t|t-1} \\ \hat{\mathbb{Y}}_{t|t-1} \end{bmatrix}, \begin{bmatrix} \mathbb{P}_{t|t-1} & \mathbb{P}_{t|t-1}\mathcal{B}' \\ \mathcal{B}\mathbb{P}'_{t|t-1} & \mathbb{F}_{t|t-1} \end{bmatrix} \right)$$

5. The filter is run over the entire sample and the moments of the conditional distribution are collected to compute the likelihood function

$$\mathcal{L}(\mathcal{Y}^{\mathcal{T}}|\Theta) = (2\pi)^{\frac{nT}{2}} \left(\prod_{t=1}^{T} |\mathbb{F}_{t|t-1}| \right)^{-\frac{1}{2}} \exp\left\{ \frac{1}{2} \sum_{t=1}^{T} (\mathbb{Y}_t - \hat{\mathbb{Y}}_{t|t-1})' \mathbb{F}_{t|t-1}^{-1} (\mathbb{Y}_t - \hat{\mathbb{Y}}_{t|t-1}) \right\}$$

2.3.2 Priors

Prior distributions describe knowledge about structural parameters of the model before observing the data. Priors are model specific and obtained through introspection, pre-sample estimates or micro-evidence. As in Del Negro and Schorfheide (2008), I divide the set of parameters Θ into two groups.

1. For the first set of parameters, I enforce dogmatic priors by using estimates from the literature. The values of these parameters are fixed throughout the estimation process. These parameters include $\Theta_1 = [\alpha, \beta, \sigma, \lambda, \epsilon_p, \theta_p, \theta_w]$.

2. The second set of parameters include the specific labor market parameters in the model and are estimated using time series data. These include $\Theta_2 = [\gamma^H, \gamma^L, \eta, \epsilon_w^H, \epsilon_w^L, \phi_p, \phi_H, \phi_L, \phi_y, \rho_r, \rho_\nu, \rho_a, \rho_z, \rho_{aH}, \rho_{aL}, \rho_p, \sigma_\nu, \sigma_a, \sigma_z, \sigma_{aH}, \sigma_{aL}, \sigma_p]$.

Table 2.1 provides a complete list of the fixed parameters in the model. The Cobb Douglas share in the production function is $\alpha = 0.33$ as in Galí et al. (2007). The discount parameter β has a value of 0.99 corresponding to a real (annualized) return on financial assets of 4% taken from Galí et al. (2007). The risk aversion parameter is $\sigma = 2$. The elasticity of product varieties is 6, implying a steady-state mark-up of 20%. The price and wage stickiness parameters are 0.75, denoting average price and wage duration of four quarters. The above parameters are the same as Ascari et al.

(2017). The fraction of high-skill workers in the economy is 36% and is calculated from the 2016 Current Population Survey.

Table 2.2 provides information on priors for key estimated parameters in Θ_2. Following Smets and Wouters (2007), priors for standard errors for the shock processes follows an Inverse Gamma distribution with mean 0.5 and four degrees of freedom. The persistence parameters for all five shock processes follow a Uniform distribution in the interval $[0, 1)$. For the labor market parameters, that is the labor supply and labor demand elasticity, I assume the same priors for both skill types. This ensures that no ex-ante differences exist between high and low-skill workers in the labor market and the data is able to predict any heterogeneity that exists between the two groups. The inverse of the Frisch elasticity parameter follows a Normal distribution with a slightly higher mean and standard deviation for high-skilled workers (2.5 and 0.5) compared to low-skilled workers (1.5 and 0.25). The wage demand elasticity for high and low-skilled workers follow a Gamma distribution with mean 5 and standard deviation 0.5. Following Del Negro and Schorfheide (2008), priors for Taylor rule coefficients follow a Gamma distribution. The inflation coefficient has a mean of 1 and standard deviation 0.25. Reaction of the interest rate to output gap and high and low-skilled wage inflation has a mean of 0.2 and standard deviation 0.1. The interest rate smoothing parameter follows a Uniform distribution in the interval $[0, 1)$.

2.3.3 Posterior Estimation

The likelihood function and priors combine to form the posterior distribution using Bayes theorem.

$$p(\Theta|\mathcal{Y}^T) \propto \mathcal{L}(\mathcal{Y}^T|\Theta)p(\Theta)$$

where we combine the likelihood function $\mathcal{L}(\mathcal{Y}^T|\Theta)$ with the prior distribution for the parameters $p(\Theta)$, to get the posterior distribution $p(\Theta|\mathcal{Y}^T)$. A MCMC algorithm, specifically the random walk Metropolis Hastings (RWMH) algorithm, is used to generate draws from the posterior distribution. Parameter point estimates are obtained via the following steps

1. A numerical optimization routine is used to obtain the posterior mode $\tilde{\Theta}$ and Hessian Σ by maximizing log of the objective function $\ln \mathcal{L}$.

2. Initialize the RW-MH chain by drawing $\Theta^{(0)}$ from $\mathcal{N}(\tilde{\Theta}, c_0\Sigma)$.

3. For $s = 1, ..., n_{sim}$, draw a candidate $\hat{\Theta}$ from the proposal distribution $\mathcal{N}(\Theta^{(s-1)}, c\Sigma)$. Denote
$$\alpha = \frac{\mathcal{L}(\hat{\Theta}|\mathcal{Y})p(\hat{\Theta})}{\mathcal{L}(\hat{\Theta}^{(s-1)}|\mathcal{Y})p(\hat{\Theta}^{(s-1)})}$$

4. With probability $min\{1, \alpha\}$ accept the candidate and set $\Theta^{(s)} = \hat{\Theta}$. Otherwise, reject the candidate and set $\Theta^{(s)} = \Theta^{(s-1)}$.

Using draws from the posterior distribution, the mean and 95% HPD intervals for the parameters are calculated as

$$E\Big[h(\Theta^{(s)}|\mathcal{Y})\Big] = \frac{1}{n_{sim}} \sum_{s=1}^{n_{sim}} h(\Theta^{(s)})$$

2.3.4 Observables

To create the final data set used for the estimation process, I use variables from various different data sources. The time period is quarterly and the sample period starts at the first quarter of 1979 and ends in the last quarter of 2007. I end the sample in 2007 as the Federal Funds Rate hits the zero-lower bound after this period and non-linearity induced by near zero interest rates can distort estimation results.

I obtain data on per-capita GDP from the Bureau of Economic Analysis (BEA). The data is in chained 2009 dollars. I use it to calculate the growth rate of output as follows

$$dy = 100 \times \Big(\frac{GDP_t - GDP_{t-1}}{GDP_{t-1}}\Big)$$

[14]Here I use csminwel optimization routine to obtain the posterior mode and Hessian. http://sims.princeton.edu/yftp/optimize/

I also obtain personal consumption expenditure also from the BEA and use it to calculate PCE inflation

$$\pi_t^p = 100 * \Big(\frac{PCE_t - PCE_{t-1}}{PCE_{t-1}}\Big)$$

The inflation data is monthly and I take quarterly averages. I obtain daily effective Federal Funds Rate from the Board of Governors of the Federal Reserve System. I calculate quarterly averages to use for the estimation process.

Current Population Survey

Unemployment rates and wages are calculated from the Current Population Survey (CPS), which is a monthly survey of employment and labor markets of approximately 60,000 households. The public use micro data files have been available at the Bureau of Labor Statistics since 1968 and the NBER has prepared extracts of these files from 1979. These extracts include data for about 30,000 individuals each month for various labor market outcomes.

In this survey, an individual reports activities of all other persons in the households. The universe of all adults 16 years or older form the U.S. non-institutional population. Each individual (household) in the CPS is interviewed for the first 4 months, ignored for the next 8 months and again interviewed for the last 4 months. The outgoing rotation group consists of households who have been asked to report their weekly earnings or weekly hours in months 4 and 8. These interviews form a single Merged Outgoing Rotation Group (MORG) file in which an individual household appears only once in any file and may reappear in the following year.

For the purpose of this study, I am interested in a few variables from the survey that I use to create the appropriate data set for the estimation process. The variables that I use are as follows

- intmonth: interview calendar month

- weight: final weight. The sum of weights in each monthly survey is representative of the U.S. non-institutional population. The outgoing rotation group is one-fourth of that population and so a single month MORG file represents one-fourth of the U.S. population. A yearly MORG file represents 3 times the U.S. population. These weights are used to calculate the employed and unemployed population and hence unemployment rates.

- earnwt: earnings weight. This sums to the total population each month and a yearly MORG file is 12 times the U.S. population. I use these weights to calculate the nominal wages.

- gradeat/grade92: highest grade of school attended for the years 1979-1991 and the highest grade completed for 1992 onwards. Between the years 1979-1988, the value for education is one more than the actual grade. For 1989-1991, the value is the actual grade. In 1992, the BLS switched from years of schooling to different measure for the highest grade completed.

- class/class94: class of workers. This variable tells me if the individual is employed in the private sector, which branch of the government or if the person is self-employed.

- esr/lfsr89/lfsr94: employment status recode for years 1979-1988 and labor force status recode for years 1989 onwards. This variable contains information on whether an individual is employed, unemployed or not in the labor force. The codes for labor force status was recoded in 1994.

- uhourse: hours worked per week.

- earnwke: earnings per week, including overtime tips and commissions.

First, I divide the population into high and low-skilled households/individuals. To do this I use the highest grade of school attended or the highest grade completed. An individual is defined as high-skilled if they at least have a Bachelors degree. For years 1979-1991, an individual is high-skilled if $gradeat \geq 16$ and for 1992 onwards if $grade92 \geq 43$.

Next, I divide the population of high and low-skilled into employed and unemployed groups. An individual in the labor force can be either employed or unemployed. I use the employment/ labor force status recode to create these groups as follows

	Employed	Unemployed	NILF
1979-88	esr=1/2	esr=3	esr=4/5/6/7
1989-93	lfsr89=1/2	lfsr89=3/4	lfsr89=5/6/7
1994-2016	lfsr94=1/2	lfsr94=3/4	lfsr94=5/6/7

I use values in the above table and final weights to calculate the population of high and low-skilled workers who are either employed or unemployed. The population of high-skilled individuals who are employed for a given month can be computed as the sum of final weights. Four times this value gives me the total U.S. non-institutional population of employed high-skill individuals (N^H) for each month. Using the same process I calculate the population of employed low-skilled households (N^L), unemployed high-skilled households (U^H) and unemployed low-skilled households (U^L). Total labor force for high-skilled is $L^H = N^H + U^H$ and for low-skilled is $L^L = N^L + U^L$. The unemployment rate for high-skilled is $u^H = \frac{U^H}{L^H}$ and for the low-skilled is $u^L = \frac{U^L}{L^L}$. To calculate the weighted average hours I use the hours worked per week and the total employed population for each skill type. Hours for each skill type is calculated by summing over all households i and using the final weights for each households

$$hours^s = \sum_i \frac{weight_i \times uhours_i}{N^s}$$

Nominal hourly wages is the ratio of earnings per week *earnwke* and hours worked per week *uhourse*, such that $hourlywage = \frac{earnwke}{uhourse}$. The weighted average hourly wages for a skill type is

$$W^s = \sum_i \frac{earnwt_i \times hourlywage_i}{\sum_i earnwt_i}$$

For each month, where i is each individual. Since *earnwt* sums to the total population each month the above equation gives me the nominal hourly wages.

2.4 Results

In this section, I discuss results from the Bayesian estimation exercise of the framework presented in the paper. Estimation results suggest the existence of labor market heterogeneity and show the differences among high and low-skill workers in the labor market.

2.4.1 Posterior Estimates

Estimation results for the model parameters show the existence of heterogeneity and the differences between high and low-skill workers in the labor market. Table 2.2 reports parameter posterior means and 95% HPD intervals. The mean for elasticity of substitution between high and low-skill in the production function is 1.27 indicating skill types are complements in firm production. This estimate is similar to results from various reduced-form papers.[15] If the firm views high and low-skill workers as complements in production, increase in wages for one skill type will result in decrease in labor demand for the other. Moreover, mean for labor demand elasticity (or the elasticity of substitution between labor services) is 2.70 for high-skill and 6.72 for low-skill, resulting in an aggregate wage mark-up of approximately 31%.[16] The elasticity

[15]Katz and Murphy (1992) find reduced-form estimates of around 1.41 for the period 1963-87 and Greenwood et al. (1997) find a value between 1.3 and 1,7 for early 60s to early 90s. More recently, Autor et al. (2008) find a value of 1.57 for 1963-2005 and Autor and Acemoglu (2010) find a value of 1.6 for 1963-87, which increases to 2.9 when the sample period is extended to 2008.

[16]Galí (2015) uses an average mark-up of 28%.

estimate suggests firms labor demand is more responsive to changes in high-skill wages. This means, with an increase in wages, firms substitute out low-skill workers faster than high-skill. Lichter et al. (2015) conducts a meta-regression analysis to find a higher labor demand elasticity for low-skill workers. This estimate also falls in line with the finding of job polarization and shifting firm demand to high-skill workers at the cost of low-skill workers.[17] On the labor supply side, the mean estimate for Frisch elasticity (inverse of γ) is 0.17 for high-skill and 0.31 for low-skill workers, suggesting the latters labor hours are more responsive to wage changes. Kimball and Shapiro (2008) find post-graduates have a significantly lower labor supply elasticity.

From the parameter estimates described above, a higher Frisch elasticity and wage demand elasticity for low-skill workers generates a flatter wage Phillips curve. The slope of the Phillips curve depends on values of the model parameters and the estimates imply a slope of -0.0039 for high-skill and -0.0028 for low-skill. This is shown in Figure 2.1 where I simulate the wage Phillips curve for high and low-skill workers in the model using the estimated parameters. In Figure 2.2 I plot the relationship between wage inflation and unemployment rates for high and low-skill workers from the data. I find the data has shows the same results that high-skill workers have a steeper wage Phillips curve and their wage inflation is more responsive to changes in their unemployment rates. The difference in Phillips curves indicate that the inflation-unemployment trade-off is stronger for high-skill workers compared to low-skill. Thus, monetary policy that uses this trade-off will have different implications on outcomes of heterogeneous agents. If the low-skill wage inflation response to unemployment is muted, the central bank should incorporate this fact while making monetary policy decisions to better stabilize outcomes of the low-skill workers.

I estimate a version of the Taylor rule given by (2.15) and find the response of monetary policy is different for high and low-skill workers. The interest rate responds to its past period value along with price inflation, output gap and wage inflation for

[17] Autor et al. (2003) find substitution of workers by machines explain 60% of shift in demand towards college educated or high-skill workers during 1970-98.

the two skill groups. Estimates show a moderate amount of interest rate smoothing with a mean value of 0.57. The mean for coefficient on price inflation is 2.23 and on output gap is 0.38. These estimates are similar to other studies. Smets and Wouters (2007) find interest rate smoothing to be 0.81 and coefficient on price inflation to be 2.03. However, I find a stronger interest rate response to output gap compared to their 0.08. The mean of the coefficients on high and low-skill wage inflation in this paper is 0.46 and 0.29 respectively. The result indicates monetary policy does respond to wage inflation in a framework where wages are sticky. Moreover, the wage inflation response of the policy rate is different for high and low-skill workers. Under the scenario where policy responds to labor market heterogeneity, I further investigate how various macroeconomic outcomes respond to certain aggregate shocks in the next section. Moreover, to analyze if this policy is optimal and the aspects of the labor market policymakers should pay attention to, I design an optimal monetary policy for the central bank in section 5.

2.4.2 Impulse Responses

I move on to discuss impulse responses to a contractionary monetary policy shock and positive technology shock, highlighting the differences in outcomes for high and low-skill workers. A monetary policy shock in this model has an immediate impact on output and prices, unlike the empirical literature that makes the "recursiveness assumption" as in chapter 1. In Figure 2.3, due to a contractionary monetary policy shock, aggregate demand falls causing a decrease in total output. This results in a fall in consumption for both high and low-skill workers. The decrease is driven by falling interest rates for high-skill who want to save more and consume less given their forward-looking nature. However, low-skill being limited in their ability to plan for the future, experience a larger decrease in consumption. This result is directly due to a fall in wage income as real wages and hours both decreases. The fall in consumption creates negative wealth effects causing labor force to increase as more workers are willing to work now. Since the decrease in consumption is larger for low-skill workers, their wealth effects are larger generating a greater increase in their labor force. Firms labor demand however does not match the increase in willingness to work causing increase in unemployment levels. Larger wealth effects and a decreasing wage income results in a greater increase in low-skill unemployment compared to high-skill. Since more workers are willing to work and the wealth effects are greater for low-skill, the unions set a lower real wage. The fall in wages is larger for the high-skill group thus reducing the gap between the wages of the two types and hence skill premium falls.

A positive technology shock has different effects on macroeconomic outcomes of high and low-skill workers. Labor decreases with a boost in technology, as seen in Figure 2.4, and is a common result when prices and wages are sticky. The intuition is with constant money supply and predetermined prices aggregate demand is unchanged in the period when the shock hits. Firms need to produce an unchanged level of output with better technology and thus require less labor causing a fall in labor demand. However, a rise in productivity drives down marginal costs and firms adjust prices downward next period, resulting in an increase in output.[18]. Since times are good labor unions choose higher wages but the fall in firm labor demand results in a decrease in wage income. Due to their inability to prepare for the future, low-skill workers experience a fall in consumption levels. On the other hand, a fall in the interest rate causes high-skill workers to save less for the future as they are forward-looking, and their consumption levels rise. Low-skill workers want to work more due to the increase in real wages driving up their labor force. This increase combined with lower firm demand for labor causes an increase in their unemployment rates. The high-skill group wants to work less due to a higher marginal rate of substitution resulting in fall in their labor force. Nevertheless, this decrease cannot offset the decrease in firm labor demand and their unemployment level rise as well. The low-skill group experiences a greater increase in unemployment due to increase in their labor force. With a boost in technology and more productive workers, unions set a higher

[18]The hump-shaped response of output is due to high persistence of the technology shock

real wage. This increase is more for the low-skill worker and hence we see a fall in the skill premium which means the gap between high and low-skill wages decreases.

Responses of outcomes to a demand shock in Figure 2.5, which takes the form of a discount rate shock, is identical to expansionary monetary policy shock. A positive shock to the discount rate increases the weight workers give to current utility. This causes a change in their preferences and consumption increases, thus causing an increase in aggregate demand and output. The expansion of output generates more labor demand for the firms and employment increases as a result. The increase in consumption causes positive wealth effects for both types of workers. Since this increase is larger for low-skill workers their wealth effects are stronger causing their labor force to decline as less workers are willing to work. This results in lower unemployment rates and this fall is larger for low-skill workers. Hence we see positive implications for low-skill worker outcomes due to a positive demand shock. Since results for this shock is qualitatively similar to expansionary monetary policy shocks, the latter would also benefit the low-skill workers.

2.4.3 Labor Market Specification

The main contribution of the paper is introduction of heterogeneity and segmentation in the labor market. To emphasize the implications of this contribution it is essential to discuss dynamics when the heterogeneity and segmented market assumption is relaxed. In this section I compare the framework presented in this paper (calling it the "baseline model) to two alternative specifications. In the first alternative specification, I relax the assumption of labor market heterogeneity by closing the two main channels through which this is introduced. This means the Frisch elasticity and labor demand elasticity is equal for high and low-skill workers. However, the high-skill can still save to prepare for the future and the low-skill are hand-to-mouth, generating different consumption levels for the skill groups. Additionally, labor markets remain segmented resulting in different labor hours and wages for the two types.[19] In the second specification, I further relax the assumption of segmented labor markets. Now high and low-skill workers are similar in all aspects, that is labor market is homogeneous, which corresponds to a standard framework with nominal price and wage rigidity as in Erceg et al. (2000).

Under homogeneous labor markets, high-skill consumption increases by a smaller amount compared to the baseline framework as seen in Figure 2.6. Once again, the baseline and segmented market response coincide indicating the importance of the savings assumption for the dynamics of consumption. A similar result is true for low-skill consumption . Unemployment rates increase for both skill groups and is highest under the baseline scenario compared to the segmented markets and homogeneous labor market specification. The dynamics of outcome variables change as the specification of the labor market changes thus hinting at the importance of the heterogeneity and market segmentation assumptions introduced by this paper. A lower unemployment rate under the homogeneous labor market specification does not necessarily mean greater economic performance. Furthermore, unemployment response to a monetary policy and technology shock shows greater negative effects for low-skill workers. Thus, monetary policy should include unemployment explicitly to mitigate these negative effects on agents. In the following section, I study the importance of skill heterogeneity for the welfare of the economy and of the heterogeneous agents. I design an optimal monetary policy for the central bank incorporating the difference in inflation-unemployment trade-off between the high and low-skill workers in the labor market.

2.5 Conclusion

In this paper, I present a new framework that captures skill heterogeneity in the U.S. labor market and study aggregate dynamics of various macroeconomic variables. Workers are either high or low-skilled and the two channels through which skill types

[19] This is unlike Galí et al. (2007) where even with rule-of-thumb behavior both types of consumers receive the same wage.

differ are their elasticity of labor supply and demand. Departing from previous literature in this area, I assume labor markets are separate for high and low-skill workers. I estimate the model using Bayesian techniques common in the DSGE literature. The data used includes aggregate macroeconomic outcomes like GDP growth, PCE inflation and the Federal Funds Rate. To specifically identify the labor market parameters I use unemployment rates and wages for high and low-skill workers.

Estimation results show the existence of heterogeneity among workers in the labor market and the differences between high and low-skill workers on the labor demand and supply side. Low-skill workers have a higher Frisch elasticity and wage elasticity of demand. This generates different wage Phillips curves by skill type as also seen in the data. High-skill workers have a steeper wage Phillips curve suggesting their unemployment-inflation trade-off is stronger than the low-skill population. Thus, the wage inflation is a good indicator of labor market tightness for high-skill but not so much for the low-skill labor market. The coefficient estimates for the Taylor-type rule indicates less than one-for-one response of the policy rate to wage inflation, the magnitude also being different for high and low-skill. The estimated impulse responses show different skill groups have varied responses to aggregate shocks. A contractionary monetary policy shock lowers the consumption and unemployment levels more for low-skill workers than high-skill.

Presence of heterogeneous labor market participants will have new implications for monetary policy. The central bank would need to care more about the low-skill workers due to the missing inflation-unemployment trade-off for this group. To study how policymakers can better stabilize outcomes of different skill groups I design an optimal Ramsey policy for the central bank in the next chapter. Analyzing optimal monetary policy for the central bank allows me to shed light on aspects of the labor market policymakers can pay attention to improve outcomes. It also allows me to quantify welfare for the overall economy and the heterogeneous agents separately to see if the policy under segmented labor markets is welfare improving.

Tables

Table 2.1.
Fixed parameter values and source

Parameter		Value	Source
Cobb-Douglas share	α	0.33	Galí et al. (2007)
Discount factor	β	0.99	Galí et al. (2004)
Fraction skilled	λ	0.36	calculated from CPS 2016
Risk aversion	σ	2	Ascari et al. (2017)
Elasticity goods	ϵ_p	6	Ascari et al. (2017)
Price stickiness	θ_p	0.75	Ascari et al. (2017)
Wage stickiness	θ_w	0.75	Ascari et al. (2017)

Table 2.2.
Priors and Posterior Estimates of Model Parameters

Parameter		Prior distribution			Posterior distribution	
		Dist.	Mean	S.D.	Mean	[0.05, 0.95]
Substitution HS-LS	η	N	1.5	0.25	1.27	[1.21, 1.36]
Inverse Frisch HS	γ^H	N	2.5	0.5	5.79	[5.38, 6.25]
Inverse Frisch LS	γ^L	N	1.5	0.25	3.23	[2.91, 3.55]
Wage elasticity HS	ϵ_w^H	G	5.00	0.5	2.70	[2.48, 2.90]
Wage elasticity LS	ϵ_w^L	G	5.00	0.5	6.72	[6.31, 7.18]
Interest smoothing	ρ_R	U	0.50	0.083	0.57	[0.51, 0.65]
MP reaction price inflation	ϕ_p	G	1	0.25	2.23	[1.75, 2.93]
MP reaction output gap	ϕ_y	G	0.20	0.10	0.38	[0.29, 0.47]
MP reaction HS wage inflation	ϕ_w^H	G	0.20	0.10	0.46	[0.14, 0.78]
MP reaction LS wage inflation	ϕ_w^L	G	0.20	0.10	0.29	[0.08, 0.53]

Note: 100,000 draws were generated from the posterior and the first 50,000 was discarded as burn-in. Based on the remaining draws the posterior means and 95% HPD interval was calculated.

Figures

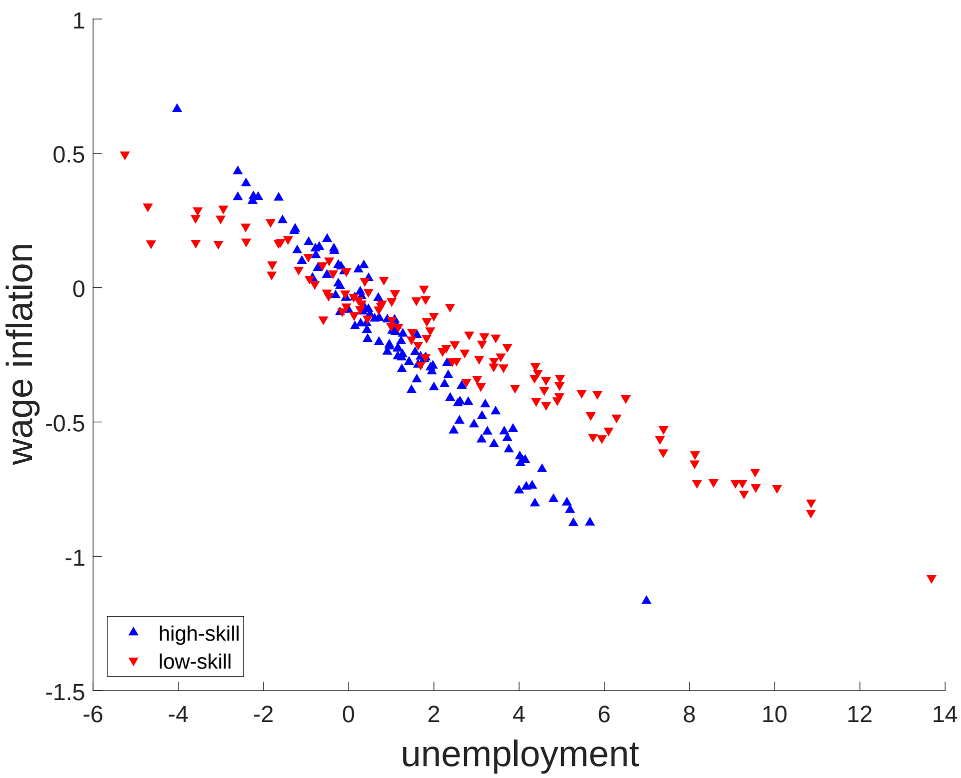
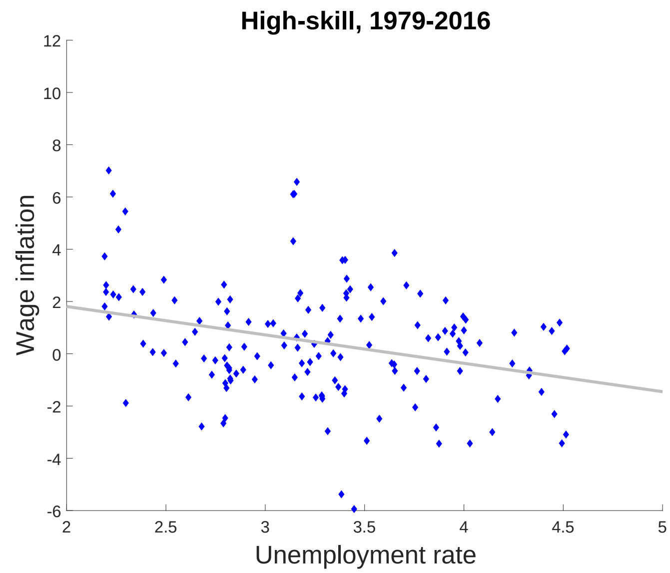
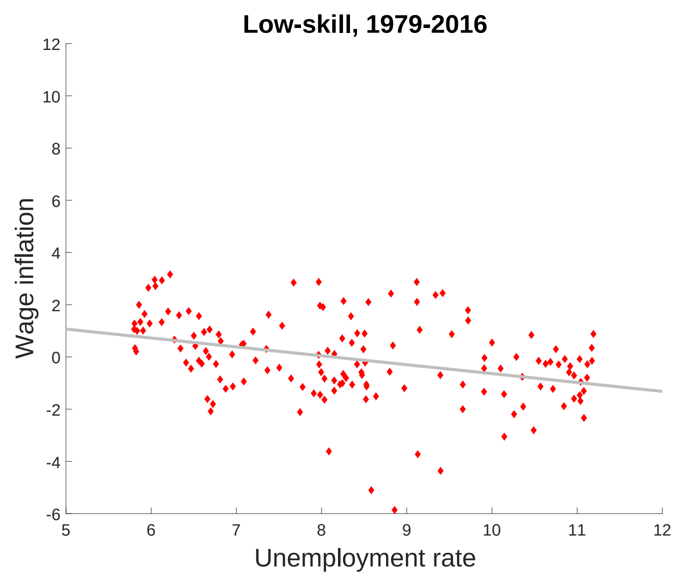


Figure 2.1. Simulated Wage Phillips Curve from the model

Note: I simulate the model using estimated parameter values and plot model generated wage inflation against unemployment. The plot shows a clear negative relationship as given by the wage Phillips curve. The slope for high-skill wage Phillips curve is -0.0039 and for low-skill is -0.0028.



(a) High-skill workers



(b) Low-skill workers

Figure 2.2. Wage Phillips Curve from data

Note: This figure shows the relationship between wage inflation and unemployment for high and low-skill workers in the data. The time period is 1979-2016. Correlation between the two series is -0.1717 for high-skill and -0.0710 for low-skill.

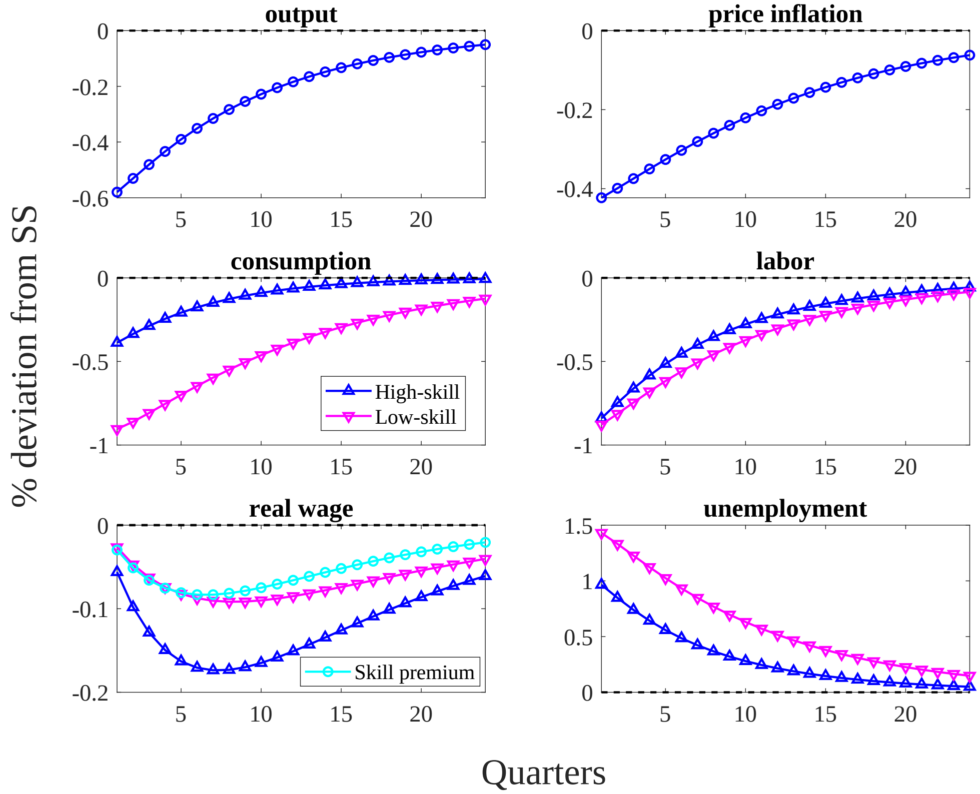


Figure 2.3. Response to Monetary Policy Shock

Note: The above impulse responses are to a contractionary monetary policy shock that takes the form of a 100 basis point increase in the annualized nominal rate. Response of variables are expressed as percent deviation from steady-state. The blue up arrow line represents response of high-skill outcomes and magenta down arrow line represents response of low-skill outcomes.

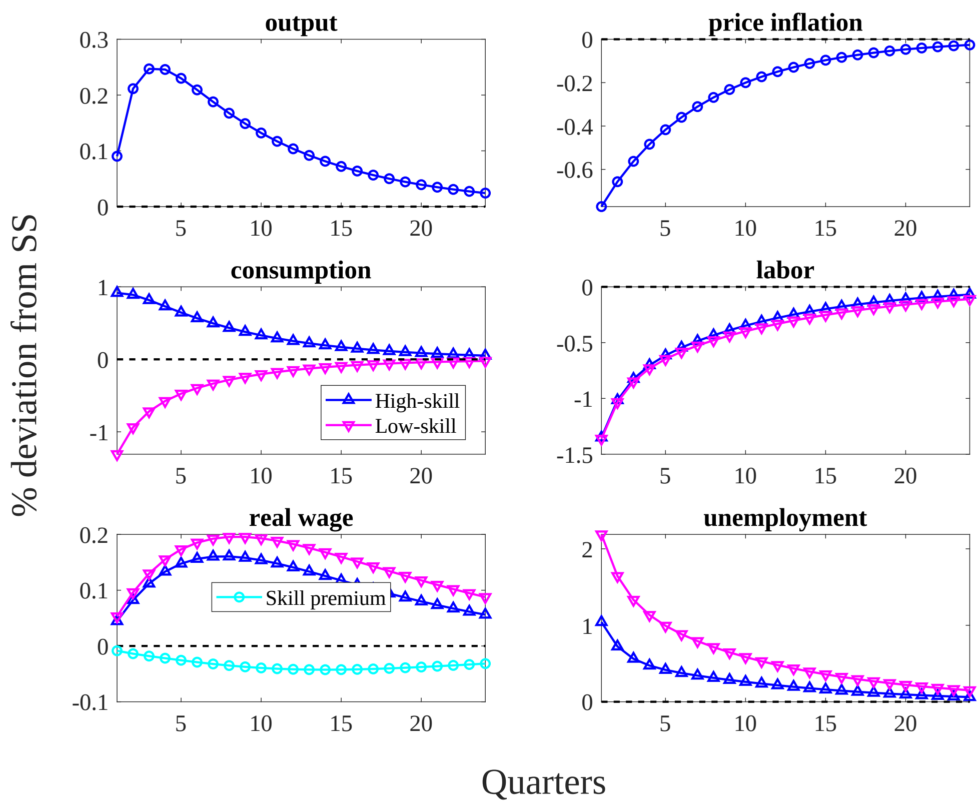


Figure 2.4. Response to Aggregate technology Shock

Note: The shock here is a 1% increase in technology. Response of variables are expressed as percent deviation from steady-state. The blue up arrow line represents response of high-skill outcomes and magenta down arrow line represents response of low-skill outcomes.

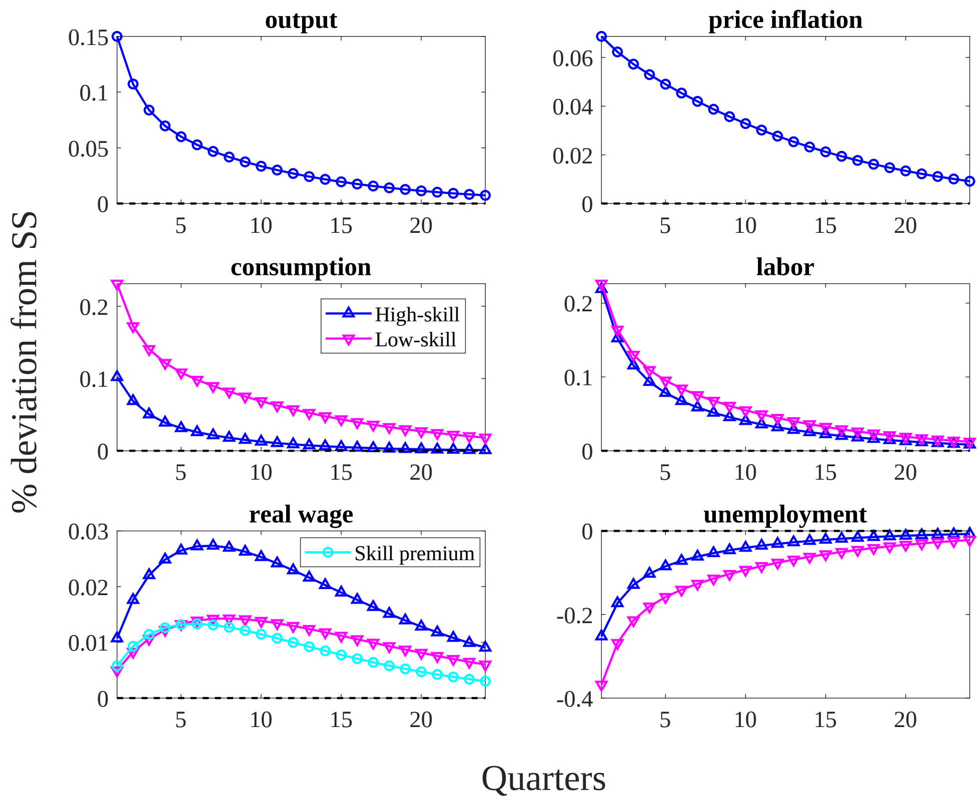


Figure 2.5. Response to Demand (discount rate) shock

Note: The shock here is an increase on impact of 1 % point in the annualized natural rate of interest. Response of variables are expressed as percent deviation from steady-state. The blue up arrow line represents response of high-skill outcomes and magenta down arrow line represents response of low-skill outcomes.

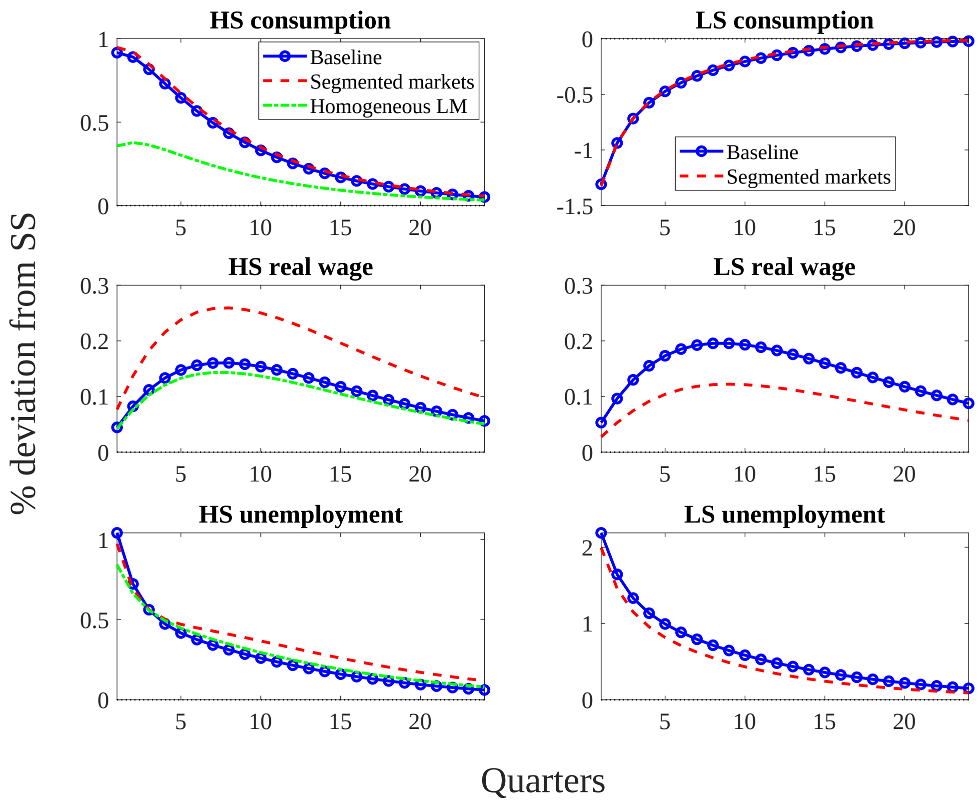


Figure 2.6. Different Labor Market Specifications: Technology Shock

Note: The shock here is a 1% increase in technology. The baseline specification follows the model presented in the paper. Segmented markets refer to the specification where Frisch elasticity and labor demand elasticity are equal for high and low-skill workers and the only difference among skill types is their forward-looking behavior. However, both types operate in separate labor markets earning different wages. Under the homogeneous labor market specification all workers are the same and are forward-looking, corresponding to a standard New Keynesian model. Response of variables are expressed as percent deviation from steady-state.

3. OPTIMAL MONETARY POLICY WITH SKILL HETEROGENEITY

3.1 Introduction

Standard models used for discussion of optimal monetary policy of central banks is centered around the assumption that labor markets are homogeneous. In these models, all agents in the economy are similar with respect to macroeconomic outcomes like unemployment rate and real wages. On the contrary, empirical evidence points to variation in labor market outcomes by demographic groups over the business cycle. For example, unemployment rates for low educated workers, African-American workers and women saw a much larger increase after the Great Recession compared to other demographic categories. Policymakers at the Federal Reserve as well as advocacy groups are all of the opinion that these labor market disparities are an important aspect in understanding overall economic growth and should be a central issue while making monetary policy decisions. This paper looks at an economy with skill differentials as the source of heterogeneity in the labor market and the design of optimal monetary policy that responds to these heterogeneous groups separately.

I use a New Keynesian model with price and wage rigidity and include skill heterogeneity among households in the labor market. This framework is the same as the one presented in chapter 2 of this dissertation. Workers in the household are either high-skilled or low-skilled and operate in separate labor markets earning different wages. The two channels through which high and low-skilled workers differ are their elasticity of labor supply and labor demand. Specifically, low-skilled workers have a higher Frisch elasticity of labor supply as their labor hours are less responsive to wage changes. This means these workers are willing to forego less consumption for more labor time than high-skill workers. Moreover, low-skilled also have a higher wage elasticity of demand that indicates that if wages were to increase firms would substitute low-skilled workers faster than high-skilled workers. These parameter differences result in different slopes for the wage Phillips curve for the high and low-skilled workers. The wage Phillips curve shows the relationship between wage inflation and unemployment rate and is an important tool used to make monetary policy decisions. In the data I find low-skilled workers have a flatter wage Phillips curve than high-skilled and the model is able to match this finding. A flatter curve means the wage inflation is almost non-responsive to fluctuations in unemployment rates. Thus, wage inflation is a good indicator of labor market tightness for the high-skill group but the same cannot be said for the low-skill labor market. Now central banks might want to incorporate this difference in inflation-unemployment trade-off between skill types while setting monetary policy to ensure that the policy benefits all groups in the economy.

With skill differentials in the labor market, optimal policy of the central bank must strike a balance between stabilization of price inflation, GDP and outcomes for high and low-skill workers separately. The monetary authority acts as a social planner and sets the path for outcomes such that utility loss due to deviation from efficient allocation, as a result of nominal price and wage rigidity, is minimized. The central bank conditions it's optimization problem on different wage Phillips curves for high and low-skill workers and I find welfare loss is reduced by half in this setting when compared to a policy that does not account for skill heterogeneity. Stabilization of outcomes depend on the composition of skill in the labor market with a greater fraction of high-skill workers implying more weight on stabilization of high-skill wage inflation. Incorporating labor misallocation shifts the focus of optimal policy from output stabilization to inflation stabilization. The weight on price inflation in the utility loss function increases by 50% and the weight on output gap decreases by 50%, when compared to a standard policy with homogeneous labor markets. In a simple model with wage rigidity the utility loss function contains a single term for inefficiencies arising from misallocation of labor services. I find two separate terms for high and low-skill workers in the loss function, for misallocation of high-skill and low-skill labor services respectively. The low-skill workers have a flatter wage Phillips curve suggesting negligible response of wage inflation to unemployment fluctuations. Thus, optimal policy tends to be more responsive to low-skill wage inflation.

The optimal policy with skill differentials can be easily implemented by an interest rate rule with unemployment rate as an argument. I develop some simple rules in this framework where the coefficient on arguments in these rules are chosen such that utility loss in the economy is minimized. Incorporating high and low-skill unemployment rates as an additional argument in the simple rule increases welfare loss by only 12% compared to the optimal policy. Moreover, this simple rule generates losses half the size of that generated by a standard Taylor rule that only responds to price inflation and output gap. Finally, I propose an interest rate rule where the policy rate responds to price inflation in the economy and high and low-skill unemployment rates to target heterogeneous aspects of the labor market. The proposed rule predicts a policy rate of around 4% in 2003-2005 compared to the Federal Funds Rate of 2% during the same period. This rule also shows a faster recovery of the policy rate since the Great Recession, predicting a rate of around 2% by 2011-2012.

Through a dynamic stochastic general equilibrium model I show optimal monetary policy under the existence of skill differentials in the labor market must account for labor market heterogeneity. With a near negligible response of low-skill wage inflation to their unemployment fluctuations, optimal monetary policy is welfare improving when skill differentials are accounted for. Moreover, an interest rate rule that uses high and low-skill unemployment rates as arguments tracks the standard Taylor (1993) rule fairly well and provides a framework that addresses criticisms from both "hawks and doves". On one hand, this addresses a criticism raised by Taylor (2009) against the Feds monetary policy saying that it has been "too easy by remaining at the zero-lower bound after the financial crisis of 2007 for longer than what was predicted by the Taylor rule. On the other hand, Bivens and Zipperer (2018) says the Federal Reserve, while raising interest rates, should also internalize benefits of low and middle wage workers who suffered the most in the aftermath of the Great Recession. This framework shows how incorporating unemployment in monetary policy can alleviate the plight of the middle and low wage workers. The evidence presented in this paper suggests accounting for skill heterogeneity among labor market participants may help policymakers achieve their macroeconomic targets of sustainable employment and stable inflation more efficiently.

3.1.1 Related literature

The most common form of household heterogeneity in the New Keynesian framework follows Gali et al. (2004); Galí et al. (2007) where they introduce "rule-of-thumb" consumers to study the implications of fiscal policy in this setting. These rule-of-thumb agents are unable to smooth consumption intertemporally. Furlanetto (2011) extends this framework to include wage rigidity and analyze it's effect for a government spending shock. He finds the assumption of sticky wages and labor market segmentation to be essential in preserving the results of the previous papers. However, these studies do not design an optimal policy of the central bank in a setting where labor markets are heterogeneous or look at welfare implications of this policy.

Bilbiie (2008) studies the design of optimal policy in a setting with sticky prices and limited asset market participation. The author explicitly models asset markets to show, with lower asset market participation the slope of the IS curve is inverted and monetary policy is less effective. Colciago (2011) and Ascari et al. (2017) show the inclusion of sticky wages in this setting preserves the usefulness of monetary policy even with asset market participation. Ascari et al. (2017) find price inflation targeting has negative welfare costs and wage inflation targeting is necessary when wages are sticky. My paper adds to this literature by analyzing optimal policy when labor markets are segmented where high and low-skill workers have different wage Phillips curves. My results show monetary policy is effective even with fewer low-skill workers, who have zero asset market participation in this case. The welfare maximizing monetary authority incorporates different response of wage inflation to unemployment fluctuations for high and low-skill workers while also optimizing worker outcomes.

Some other papers that look at optimal policy in New Keynesian models include Ravenna and Walsh (2012). In this paper the authors design optimal policy in a setting where worker heterogeneity is introduced through different efficiency levels in a search and matching framework. Gornemann et al. (2012) use an incomplete markets model with labor market frictions to study the distributional effects of monetary policy. Luetticke (2018) has a model with incomplete markets and sticky prices to study the transmission of policy when there is heterogeneity in the marginal propensity to consume and invest. Faia (2008) builds a model with sticky prices, search and matching frictions and wage rigidity in the labor market to study optimal monetary policy rules.

3.2 A New Keynesian Model with Heterogeneous Labor Markets

The model presented in this chapter is the same framework presented in chapter 2 of this dissertation and hence most of this section is repetitive. The channels through which high and low-skill workers differ are their labor supply and demand elasticity. Following Galí et al. (2007), I assume high-skill workers are forward-looking, consume their permanent income and save for the future. Low-skill workers follow a "rule-of-thumb" of consuming their current income every period.

3.2.1 Households

Households are the same as in chapter 2. There is a continuum of households of unit mass indexed by $(j, h) \in [0, 1] \times [0, 1]$. j represents the occupation or labor service in which a worker in the household specializes and h is the disutility from working. I assume labor services are heterogeneous and can be of two types, high-skill (H) or low-skill (L). λ is the fraction of workers in the economy who are high-skill, such that $j \in [0, \lambda]$ are high-skill workers who are forward-looking and save to smooth consumption over time. $j \in [\lambda, 1]$ are the low-skill workers who are hand-to-mouth. There are two representative households in this framework, one providing high-skill labor and the other low-skill labor.

The period utility for a household of type $s \in \{H, L\}$ is given by

$$U(C_t^s, N_t^s(j); Z_t) = \Big(\frac{C_t^{s\,1-\sigma}}{1-\sigma} - \chi \frac{N_t^s(j)^{1+\gamma^s}}{1+\gamma^s}\Big) Z_t \tag{3.1}$$

where $C_t^s = \Big(\int_0^1 C_t^s(i)^{\frac{\epsilon_t^p - 1}{\epsilon_t^p}} di\Big)^{\frac{\epsilon_t^p}{\epsilon_t^p - 1}}$ is the consumption index for each skill type and $C_t^s(i)$ is the amount of good i consumed, $i \in [0, 1]$. The time-varying elasticity of substitution for product varieties is given by ϵ_t^p. $z_t = \ln(Z_t)$ is a discount rate shock (or demand shock) common to all households which follows an AR(1) process given by $z_t = \rho_z z_{t-1} + \varepsilon_t^z$ where $\rho_z \in [0, 1]$ and ε_t^z is a white noise term with mean zero and standard deviation σ_z. γ^s determines the Frisch elasticity of labor supply for skill type s. The Frisch elasticity denotes how responsive a worker's labor supply is to changes in wages. Each type of household seeks to maximize the net present value of discounted utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t^s, N_t^s(j); Z_t)$$

subject to a budget constraint. Workers in the high-skill household have access to a risk-free nominal bond that allows them to perfectly smooth consumption and their budget constraint is given by

$$\int_0^1 P_t(i) C_t^H(i) di + \frac{B_t^H}{1+i_t} \leq B_{t-1}^H + \int_0^1 W_t^H(j) N_t^H(j) di + D_t^H$$

where $P_t(i)$ is the price of good i, $W_t^H(j)$ is the nominal wage for a high-skill worker with occupation j, i_t is the nominal interest rate in the economy, B_t^H is the bond holding in period t and D_t^H is the dividend received from ownership of firms. The household discounts future at the rate $\beta \in [0, 1]$. Low-skill workers in the household do not have access to nominal bonds and consume labor income every period. Their household budget constraint is given by

$$\int_0^1 P_t(i) C_t^L(i) di = \int_0^1 W_t^L(j) N_t^L(j) di$$

Households must choose the optimal amount of consumption expenditure among different goods. The solution to this problem yields a set of demand equations (log-linearized) for each type of good i for high and low-skill households[1]

$$c_t^H(i) = -\epsilon_t^p(p_t(i) - p_t) + c_t^H \tag{3.2}$$

$$c_t^L(i) = -\epsilon_t^p(p_t(i) - p_t) + c_t^L$$

where the price index for goods is $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon_t^p} di \right)^{\frac{1}{1-\epsilon_t^p}}, \quad p_t \equiv \ln(P_t)$.

The high-skill workers are forward-looking and save for the future that allows them to smooth consumption. Their consumption-savings decision leads to an Euler equation given by

$$c_t^H = \mathbb{E}_t\{c_{t+1}^H\} - \frac{1}{\sigma}\Big(i_t - \rho - E_t\{\pi_{t+1}^p\} - (1-\rho_z)z_t\Big) \tag{3.3}$$

where $\pi_t^p \equiv p_t - p_{t-1}$ denotes price inflation and $\rho \equiv -\ln(\beta)$. The low-skill workers are hand-to-mouth and consume all their labor income every period. They do not have a consumption-savings decision to make. Since labor markets are monopolistically competitive, workers in households form labor unions who have a market power and set wages in each market. A standard model with only price rigidity has a labor supply condition from the household's problem. When wage rigidity is included in the framework, households supply labor that meets the firm labor demand and the former condition is replaced by a wage inflation equation discussed in the next subsection.

Wage Setting

Wages are set by labor unions in a monopolistically competitive market. Each household with occupation j is a labor union that pools across all labor disutility $h \in [0, 1]$. There is a continuum of high-skill labor unions $j \in [0, \lambda]$ and a continuum of low-skill labor unions $j \in [\lambda, 1]$. In the presence of nominal wage rigidities, the labor union of type $s \in \{H, L\}$ can update wage in period t with probability $1 - \theta_w$

[1] Lower case letters denote logs.

to choose optimal wage rate $W_t^{s*}(j)$. If wage is not updated, the past period wage prevails. Thus, labor unions face a dynamic utility maximization problem, and set wages such that they meet firm labor demand for each skill type.[2]

High and low-skill labor unions operate in separate labor markets, set different wages and face distinct optimization problems. The resulting optimality condition provides separate equations for high-skill wage inflation $\pi_t^{Hw} \equiv w_t^H - w_{t-1}^H$ and low-skill wage inflation $\pi_t^{Lw} \equiv w_t^L - w_{t-1}^L$ given by

$$\pi_t^{Hw} = \beta \mathbb{E}_t\{\pi_{t+1}^{Hw}\} - \Theta_w^H(\mu_t^{Hw} - \mu^{Hw}) \tag{3.4}$$

and

$$\pi_t^{Lw} = \beta \mathbb{E}_t\{\pi_{t+1}^{Lw}\} - \Theta_w^L(\mu_t^{Lw} - \mu^{Lw}) \tag{3.5}$$

where the average (log) wage mark-up for type s is $\mu_t^{sw} = (w_t^s - p_t) - mrs_t^s$ and $mrs_t^s = \xi + \sigma c_t^s + \gamma^s n_t^s$, $\xi \equiv \ln(\chi)$. Due to market power of labor unions, the average mark-up is the difference between real wage and the households marginal rate of substitution. If nominal rigidity in wages are absent and wage is flexible, the desired log wage mark-up is $\mu^{sw} = \ln\left(\frac{\epsilon_w^s}{\epsilon_w^s - 1}\right)$ where ϵ_w^s denotes the wage elasticity of demand.

Unemployment

Following Galí (2011), I introduce unemployment in this model, which acts as a driving force of wage inflation in the U.S. economy. Unemployment arises in this framework as a result of labor unions' market power. In the absence of monopolistic competition and sticky wages, real wage equals the marginal rate of substitution for a household of type $s \in \{H, L\}$ and this level of employment is a "shadow" labor force denoted by $\bar{N}_t$. If market frictions were non-existent, that is labor markets were perfectly competitive, a marginal household of type s with occupation j has the following labor supply condition

$$\frac{W_t^s(j)}{P_t} = \chi (C_t^s)^\sigma \bar{N}_t^s(j)^{\gamma^s}$$

[2]The firm labor demand schedules (3.10) are discussed in section 3.2.2.

Log-linearization of above equation and integrating over j yields a participation equation for skill-type s as

$$w_t^s - p_t = \xi + \sigma c_t^s + \gamma^s \bar{n}_t^s$$

where $\bar{N}_t^H \equiv \int_0^\lambda \bar{N}_t^H(j)dj$ and $\bar{N}_t^L \equiv \int_\lambda^1 \bar{N}_t^L(j)dj$ and $\bar{n}_t^s \equiv \ln(\bar{N}_t^s)$.

Labor unions set wages as a mark-up over the competitive wage in frictional labor markets and maintain employment below the “shadow” labor force level. This gives rise to unemployment, which is the difference between the labor force and employment provided by the unions, and would not exist in the absence of monopolistic competition and nominal wage rigidity. The unemployment rate can be defined as log difference between the “shadow” labor force and average employment with market frictions

$$u_t^s \equiv \bar{n}_t^s - n_t^s$$

Combining this definition with the average wage mark-up $\mu_t^{sw} = (w_t^s - p_t) - (\xi + \sigma c_t^s + \gamma^s n_t^s)$ and the labor participation equation, the unemployment rate can be written as

$$\gamma^s u_t^s = \mu_t^{sw} \tag{3.6}$$

The above equation clearly shows that unemployment in this model arises from the labor unions’ market power given by the mark-up of real wage over marginal rate of substitution due to monopolistic competition and sticky wages. Low-skill workers have a lower marginal rate of substitution and hence their wage inflation is less responsive to unemployment fluctuations. Fluctuations in unemployment arise from variations in the wage mark-up. Following (3.6), the natural rate of unemployment is given as $u^{sn} = \frac{\mu^{sw}}{\gamma^s}$, which prevails when wages are flexible. Thus, unemployment can be generated even when sticky wages are absent and only market imperfection exists. The natural rate of unemployment is increasing in the Frisch elasticity (determined by γ^s) and decreasing in the wage elasticity of demand (or increasing in the mark-up given by μ^{sw}).

Using the definition of unemployment, the wage inflation equation can be re-written as

$$\pi_t^{sw} = \beta E_t\{\pi_{t+1}^{sw}\} - \Theta_w^s \gamma^s (u_t^s - u^{sn}) \qquad s \in \{H, L\} \tag{3.7}$$

The above formulation of the wage inflation equation is often referred to as the New Keynesian Wage Phillips curve. It is a relationship between wage inflation and unemployment in the economy. The ratio between demand and supply elasticity is key in understanding unemployment rate differences among high and low-skilled workers. If the high-skilled earn a higher wage mark-up than low-skilled, thus having a lower wage elasticity of demand ($\epsilon_w^H < \epsilon_w^L$), the above equations imply that the former have a lower natural unemployment rate than the latter. Again, if high-skilled workers have a flatter labor supply curve, which means a lower Frisch elasticity ($\gamma^H > \gamma^L$), their flexible price unemployment rates are lower than the low-skilled workers. Slopes of the New Keynesian Wage Phillips curve depends on the demand and supply elasticity for the workers $\Theta_w^H = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\gamma^H \epsilon_w^H)}$ and $\Theta_w^L = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\gamma^L \epsilon_w^L)}$. If the above assumptions about elasticity are true, the high-skilled workers face a steeper wage Phillips curve.

3.2.2 Firms

Firms are the same as in chapter 2. There is a continuum of monopolistically competitive firms of unit mass indexed by $i \in [0, 1]$, each producing a differentiated product. Each firm hires both high and low-skill labor, which is aggregated into a labor input index using CES technology as in Goldin and Katz (2007). The production function of the firm is given by

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{3.8}$$

where

$$N_t(i) = \Big[\lambda\big(N_t^H(i)\big)^{\frac{\eta-1}{\eta}} + (1-\lambda)\big(N_t^L(i)\big)^{\frac{\eta-1}{\eta}}\Big]^{\frac{\eta}{\eta-1}} \tag{3.9}$$

$$N_t^H(i) = \Big(\frac{1}{\lambda}\int_0^{\lambda} N_t^H(i,j)^{\frac{\epsilon_w^H-1}{\epsilon_w^H}} dj\Big)^{\frac{\epsilon_w^H}{\epsilon_w^H-1}}$$

$$N_t^L(i) = \Big(\frac{1}{1-\lambda}\int_\lambda^1 N_t^L(i,j)^{\frac{\epsilon_w^L-1}{\epsilon_w^L}}dj\Big)^{\frac{\epsilon_w^L}{\epsilon_w^L-1}}$$

A_t is an aggregate technology shock which follows an AR(1) process $a_t = \rho_a a_{t-1} + \varepsilon_t^a$, $a_t \equiv \ln(A_t)$ and ε_t^a is a white noise process with mean zero and standard deviation σ_a. The decreasing returns to scale parameter in the production function is $\alpha \in [0,1]$. The elasticity of substitution between high-skill and low-skill labor in firm production is η. The wage elasticity of demand for high-skill and low-skill occupations are ϵ_w^H and ϵ_w^L, respectively. This elasticity represents substitution between the j occupations for each skill type.

The solution to a firms' cost minimization problem leads to a set of demand schedules for high and low-skill labor

$$n_t^H(i,j) = -\epsilon_w^H(w_t^H(j) - w_t^H) + n_t^H(i) \text{ and } n_t^H(i) = -\eta(w_t^H - w_t) + n_t(i)$$

$$n_t^L(i,j) = -\epsilon_w^L(w_t^L(j) - w_t^L) + n_t^L(i) \text{ and } n_t^L(i) = -\eta(w_t^L - w_t) + n_t(i) \tag{3.10}$$

The average nominal wages for the high and low-skilled households are aggregated as follows

$$W_t^H = \Big(\frac{1}{\lambda}\int_0^\lambda W_t^H(j)^{1-\epsilon_w^H}dj\Big)^{\frac{1}{1-\epsilon_w^H}}$$

$$W_t^L = \Big(\frac{1}{1-\lambda}\int_\lambda^1 W_t^L(j)^{1-\epsilon_w^L}dj\Big)^{\frac{1}{1-\epsilon_w^L}}$$

The overall wage index for the economy is

$$W_t = \Big[\lambda\Big(W_t^H\Big)^{1-\eta} + (1-\lambda)\Big(W_t^L\Big)^{1-\eta}\Big]^{\frac{1}{1-\eta}}$$

where $w_t \equiv \ln(W_t)$, $w_t^H \equiv \ln(W_t^H)$ and $w_t^L \equiv \ln(W_t^L)$.

Price Setting

The goods market is characterized by monopolistic competition. Each firm sets a price at which to sell it's product in the presence of nominal rigidities. A firm can update it's price in period t with probability $1-\theta_p$ and choose the optimal price $P^*(t)$. If a firm cannot update, the price is same as last period. Thus a firm faces a dynamic profit maximization problem, and chooses price to meet household demand for products given by equation (3.2). The resulting optimality condition is an inflation equation for prices

$$\pi_t^p = \beta E_t \left\{ \pi_{t+1}^p \right\} - \Theta_p(\mu_t^p - x_t^p) \tag{3.11}$$

where

$$\mu_t^p = \ln(1-\alpha) - rw_t - \frac{\alpha}{1-\alpha} y_t + \frac{1}{1-\alpha} a_t \tag{3.12}$$

is the average price mark-up in the economy and $\Theta_p = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\epsilon^p}$. In the absence of sticky prices the desired mark-up $x_t^p = \ln\left(\frac{\epsilon_t^p}{\epsilon_t^p - 1}\right)$ is time-varying and follows an AR(1) process with mean $\mu^p = \ln(\frac{\epsilon^p}{\epsilon^p - 1})$, autoregressive coefficient ρ_p and an innovation term ε_t^p.

The above equation is the New Keynesian Phillips curve and is similar to one obtained in a standard New Keynesian model with homogeneous labor. If average mark-up falls short of the desired level, firms update prices such that the mark-up adjusts and drives up inflation.

3.2.3 Market Clearing and Equilibrium

Goods Market

Goods market clearing requires quantity of each good produced to be equal to quantity demanded. This means total amount of a good produced by firm i must equal the amount of consumption for that good by the households. So in equilibrium we have

$$Y_t(i) = C_t(i)$$

Aggregate output is defined as $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon_t^p - 1}{\epsilon_t^p}} di \right)^{\frac{\epsilon_t^p}{\epsilon_t^p - 1}}$ and consumption of good i by high and low-skilled household is $C_t(i) = \lambda C_t^s(i) + (1-\lambda) C_t^u(i)$.

Making use of household demand for product varieties, the output market clearing can be written as

$$Y_t = C_t(1 + \Delta_t)$$

where $C_t = \lambda C_t^s + (1-\lambda)C_t^u$ and $\Delta_t = \left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon_t^p} di\right)^{\frac{\epsilon_t^p}{\epsilon_t^p - 1}}$.[3]

Labor Market

The average employment across all high-skill and low-skill households is

$$N_t^H = \frac{1}{\lambda}\int_0^1 \int_0^\lambda N_t^H(i,j)djdi \quad \text{and} \quad N_t^L = \frac{1}{1-\lambda}\int_0^1 \int_\lambda^1 N_t^L(i,j)djdi$$

Given firm demand, the labor market clearing condition is

$$N_t^H = \frac{1}{\lambda}\Big(\frac{W_t^H}{W_t}\Big)^{-\eta}\Big(\frac{Y_t}{A_t}\Big)^{\frac{1}{1-\alpha}}\Delta_t^H \Delta_t^p$$

$$N_t^L = \frac{1}{1-\lambda}\Big(\frac{W_t^L}{W_t}\Big)^{-\eta}\Big(\frac{Y_t}{A_t}\Big)^{\frac{1}{1-\alpha}}\Delta_t^L \Delta_t^p$$

where $\Delta_t^H = \int_0^\lambda \left(\frac{W_t^H(j)}{W_t^H}\right)^{-\epsilon_w^H} dj$ is the high-skill wage dispersion, $\Delta_t^L = \int_\lambda^1 \left(\frac{W_t^L(j)}{W_t^L}\right)^{-\epsilon_w^L} dj$ is low-skill wage dispersion and $\Delta_t^p = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon_t^p}{1-\alpha}}$ is price dispersion.

Output and Wage Gap

Output gap $\tilde{y}_t$ is defined as the (log) deviation between output and its natural or flexible price counterpart y_t^n, such that $\tilde{y}_t \equiv y_t - y_t^n$. Real wage for skill type s is $\omega_t^s \equiv w_t^s - p_t$, which allows me to define the real wage gap as $\tilde{\omega}_t^s \equiv \omega_t^s - \omega_t^{sn}$. Similar to natural level of output, ω_t^{sn} is the natural or flexible price level of real wage for skill type s. The deviation of price mark-up from it's natural counterpart can now be expressed in terms of output and wage gap as

$$\mu_t^p - x_t = -\tilde{\omega}_t - \Big(\frac{\alpha}{1-\alpha}\Big)\tilde{y}_t$$

Substituting the above expression into the price inflation equation (2.11) yields

$$\pi_t^p = \beta E_t\left\{\pi_{t+1}^p\right\} + \Theta_p \tilde{\omega}_t + \kappa_p \tilde{y}_t \tag{3.13}$$

[3]In the neighborhood of zero inflation steady-state, Δ_t is zero up to a first-order approximation Galí (2015). This means $Y_t = C_t$.

where $\kappa_p = \frac{\alpha}{1-\alpha}\Theta_p$.

Similarly, the deviation of average wage mark-up from its natural counterpart can be expressed in terms of wage and output gap as

$$\mu_t^{sw} - \mu^{sn} = (1-\sigma)\tilde{\omega}_t^s + \eta(\sigma+\gamma^s)\Big(\tilde{\omega}_t^s - \tilde{\omega}_t\Big) - \frac{\sigma+\alpha}{1-\alpha}\tilde{y}_t$$

allowing me to write the wage inflation inflation for skill type s as

$$\pi_t^{sw} = \beta E_t\left\{\pi_{t+1}^{sw}\right\} + \kappa_1^s\tilde{\omega}_t^s + \kappa_2^s\Big(\tilde{\omega}_t^s - \tilde{\omega}_t\Big) + \kappa_3^s\tilde{y}_t \tag{3.14}$$

where $\kappa_1^s = (1-\sigma)\Theta^s, \quad \kappa_2^s = \eta(\sigma+\gamma^s)\Theta^s, \quad \kappa_3^s = \left(\frac{\sigma+\alpha}{1-\alpha}\right)\Theta^s$.

3.3 Optimal Monetary Policy and Welfare

In this section, I describe the design of an optimal policy that the central bank implements with heterogeneous and segmented labor market. High and low-skill workers operate in separate labor markets. With different inflation-unemployment trade-off for high and low-skill workers, central bank must decide if the socially optimal policy should recognize the existence of heterogeneous labor to increase welfare in the economy.

3.3.1 Efficient Allocation

There exist two sources of sub-optimality in a New Keynesian model, first due to market power in goods and labor market and second due to nominal rigidity in price and wages. The natural allocation, which is the equilibrium under flexible price and wage, is optimal and efficient. The efficient allocation in a standard New Keynesian model with homogeneous labor is determined through the social planner problem in which the planner maximizes a representative households welfare given technology and preferences. In this paper, with skill heterogeneity in the labor market, there is one representative household for each skill type. The social planner must now maximize welfare of the economy by minimizing utility loss for each type of household.

The problem faced by a benevolent central bank is to maximize overall utility, which is a weighted sum of utility of the two skill types, given as follows

$$\begin{aligned} \max \quad & U(C_t, N_t(j); Z_t) \equiv \lambda U^H(C_t^H, N_t^H(j); Z_t) + (1-\lambda) U^L(C_t^L, N_t^L(j); Z_t) \\ \text{subject to} \quad & C_t(i) = A_t N_t(i)^{1-\alpha} \\ & N_t = \int_0^1 N_t(i) di \end{aligned}$$

The Pareto-optimality conditions are

$$\begin{aligned} -\frac{U_{n,t}^H}{U_{c,t}^H} &= MPN_t \\ -\frac{U_{n,t}^L}{U_{c,t}^L} &= MPN_t \end{aligned}$$

where

$$MPN_t \equiv (1-\alpha) A_t N_t^{-\alpha} \left(\lambda \Big(E_t \frac{N_t^H}{N_t} \Big)^{-\frac{1}{\eta}} + (1-\lambda) \Big(\frac{N_t^L}{N_t} \Big)^{-\frac{1}{\eta}} \right)$$

Optimal price and wage setting in the absence of nominal rigidities imply

$$\frac{W_t^H}{P_t} = -\frac{U_{n,t}^H}{U_{c,t}^H} \mathcal{M}_w^H \quad \text{and} \quad \frac{W_t^L}{P_t} = -\frac{U_{n,t}^L}{U_{c,t}^L} \mathcal{M}_w^L$$

and

$$P_t = \mathcal{M}_p \frac{(1-\tau) W_t}{(1-\alpha) A_t N_t^{-\alpha}}$$

where $\mathcal{M}_p = \frac{\epsilon_p}{\epsilon_p - 1}$ is the price mark-up and $\mathcal{M}_w^s = \frac{\epsilon_w^s}{\epsilon_w^s - 1}$ is the wage mark-up for skill type $s \in \{H, L\}$. The above condition results in an efficient level of activity in the steady-state. The employment subsidy $\tau = 1 - \frac{1}{\mathcal{M}_p \mathcal{M}_w \Phi_N}$, funded by the government through lumpsum taxes, ensures the optimality conditions above are satisfied. With this subsidy the distortions caused by market power in goods and labor market are exactly offset. However, there arises a distortion due to misallocation of labor among the high and low-skill labor markets given by Φ_N. In contrast to a standard model as in Erceg et al. (2000), for the natural allocation to be efficient the central bank additionally needs to be aware of the relative employment of high and low-skill workers in the steady-state. $\mathcal{M}_w$ is a composite wage mark-up that aggregates high and low-skill wage mark-up. If labor market is homogeneous, where $\lambda = 1$ and $\epsilon_w^H = \epsilon_w^L$, the steady-state distortion term is 1 and we get $\tau = 1 - \frac{1}{\mathcal{M}_p \mathcal{M}_w}$ as in Galí (2015) and Erceg et al. (2000).

3.3.2 Optimal Policy Problem

The employment subsidy funded by the government addresses the suboptimality created by monopolistic competition in the goods and labor market. The second source of suboptimality, due to sticky prices and wages, results in a gap between the optimal and natural allocation. In such a situation, the central bank targets this gap and sets path for outcomes that minimizes the utility losses for all agents in the economy. Following Woodford (2011), I derive a second-order approximation to the welfare loss for the economy due to deviation from the efficient allocation. Utility for the entire economy is a convex combination of utility for the two skill types, where I use the fraction of each skill type as Pareto weights. The utility loss is given by

$$\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Psi_1 (\pi_t^p)^2 + \Psi_2 (\pi_t^{Hw})^2 + \Psi_3 (\pi_t^{Lw})^2 + \Psi_4 (\tilde{y}_t)^2 \right] \tag{3.15}$$

The derivation of the above expression is described in Appendix B.2 along with description of the weights on each policy term. As in Erceg et al. (2000), the first term in equation (3.15) is associated with utility loss due to inefficient allocation of labor by firms and the last term is due to inefficiency in the level of output. Additionally, there arises utility loss from inefficient allocation of labor services. Since there are high and low-skill workers in the economy, I find two separate terms, one that targets inefficiency in high-skill labor services and another that targets the low-skill labor services.

The weights on each policy variable depends on structural parameters of the model. The values of these parameters are given by the estimates presented in chapter 2 of this thesis.[4] As high and low-skill workers become more substitutable in the production process, that is $\eta \to \infty$, welfare loss decreases. This takes place in two ways, firstly because inefficiency in the allocation of labor by firms decrease since firms consider high and low-skill workers similar and this makes their labor hiring decisions more efficient. Secondly, inefficiency in allocation of labor services also reduces as there is only one skill type in the market now. With the reduction of labor misallocation of high and low-skill workers in both labor markets, weights of the price and wage inflation terms become smaller thus reducing welfare loss overall. Estimates of Frisch elasticity and wage elasticity of demand are larger for the low-skill thus increasing the weight on low-skill wage inflation in the loss function.

The optimal policy problem under full commitment faced by the central bank is

$$\begin{aligned}
\max \qquad & \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Psi_1(\pi_t^p)^2 + \Psi_2(\pi_t^{Hw})^2 + \Psi_3(\pi_t^{Lw})^2 + \Psi_4(\tilde{y}_t)^2 \right] \\
\text{subject to } \quad & \pi_t^p = \beta E_t \left\{ \pi_{t+1}^p \right\} + \Theta_p \tilde{\omega}_t + \kappa_p \tilde{y}_t \\
& \pi_t^{Hw} = \beta E_t \left\{ \pi_{t+1}^{Hw} \right\} + \kappa_1^H \tilde{\omega}_t^H + \kappa_2^H \left(\tilde{\omega}_t^H - \tilde{\omega}_t \right) + \kappa_3^H \tilde{y}_t \\
& \pi_t^{Lw} = \beta E_t \left\{ \pi_{t+1}^{Lw} \right\} + \kappa_1^L \tilde{\omega}_t^L + \kappa_2^L \left(\tilde{\omega}_t^L - \tilde{\omega}_t \right) + \kappa_3^L \tilde{y}_t
\end{aligned}$$

To account for labor allocation in two separate markets the central bank, while optimally choosing the path for outcomes in the economy, faces a trade-off between stabilization of inflation and output gap. To make all agents in the economy better-off, the central bank stabilizes inflation at the cost of output gap. With heterogeneous labor markets, an additional trade-off arises where the monetary authority must also consider wage inflation of the high and low-skill workers. The lower bound of zero welfare losses, by setting $\pi_t^p = \pi_t^{sw} = \tilde{y}_t = 0$, is no longer attainable and optimal policy must balance output, price inflation and wage inflation for both skill types jointly. The Frisch elasticity and wage elasticity of demand for the two skill types generate a flatter wage Phillips curve for the low-skill. The wage inflation for low-skill workers is less responsive to fluctuations in their unemployment level. Since optimal

[4] Table 3.1 provides a complete list of information on all structural parameters of the model.

monetary policy is set conditional on the wage Phillips curve, the central bank tries to correct for this non-response of low-skill wage inflation by stabilizing outcomes of low-skill workers at the cost of high-skill workers.

With skill differentials in the labor market, optimal monetary policy must strike a balance between price inflation, output and wage inflation for the two skill types. To further investigate this policy trade-off, I discuss two limiting cases. In the first case I consider prices to be flexible where $\Theta_p \rightarrow \infty$ and we get a vertical Phillips curve. With flexible prices, no market power exists in the goods market and the term associated with inefficient allocation of labor by the firm disappears from the loss function. There still exists monopoly power in the labor market leading to inefficient allocation of labor services. In this paper however, there are two labor markets with a wage Phillips curve for each market and setting $\pi_t^w = 0 = \tilde{y}_t$ violates optimality conditions. Unlike Erceg et al. (2000), in a setting with heterogeneous labor market agents, stabilizing only wage inflation and output gap can no longer generate zero welfare losses even with flexible prices.

Secondly, I consider wages in both labor markets to be flexible and $\Theta^H \rightarrow \infty$ and $\Theta^L \rightarrow \infty$. Labor unions have no market power and the terms associated with inefficient allocation of labor services no longer exist in the loss function. However, due to market segmentation firms still choose optimal number of high and low-skill workers for production and a wage differential still exists in the economy. Setting $\pi_t^p = 0 = \tilde{y}_t$ violates equilibrium conditions and zero welfare losses cannot be obtained by stabilizing price inflation and output even when wages are flexible. Therefore, a strict price inflation targeting policy or a strict wage inflation targeting policy is suboptimal even in limiting cases of flexible prices and wages. Central banks must consider a combination of price inflation and wage inflation stabilization.

3.3.3 Comparing Optimal Policy and Taylor-type Rule

Under the optimal policy problem, the central bank sets the path for outcomes in the economy by minimizing utility losses due to deviation from the efficient allocation. The instrument for the central bank is the nominal interest rate. In this section, I discuss the results obtained from the optimal policy problem. I compare these results to a Taylor-type rule, where the nominal interest rate responds to price inflation, output gap and high and low-skill wage inflation.[5] I compare the dynamics of macroeconomic variables to a 1% increase in technology under the optimal policy and the Taylor-type rule and calculate welfare losses in the economy under the two policies. For parameter values I use the results obtained from the Bayesian estimation exercise from chapter 2 and are presented in table 3.1.

Impulse Responses

Impulse responses to a 1% increase in technology shows the optimal policy stabilizes macroeconomic outcomes of the low-skill better than the Taylor-type rule. The percentage deviation of output from steady-state, as seen in Figure 3.1, is positive under both policies, but the rise is longer and more persistent under the optimal policy. This deviation is negative for price inflation and smaller under the optimal policy. Inflation reverts back to its steady-state value faster under the optimal policy than under the Taylor-type rule. This is a result of the inflation-output trade-off faced by the central bank in which inflation stabilization is given more significance at the cost of output stabilization. The variance of inflation decreases from 0.13 to 0.03 when we move from the Taylor-type rule to optimal policy. On the other hand, variance of output increases from 0.86 to 1.95. Moving on to skill specific outcomes, deviation of high-skill consumption from steady-state rises due to falling interest rates. Low-skill consumption falls due to fall in wage income as these workers are hand-to-mouth and consume current income every period. This result holds for the first five quarters after

[5]This Taylor-type rule is the same as the one estimated in chapter 2

the shock hits. High-skill consumption shows a longer and more persistent deviation from steady-state under the optimal policy after the first five quarters. As for low-skill consumption, optimal policy results in a lagged increase making these workers better off than under the Taylor-type policy. The optimal policy improves consumption for the low-skill by compromising on stability of consumption for high-skill.

Unemployment rates for both skill types show an initial increase, almost of the same magnitude as the Taylor-type rule. However these outcomes approach steady-state faster under the optimal policy. This result is expected as optimal policy targets misallocation of labor among high and low-skill labor markets. There is also greater real wage adjustment under the optimal policy due to larger increases in nominal wages and fall in unemployment even with muted deflation. With different wage Phillips curves for high and low-skill workers the central bank tries to correct for the unemployment-inflation trade-off that is almost non-existent for the low-skill with the optimal policy. This results in similar responses of high and low-skill real wages under the optimal policy. Moreover, the optimal policy results in low-skill unemployment reverting to steady-state levels in about the same time as the high-skill unemployment rate, even though the initial increase for the former was greater. The optimal policy puts more weight on low-skill outcomes since their wage Phillips curve suggests inflation is not very informative about their labor market tightness.

Welfare

To further analyze the performance of optimal policy I evaluate and compare utility losses under the two policies using the loss function (3.15). These losses are generated as a result of deviation from the efficient allocation due to nominal rigidities. Welfare losses are reported in table 3.2 and expressed as a percentage of steady-state consumption. The loss is considerably lower under the optimal policy compared to the Taylor-type rule. The agents in the economy are willing to forego 2.2% of their consumption under the Taylor rule to go back to the efficient friction-less benchmark.

In comparison, they would only require to forego 0.73% of their consumption to arrive at the efficient allocation under the optimal policy. The high-skill workers experience a welfare gain under the Taylor-type rule, of 0.3% of consumption, but a negligible welfare loss under the optimal policy of 0.02% of consumption. On the other hand, low-skill workers lose 2.4% of their consumption under the Taylor rule compared to a loss of only 0.8% under the optimal policy. Overall the economy loses an additional 1.5% of consumption under the naive policy compared to when policy incorporates labor heterogeneity.

It is clear from the impulse responses and welfare calculations that optimal policy calls for a stronger unemployment stabilization for both high and low-skill workers. The second moments for the variables presented in table 3.3 show an increase in the variance of output going from the Taylor-type rule to optimal policy. Volatility of high-skill unemployment and labor is almost similar between both policies, but the volatility of these outcomes for low-skill workers decreases under the optimal policy. Even though the Taylor-type rule responds to skill outcomes separately, it is unable to mimic the optimal policy in stabilization of unemployment. These results point to a more stable unemployment rate, especially for the low-skill workers. In the next section, I explore whether including unemployment rate for high and low-skill workers as an argument in a policy rule can improve welfare in an economy with skill heterogeneity when the economy is subjected to a positive technology shock.

3.3.4 Optimal Simple Rules

The framework developed in this paper has been analyzed under two types of policies. In chapter 2 I propose a Taylor-type rule where the nominal interest rate responds to wage inflation for each skill type along with price and wage inflation. Estimation of the coefficients of this rule shows that in the U.S. economy response of monetary policy can be significantly different for the high and low-skill workers. However, welfare losses under this policy rule is substantially large. In this chapter I design the optimal policy in which the central bank minimizes utility loss using the nominal interest rate as an instrument. This naturally generates lower welfare losses compared to the Taylor-type rule and calls for greater unemployment stabilization for high and low-skill workers. Since the analysis under the optimal policy shows lower volatility of unemployment can lead to welfare gains, in this section I talk about some simple rules with unemployment that generate similar welfare gains as the optimal policy. A potential advantage of using unemployment as an argument in the interest rate rule is its observability to the policymaker.

The simple interest rate rules take the form of

$$i_t = \hat{\rho}_R i_{t-1} + (1 - \hat{\rho}_R)(\rho + \hat{\phi}_\pi \pi_t^p + \hat{\phi}_y \hat{y}_t + \hat{\phi}_u^H \hat{u}_t^H + \hat{\phi}_u^L \hat{u}_t^L + \hat{\phi}_w^H \pi_t^{Hw} + \hat{\phi}_w^L \pi_t^{Lw}) \quad (3.16)$$

where the coefficients are chosen such that they minimize the unconditional period utility loss given by

$$\Psi_1 var(\pi_t^p) + \Psi_2 var(\pi_t^{Hw}) + \Psi_3 var(\pi_t^{Lw}) + \Psi_4 var(\hat{y}_t)$$

Under a 1% increase in technology the coefficients resulting from the optimization routine for various specifications of (3.16) are presented in table 3.4. Along with the values for each coefficient I also present the utility loss as a percentage of steady-state consumption and the implied utility loss expressed as a ratio to the loss under the optimal policy (reported in table 3.2). Rows (a) and (b) show results from a nave rule that only responds to price inflation and output gap, with interest rate smoothing in row (b). The coefficient on inflation is large and positive whereas the coefficient on output gap is small and negative. These results are similar to Galí (2010) where the author also finds a small negative coefficient on output gap. The utility loss under this simple rule is large and twice the size of that generated by the optimal policy. Agents in this economy must forego approximately 1.5% of their consumption to return to the efficient allocation. This result is expected as the simple rules in (a) and (b) does not address the inefficiencies in outcomes of high and low-skill workers in the economy.

The specifications in rows (c) and (d) include unemployment gap of high and low-skill workers are arguments in the simple rule. I find a small amount of interest rate smoothing. The loss-minimizing coefficient on inflation is positive and close to 1 whereas the coefficient on output gap is large and negative. The negative value on output gap is also seen in Galí (2010) once unemployment is included as an argument in the simple rule, however the author uses aggregate unemployment whereas I use unemployment by skill type. The coefficient on high-skill unemployment is large and negative however the coefficient for low-skill unemployment is small and positive. Utility losses are reduced by a substantial amount when the policy rule includes unemployment rates. Agents would only have to forego 0.8% of their consumption to return to the efficient allocation, compared to the 1.5% under the naive policy. Moreover, this rule generates a welfare loss that is only 12-14% higher when compared to the optimal policy. This is because the policy rule incorporates labor market inefficiencies by targeting unemployment that the nave rule in rows (a) and (b) failed to recognize. Specifications (e) and (f) show the central bank does not respond to wage inflation with coefficients being negligibly small. Welfare loss in this case is almost the same as specifications (c) and (d) thus proving the robustness of the simple rule with unemployment.

Using the loss-minimizing coefficients for the specification in (c), I compare the response of outcomes under this simple rule to the optimal policy as shown in figure 3.2. For the aggregate macroeconomic variables response of price inflation is identical. However, output shows a strong increase for the first six quarters after which it tracks the response under the optimal policy. For skill specific outcomes, response of real wage is identical whereas unemployment shows a smaller increase under the simple rule for both skill types. High-skill consumption shows a large initial increase due to decreasing interest rates but tracks the optimal policy from quarter 6. Low-skill consumption shows a smaller initial decrease and the lagged increase after quarter 6. Overall, the simple rule is a reasonable approximation of the optimal policy.

3.3.5 A Simple Rule with Unemployment

Finally, to show how accounting for heterogeneous agents in the economy can help policymakers, I propose an interest rate rule that responds to unemployment rates for high and low-skill workers. I generalize the simple rule specified in the previous section and compare its performance to a standard Taylor (1993) rule and the Federal Funds Rate during the Greenspan-Bernanke-Yellen era of the U.S. Federal Reserve.

The proposed empirical rule is

$$i_t = 1 + 1.5\pi_t - 1.6\hat{u}_t^H + 0.4\hat{u}_t^L \tag{3.17}$$

where i_t is the quarterly policy rate and π_t denotes quarterly inflation. As a measure of inflation, I use PCE (Personal Consumption Expenditure) inflation obtained from the Bureau of Economic Analysis. As in a Taylor (1993) rule, the proposed rule above implies an annual inflation target of 2% and a steady-state real interest rate of 2%. Additionally, I also include unemployment rates for high and low-skill workers. The policy rate responds countercyclically to high-skill unemployment and procyclically to low-skill unemployment and are chosen keeping in mind the results from the previous section. For target unemployment rates I use the mean of high and low-skill unemployment over the sample period. Since unemployment and output gap are related through Okuns law I do not include the latter as an argument in the proposed rule. Furthermore, in the previous section we noted that an optimized simple rule with unemployment has a negative coefficient on output gap.

In figure 3.3, I compare the proposed empirical rule to the Federal Funds Rate over the period 1987Q1-2015Q4 and the "standard" Taylor rule given by

$$i_t = 1 + 1.5\pi_t + \hat{y}_t \tag{3.18}$$

As seen in the figure, the Taylor (1993) rule tracks the Federal Funds rate reasonably well. However, two main criticisms have emerged against the policy of the Federal Reserve in recent years. The first criticism raised by Taylor (2009) and others say that over the period 2003-05 the Federal Funds Rate has been lower than what is proposed by the Taylor rule. In the figure you can see the Federal Funds Rate is around 2% over this period whereas the Taylor rule predicts a policy rate of around 4%. Moreover, since the financial crisis of 2007, the Federal Funds Rate has remained at its zero-lower bound level far longer and should have recovered sooner. It is clear from the figure that the Federal Funds Rate has remained close to zero since 2009 whereas the policy rate under the Taylor rule becomes positive after 2013. Like the Taylor rule, the proposed simple rule with unemployment also suggests the policy rate should have recovered earlier, but the recovery should have started around 2010 with the policy rate reaching 2% by 2011 and 3% by 2016. Economists argue about the recent rate hikes by the Fed saying that on one hand it's required so that there's enough room for a decrease when the next recession hits.[6] But with increasing the Federal Funds rate over a short period of time also causes concern for the economy overheating, which makes the next recession more likely. The simple rule with unemployment proposed here addresses this issue by showing the policy rate increase should have started around 2010 thus mitigating the effects of an over-heating economy due to persistent rate hikes.

The second criticism addresses the full employment aspect of the Federal Reserve's mandate. Economists point out that with weak wage growth in recent years the economy is not at full employment even though aggregate unemployment rates suggest otherwise.[7] Bivens and Zipperer (2018) show extended periods of low unemployment can help boost wage growth and shrink disparities in the labor market, a consequence of the Great Recession. However, they do not find significant improvement in outcomes of the groups that suffered most in the aftermath of the Great Recession, even though aggregate unemployment and labor force participation shows significant improvement. The proposed simple rule addresses this concern as monetary policy responds to unemployment rates of two kinds of workers separately. If the policy rate responds to their unemployment separately I show there is improvement in welfare,

[6]Bivens (2018a)

[7]Bivens (2018b); White (2017)

even with rising interest rates. Overall, the evidence presented above suggest paying attention to heterogeneity in the labor market and targeting unemployment rates for high and low-skill workers may help the Federal Reserve better fulfill their dual mandate of achieving maximum employment for all demographic groups in the economy while also keeping inflation low.

3.4 Conclusion

In this chapter I use the framework built in chapter 2 to analyze an optimal monetary policy and welfare when labor markets have heterogeneous agents. With workers in the labor market being either high or low-skilled and differing in their labor supply and demand elasticity, the wage Phillips curve for the two skill types are different. Specifically, high-skill workers have a steeper wage Phillips curve than the low-skill workers. This indicates the former's wage inflation is more responsive to fluctuations in unemployment levels than the latter. Given that wage inflation fails to be a good measure of labor market tightness for low-skill workers, policymakers at central banks would want to incorporate this difference in their decisions such that the new policy would benefit all sections of the economy.

Presence of heterogeneous labor market participants and their different unemployment-inflation trade-off has important implications for the design of monetary policy. To study how policymakers can better stabilize outcomes of different skill groups I design an optimal Ramsey policy for the central bank. In this problem the monetary authority sets the equilibrium path for agents outcomes by maximizing welfare for all agents. Due to a flat wage Phillips curve for low-skill workers and almost no response of low-skill wage inflation to unemployment fluctuations, optimal policy puts more weight on low-skill outcomes. An additional trade-off arises for the central bank where the policy must strike a balance between stabilization of price inflation, GDP and wage inflation for both skill types.

Comparing this optimal monetary policy to a Taylor rule I find macroeconomic outcomes are better stabilized under the optimal policy. There arises an inflation-output gap trade-off for the central bank and this policy has a greater focus on inflation stabilization as opposed to output stabilization. This can be seen in the impulse response functions of these two outcomes to a positive technology shock where output takes a longer time to return to its steady-state level compared to inflation. I find the optimal policy also stabilizes outcomes of the different skill types better than a Taylor rule. Moreover, this policy makes the low-skill workers better off by returning their unemployment and real wage to steady-state levels faster. In terms of welfare, the economy loses an additional 1.5% of their consumption under the naive policy. I find a clear redistribution of welfare where the central bank makes the low-skill workers better-off by making the high-skill workers slightly worse-off. High-skill workers get about 0.3% less consumption under the optimal policy but the low-skill workers get 1.4% more consumption.

I further show the optimal policy can be implemented using a simple interest rate rule with unemployment rates of high and low-skill workers. Welfare losses reduce by half compared to a policy that does not account for skill heterogeneity. A policy rule with unemployment performs as well as the Taylor rule and addresses some of the criticisms raised against the Federal Reserve's recent policy. Through the analysis of monetary policy conducted in this paper, differences among labor market participants seem to be key. This paper provides a framework through which policymakers at central banks will be better able to assess economic performance and improve outcomes of disadvantaged groups in the economy.

Tables

Table 3.1.
Parameter values and source

Parameter		Value	Source
Cobb-Douglas share	α	0.33	Galí et al. (2007)
Discount factor	β	0.99	Galí et al. (2004)
Fraction skilled	λ	0.36	calculated from CPS 2016
Risk aversion	σ	2	Ascari et al. (2017)
Elasticity goods	ϵ_p	6	Ascari et al. (2017)
Price stickiness	θ_p	0.75	Ascari et al. (2017)
Wage stickiness	θ_w	0.75	Ascari et al. (2017)
Substitution HS-LS	η	1.27	Estimated
Inverse Frisch HS	γ^H	5.79	Estimated
Inverse Frisch LS	γ^L	3.23	Estimated
Wage Elasticity HS	ϵ_w^H	2.70	Estimated
Wage Elasticity LS	ϵ_w^L	6.72	Estimated

Table 3.2.
Welfare comparison

	Taylor-type Rule	Optimal Policy	Frictionless
Overall	-2.17	-0.73	0
High-skill	0.30	0.04	0
Low-skill	-2.39	-0.79	0

Note: Optimal policy refers to the case when the central banks implement the Ramsey policy and sets the path of variables by maximizing welfare. Under each case, the welfare loss/gain is reported as a percentage of steady-state consumption.

Table 3.3.
Second-order moments

	Taylor-type Rule		Optimal Policy	
	$\sigma(x)$	$\rho(x,y)$	$\sigma(x)$	$\rho(x,y)$
Output	0.92	1	1.40	1
HS unemployment	1.53	0.93	1.57	0.39
LS unemployment	3.33	0.94	3.09	0.44
HS employment	1.99	-0.94	1.93	-0.55
LS employment	2.18	-0.99	1.96	-0.53

Note: Under the optimal policy the central bank sets path of outcomes by minimizing utility losses. The Taylor-type rule is the one presented in section 3. Under each policy the standard deviation of the outcome and it's correlation with output is presented.

Table 3.4.
Optimal Simple Rules

	$\hat{\rho}_R$	$\hat{\phi}_\pi$	$\hat{\phi}_y$	$\hat{\phi}_u^H$	$\hat{\phi}_u^L$	$\hat{\phi}_w^H$	$\hat{\phi}_w^L$	**Loss**
(a)		2.37	-0.07					-1.73 (2.37)
(b)	0.6	3.46	-0.08					-1.48 (2.03)
(c)		1.07	-0.21	-0.30	0.07			-0.83 (1.14)
(d)	0.19	1.03	-0.20	-0.37	0.10			-0.82 (1.12)
(e)		1.07	-0.21	-0.30	0.07	-1.42×10^{-5}	-1.24×10^{-5}	-0.83 (1.14)
(f)	0.19	1.03	-0.20	-0.37	0.10	8.44×10^{-5}	9.69×10^{-5}	-0.81 (1.11)

Note: The coefficients of the simple rules are chosen such that the loss function presented in section 5 is minimized. Under each specification, the utility loss is reported as a percentage of steady-state consumption. In parenthesis I report the loss relative to that under the optimal policy.

Figures

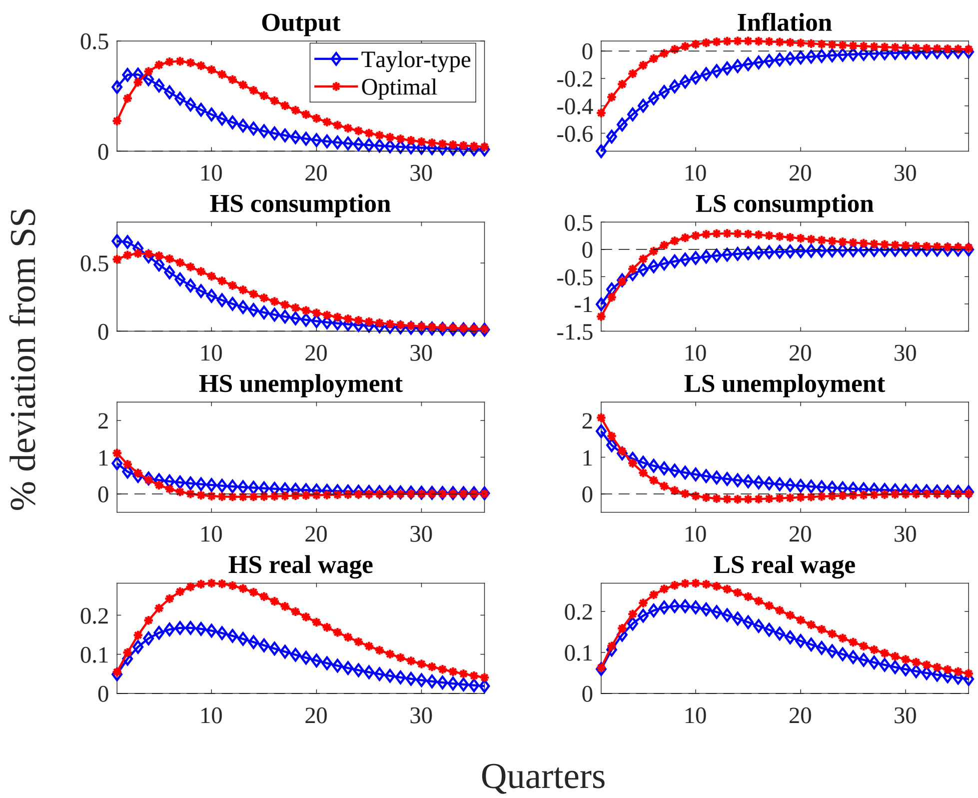


Figure 3.1. Comparing Taylor Rule and Optimal Monetary Policy

Note: The shock here is a 1% increase in technology. Under the optimal policy a central bank solves the Ramsey problem. The path of outcomes are set by the central bank such that welfare of the economy is maximized. The optimal policy is compared responses of outcomes when the interest rate follows the Taylor rule. Variables are expressed as percent deviation from steady-state. Price inflation is expressed in annual terms.

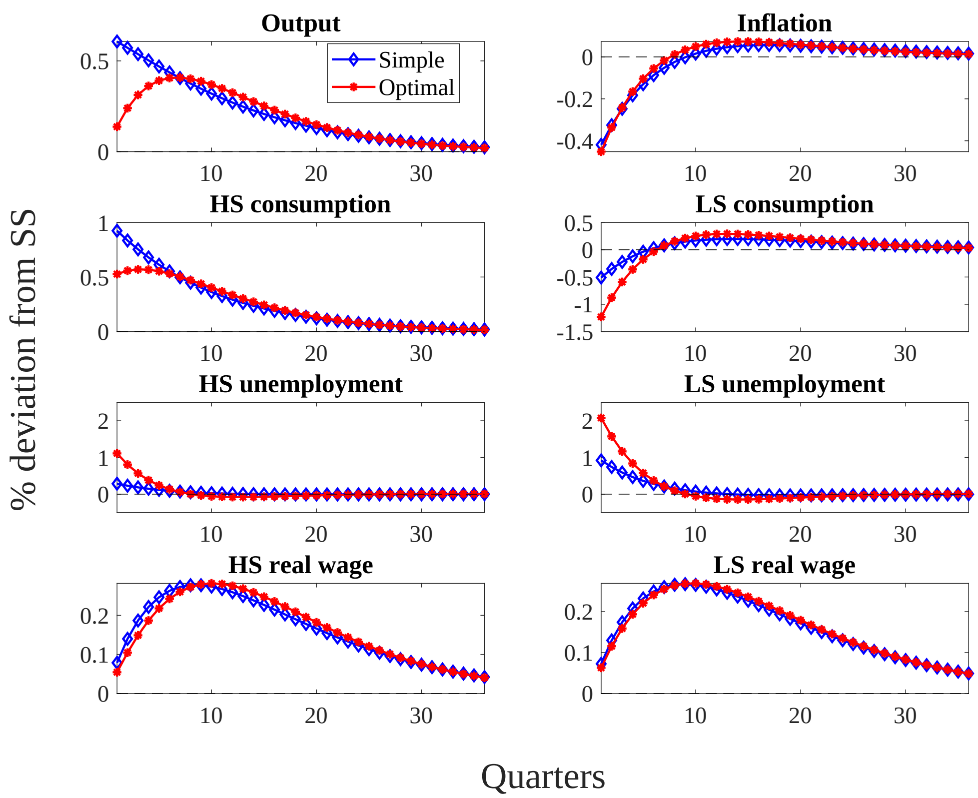


Figure 3.2. Comparing Simple Rule and Optimal Monetary Policy

Note: The shock here is a 1% increase in technology. Under the optimal policy a central bank solves the Ramsey problem. The path of outcomes are set by the central bank such that welfare of the economy is maximized. The optimal policy is compared responses of outcomes when the interest rate follows a simple rule with unemployment. Variables are expressed as percent deviation from steady-state. Price inflation is expressed in annual terms.

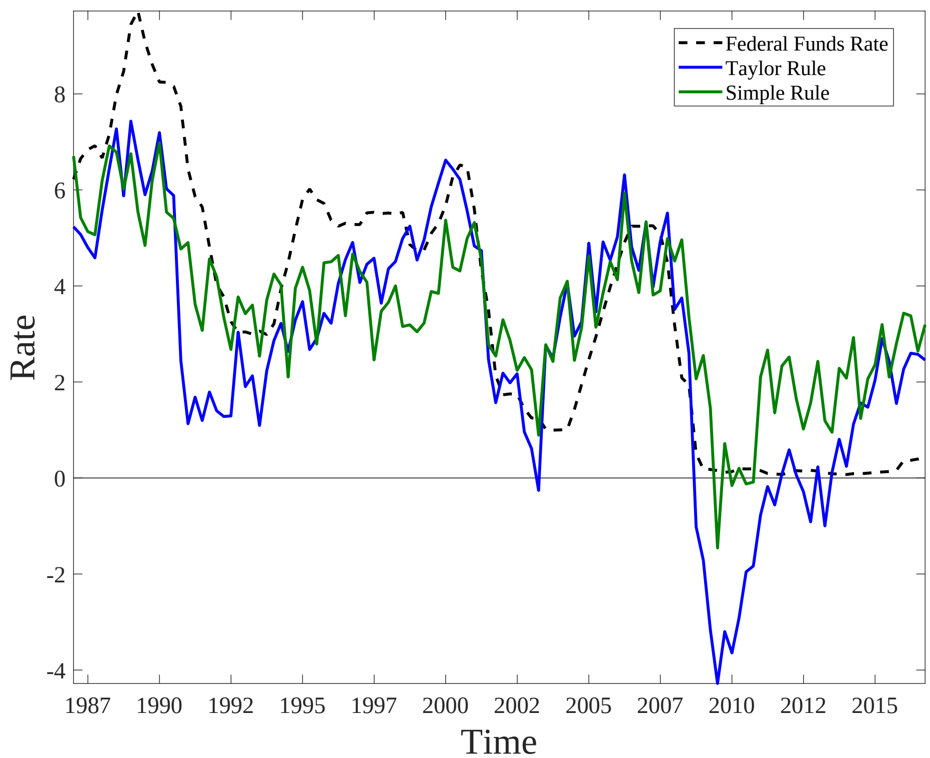


Figure 3.3. Performance of Proposed Simple Rule with Unemployment

Note: The simple rule proposed in section 3.3.5 equation (3.17) is compared to the Federal Funds Rate and the "standard" Taylor rule equation (3.18)

BIBLIOGRAPHY

Bibliography

Ascari, G., Colciago, A., and Rossi, L. (2017). Limited asset market participation, sticky wages, and monetary policy. *Economic Inquiry*, 55(2):878–897.

Autor, D. and Acemoglu, D. (2010). Skills, tasks, and technologies: Implications for employment and earnings. *NBER Working Paper*, (16082).

Autor, D. H., Katz, L. F., and Kearney, M. S. (2008). Trends in us wage inequality: Revising the revisionists. *The Review of economics and statistics*, 90(2):300–323.

Autor, D. H., Levy, F., and Murnane, R. J. (2003). The skill content of recent technological change: An empirical exploration. *The Quarterly journal of economics*, 118(4):1279–1333.

Barakchian, S. M. and Crowe, C. (2013). Monetary policy matters: Evidence from new shocks data. *Journal of Monetary Economics*, 60(8):950–966.

Barro, R. J. (1977). Unanticipated money growth and unemployment in the united states. *The American Economic Review*, 67(2):101–115.

Bernanke, B. S., Boivin, J., and Eliasz, P. (2005). Measuring the effects of monetary policy: a factor-augmented vector autoregressive (favar) approach. *The Quarterly journal of economics*, 120(1):387–422.

Bilbiie, F. O. (2008). Limited asset markets participation, monetary policy and (inverted) aggregate demand logic. *Journal of Economic Theory*, 140(1):162–196.

Bivens, J. (2018a). How new fed chair jerome powell should get ready for the next recession [blog post]. *https://www.epi.org/blog/how-new-fed-chair-jerome-powell-should-get-ready-for-the-next-recession/*.

Bivens, J. (2018b). A long spell of very low unemployment would raise wages-even in the face of employers' monopsony power [blog post]. *https://www.epi.org/blog/a-long-spell-of-very-low-unemployment-would-raise-wages-even-in-the-face-of-employers-monopsony-power/.*

Bivens, J. and Zipperer, B. (2018). The importance of locking in full employment for the long haul. *Economic Policy Institute*.

Blanchard, O. J. and Kahn, C. M. (1980). The solution of linear difference models under rational expectations. *Econometrica: Journal of the Econometric Society*, pages 1305–1311.

Campbell, J. Y. and Mankiw, N. G. (1989). Consumption, income, and interest rates: Reinterpreting the time series evidence. *NBER macroeconomics annual*, 4:185–216.

Carpenter, S. B. and Rodgers, W. M. (2004). The disparate labor market impacts of monetary policy. *Journal of Policy Analysis and Management*, 23(4):813–830.

Christiano, L. J. (2002). Solving dynamic equilibrium models by a method of undetermined coefficients. *Computational Economics*, 20(1-2):21–55.

Christiano, L. J., Eichenbaum, M., and Evans, C. L. (1999). Monetary policy shocks: What have we learned and to what end? *Handbook of macroeconomics*, 1:65–148.

Coibion, O. (2012). Are the effects of monetary policy shocks big or small? *American Economic Journal: Macroeconomics*, 4(2):1–32.

Coibion, O., Gorodnichenko, Y., Kueng, L., and Silvia, J. (2017). Innocent bystanders? monetary policy and inequality. *Journal of Monetary Economics*, 88:70–89.

Colciago, A. (2011). Rule-of-thumb consumers meet sticky wages. *Journal of money, credit and banking*, 43(2-3):325–353.

Del Negro, M. and Schorfheide, F. (2008). Forming priors for dsge models (and how it affects the assessment of nominal rigidities). *Journal of Monetary Economics*, 55(7):1191–1208.

Elsby, M. W., Hobijn, B., and Sahin, A. (2010). The labor market in the great recession. Technical report, National Bureau of Economic Research.

Erceg, C. J., Henderson, D. W., and Levin, A. T. (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of monetary Economics*, 46(2):281–313.

Faia, E. (2008). Optimal monetary policy rules with labor market frictions. *Journal of Economic dynamics and control*, 32(5):1600–1621.

Furlanetto, F. (2011). Fiscal stimulus and the role of wage rigidity. *Journal of Economic Dynamics and Control*, 35(4):512–527.

Galí, J. (2010). Unemployment fluctuations and stabilization policies: A new keynesian perspective. *Zeuthen lectures delivered March*.

Galí, J. (2011). The return of the wage phillips curve. *Journal of the European Economic Association*, 9(3):436–461.

Galí, J. (2015). *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*. Princeton University Press.

Gali, J., López-Salido, J. D., and Vallés, J. (2004). Rule-of-thumb consumers and the design of interest rate rules. Technical report, National Bureau of Economic Research.

Galí, J., López-Salido, J. D., and Vallés, J. (2007). Understanding the effects of government spending on consumption. *Journal of the European Economic Association*, 5(1):227–270.

Gertler, M. and Karadi, P. (2015). Monetary policy surprises, credit costs, and economic activity. *American Economic Journal: Macroeconomics*, 7(1):44–76.

Goldin, C. and Katz, L. F. (2007). Long-run changes in the us wage structure: Narrowing, widening, polarizing. Technical report, National Bureau of Economic Research.

Gornemann, N., Kuester, K., and Nakajima, M. (2012). Monetary policy with heterogeneous agents.

Greenwood, J., Hercowitz, Z., and Krusell, P. (1997). Long-run implications of investment-specific technological change. *The American Economic Review*, pages 342–362.

Gürkaynak, R. S., Sack, B., and Swanson, E. (2005). The sensitivity of long-term interest rates to economic news: Evidence and implications for macroeconomic models. *American economic review*, 95(1):425–436.

Herbst, E. P. and Schorfheide, F. (2015). *Bayesian estimation of DSGE models*. Princeton University Press.

Hetzel, R. L. (2018). *The evolution of US monetary policy*. Springer.

Jordà, Ò. (2005). Estimation and inference of impulse responses by local projections. *American economic review*, 95(1):161–182.

Juhn, C., Murphy, K. M., and Pierce, B. (1993). Wage inequality and the rise in returns to skill. *Journal of political Economy*, 101(3):410–442.

Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary policy according to hank. *American Economic Review*, 108(3):697–743.

Katz, L. F. and Murphy, K. M. (1992). Changes in relative wages, 1963–1987: supply and demand factors. *The quarterly journal of economics*, 107(1):35–78.

Kimball, M. S. and Shapiro, M. D. (2008). Labor supply: Are the income and substitution effects both large or both small? Technical report, National Bureau of Economic Research.

Krusell, P., Ohanian, L. E., Ríos-Rull, J.-V., and Violante, G. L. (2000). Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica*, 68(5):1029–1053.

Lester, R. (2014). Heterogeneous skill and wage rigidity in a dsge model.

Lichter, A., Peichl, A., and Siegloch, S. (2015). The own-wage elasticity of labor demand: A meta-regression analysis. *European Economic Review*, 80:94–119.

Lindquist, M. J. (2004). Capital–skill complementarity and inequality over the business cycle. *Review of Economic Dynamics*, 7(3):519–540.

Luetticke, R. (2018). Transmission of monetary policy with heterogeneity in household portfolios.

Owyang, M. T. and Ramey, G. (2004). Regime switching and monetary policy measurement. *Journal of Monetary Economics*, 51(8):1577–1597.

Powell, J. H. et al. (2018). Monetary policy at a time of uncertainty and tight labor markets: A speech at "price and wage-setting in advanced economies", an ecb forum on central banking, sintra, portugal, june 20, 2018. Technical report.

Prasad, E. (1996). Skill heterogeneity and the business cycle. *Canadian Journal of Economics*, pages 910–929.

Ramey, V. A. (2016). Macroeconomic shocks and their propagation. In *Handbook of macroeconomics*, volume 2, pages 71–162. Elsevier.

Ravenna, F. and Walsh, C. E. (2012). Screening and labor market flows in a model with heterogeneous workers. *Journal of Money, Credit and Banking*, 44:31–71.

Riley, R. and Young, G. (2007). Skill heterogeneity and equilibrium unemployment. *Oxford Economic Papers*, 59(4):702–725.

Romer, C. D. and Romer, D. H. (1989). Does monetary policy matter? a new test in the spirit of friedman and schwartz. *NBER macroeconomics annual*, 4:121–170.

Romer, C. D. and Romer, D. H. (2004). A new measure of monetary shocks: Derivation and implications. *American Economic Review*, 94(4):1055–1084.

Seguino, S. and Heintz, J. (2012). Monetary tightening and the dynamics of us race and gender stratification. *American Journal of Economics and Sociology*, 71(3):603–638.

Sims, C. A. (1972). Money, income, and causality. *The American economic review*, 62(4):540–552.

Sims, C. A. (2002). Solving linear rational expectations models. *Computational economics*, 20(1):1–20.

Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American economic review*, 97(3):586–606.

Taylor, J. B. (1993). Discretion versus policy rules in practice. In *Carnegie-Rochester conference series on public policy*, volume 39, pages 195–214. Elsevier.

Taylor, J. B. (2009). The financial crisis and the policy responses: An empirical analysis of what went wrong. Technical report, National Bureau of Economic Research.

Thorbecke, W. (2001). Estimating the effects of disinflationary monetary policy on minorities. *Journal of Policy Modeling*, 23(1):51–66.

White, G. B. (2017). Full employment: Are we there yet? *The Atlantic*. *https://www.theatlantic.com/business/archive/2017/05/full-employment/528339/*.

Woodford, M. (2011). *Interest and prices: Foundations of a theory of monetary policy.* princeton university press.

Yellen, J. L. (2017). Inflation, uncertainty, and monetary policy. *Business Economics*, 52(4):194–207.

Zago, R. (2018). Job polarization, skill mismatch and the great recession.

APPENDICES

A. APPENDIX: Skill Heterogeneity in an Estimated DSGE Model

A.1 Derivation of key model equations

A.1.1 Households

Demand for product varieties

$$\max_{C_t^s(i)} \quad C_t^s = \left(\int_0^1 C_t^s(i)^{\frac{\epsilon_t^p - 1}{\epsilon_t^p}} \right)^{\frac{\epsilon_t^p}{\epsilon_t^p - 1}}$$
$$\text{s.t.} \qquad \int_0^1 P_t(i) C_t^s(i) di = I_t^s$$

where I_t^s is the total household income for skill type $s \in \{H, L\}$.

The first-order condition with respect to $C_t^s(i)$ yields

$$\frac{C_t^s(i)}{C_t^s(k)} = \left(\frac{P_t(i)}{P_t(k)} \right)^{-\epsilon_t^p} \qquad \forall i, k \in [0, 1]$$

The price index is defined as $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon_t^p} \right)^{\frac{1}{1-\epsilon_t^p}}$ such that household demand for good i is given by

$$C_t^s(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_t^p} C_t^s$$

Log-linearization of the above equation gives

$$c_t^s(i) = -\epsilon_t^p (p_t(i) - p_t) + c_t^s$$

Wage inflation equations

The optimality condition of the household wage setting problem for skill type $s \in \{H, L\}$ is

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k \mathbb{E}_t\Big\{N^s_{t+k|t}(C^s_{t+k})^{-\sigma^s} Z_{t+k}\Big(\frac{W^{s*}_t}{P_{t+k}} - \frac{\epsilon^s_w}{\epsilon^s_w - 1} MRS^s_{t+k|t}\Big)\Big\} = 0$$

Under perfect foresight zero inflation steady-state, $\frac{W^{s*}}{P} = \frac{W^s}{P} = \frac{\epsilon^s_w}{\epsilon^s_w - 1} MRS^s$. Thus, under full wage flexibility and in the absence of nominal wage rigidities, the wedge between real wage and marginal rate of substitution is the desired gross mark-up. Log-linearization of the optimality condition around zero inflation steady-state yields

$$w^{s*}_t = \mu^{sw} + (1-\beta\theta_w)\sum_{k=0}^{\infty}\mathbb{E}_t\left\{mrs_{t+k|t} + p_{t+k}\right\}$$

where $mrs_{t+k|t} = \sigma^s c^s_{t+k} + \gamma^s n^s_{t+k|t}$ and $\mu^{sw} = \ln\left(\frac{\epsilon^s_w}{\epsilon^s_w - 1}\right)$. The relation between average marginal rate of substitution for the economy and household specific MRS is

$$\begin{aligned} mrs_{t+k|t} &= mrs_{t+k} + \gamma^s(n^s_{t+k|t} - n_{t+k}) \\ &= mrs_{t+k} - \gamma^s\epsilon^s_w(w^{s*}_t - w^s_{t+k}) \end{aligned}$$

where the second equality is reached using the firm demand for labor. Substituting this equation into the log-linearized optimality condition and re-arranging terms yields

$$w^{s*}_t = \frac{1-\beta\theta_w}{1+\gamma^s\epsilon^s_w}\sum_{k=0}^{\infty}\mathbb{E}_t\left\{\mu^{sw} + mrs_{t+k} + \gamma^s\epsilon^s_w w^s_{t+k} + p_{t+k}\right\}$$

The recursive formulation of the above equation is combined with wage dynamics $w^s_t = \theta_w w^s_{t-1} + (1-\theta_w)w^{s*}_t$ to give us a wage inflation equation

$$\pi^{sw}_t = \beta E_t\{\pi^{sw}_{t+1}\} - \Theta^s_w(\mu^{sw}_t - \mu^{sw})$$

A.1.2 Firms

Demand for labor varieties

$$\begin{aligned}
&\min_{N_t^H(i,j),N_t^L(i,j)} && \int_0^\lambda W_t^H(j)N_t^H(i,j)dj + \int_\lambda^1 W_t^L(j)N_t^L(i,j)dj \\
&\text{s.t.} && N_t(i) = \Big[\lambda\big(A_t^H N_t^H(i)\big)^{\frac{\eta-1}{\eta}} + (1-\lambda)\big(A_t^L N_t^L(i)\big)^{\frac{\eta-1}{\eta}}\Big]^{\frac{\eta}{\eta-1}} \\
& && N_t^H(i) = \Big(\frac{1}{\lambda}\int_0^\lambda N_t^H(i,j)^{\frac{\epsilon_w^H-1}{\epsilon_w^H}}dj\Big)^{\frac{\epsilon_w^H}{\epsilon_w^H-1}} \\
& && N_t^L(i) = \Big(\frac{1}{1-\lambda}\int_\lambda^1 N_t^L(i,j)^{\frac{\epsilon_w^L-1}{\epsilon_w^L}}dj\Big)^{\frac{\epsilon_w^L}{\epsilon_w^L-1}}
\end{aligned}$$

The first-order condition with respect to $N_t^H(i,j)$ yields

$$\frac{N_t^H(i,j)}{N_t^H(i,k)} = \left(\frac{W_t^H(j)}{W_t^H(k)}\right)^{-\epsilon_w^H} \qquad \forall j,k \in [0,\lambda]$$

The high-skill wage index is defined as $W_t^H = \Big(\frac{1}{\lambda}\int_0^\lambda W_t^H(j)^{1-\epsilon_w^H}dj\Big)^{\frac{1}{1-\epsilon_w^H}}$ such that the firm demand is

$$N_t^H(i,j) = \left(\frac{W_t^H(j)}{W_t^H}\right)^{-\epsilon_w^H} N_t^H(i)$$

The first-order condition with respect to $N_t^L(i,j)$ yields

$$\frac{N_t^L(i,j)}{N_t^L(i,k)} = \left(\frac{W_t^L(j)}{W_t^L(k)}\right)^{-\epsilon_w^L} \qquad \forall j,k \in [\lambda,1]$$

The low-skill wage index is defined as $W_t^L = \Big(\frac{1}{1-\lambda}\int_\lambda^1 W_t^L(j)^{1-\epsilon_w^L}dj\Big)^{\frac{1}{1-\epsilon_w^L}}$ such that the firm demand is

$$N_t^L(i,j) = \left(\frac{W_t^L(j)}{W_t^L}\right)^{-\epsilon_w^L} N_t^L(i)$$

It can be shown that $\int_0^\lambda W_t^H(j)N_t^H(i,j)dj = \lambda W_t^H N_t^H$ and $\int_\lambda^1 W_t^L(j)N_t^L(i,j)dj = (1-\lambda)W_t^L N_t^L$. The objective function that the firm minimizes can be re-written as $\lambda W_t^H N_t^H + (1-\lambda)W_t^L N_t^L$.

The first-order conditions with respect to $N_t^H(i)$ and $N_t^L(i)$ yields

$$\frac{N_t^H(i)}{N_t^L(i)} = \left(\frac{W_t^H}{W_t^L}\right)^{-\eta}\left(\frac{A_t^H}{A_t^L}\right)^{\eta-1}$$

The overall wage index is defined as $W_t = \left[\lambda\left(\frac{W_t^H}{A_t^H}\right)^{1-\eta} + (1-\lambda)\left(\frac{W_t^L}{A_t^L}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$ such that

$$N_t^H(i) = (A_t^H)^{\eta-1}\left(\frac{W_t^H}{W_t}\right)^{-\eta} N_t(i) \quad \text{and} \quad N_t^L(i) = (A_t^L)^{\eta-1}\left(\frac{W_t^L}{W_t}\right)^{-\eta} N_t(i)$$

New Keynesian Phillips Curve

The optimality condition for the profit maximization problem faced by the firm is given by

$$\sum_{k=0}^{\infty}(\theta_p)^k E_t\left\{\Lambda_{t,t+k}Y_{t+k|t}\left(P_t^* - \frac{\epsilon_t^p}{\epsilon_t^p - 1}MC_{t+k|t}\right)\right\} = 0$$

Under perfect foresight zero inflation steady-state $\Lambda_{t,t+k} = \beta^k$ and $\frac{P_t^*}{P_{t-k}} = \frac{P_t}{P_{t-k}} = 1$. All firms produce the same amount of output and face same marginal costs, that it $Y_{t+k|t} = Y$ and $MC_{t+k|t} = MC_{t+k} = MC_t$. Thus, in steady-state, price is a constant mark-up over marginal cost $P_t = \frac{\epsilon^p}{\epsilon^p - 1}MC_t$. Log-linearization of the optimality condition around zero inflation steady-state yields

$$p_t^* = (1-\beta\theta_p)\sum_{k=0}^{\infty}(\beta\theta_p)^k E_t\left\{mc_{t+k|t} + x_t^p\right\}$$

where $mc_{t+k|t} \equiv \ln MC_{t+k|t}$ and $x_t^p = \ln\left(\frac{\epsilon_t^p}{\epsilon_t^p - 1}\right)$. The relation between firm specific and economy wide marginal cost can be written as

$$\begin{aligned} mc_{t+k|t} &= mc_{t+k} + \alpha\left(n_{t+k|t} - n_{t+k}\right) \\ &= mc_{t+k} + \frac{\alpha}{1-\alpha}\left(y_{t+k|t} - y_{t+k}\right) \\ &= mc_{t+k} - \frac{\alpha\epsilon^p}{1-\alpha}\left(p_t^* - p_{t+k}\right) \end{aligned}$$

where the second equality results from using the production function and the third equality from combining the household demand for product varieties and output market clearing condition. Substituting this into the log-linearized optimality condition and re-arranging terms yields

$$p_t^* = (1-\beta\theta_p)\sum_{k=0}^{\infty} E_t\left\{p_{t+k} - \Theta(\mu_t^p - x_t^p)\right\}$$

where $\Theta_p = \frac{1-\alpha}{1-\alpha+\alpha\epsilon^p}$ and $\mu_p(t) = p(t) - mc(t)$. The recursive formulation of the above equation is combined with price dynamics $p_t = \theta_p p_{t-1} + (1-\theta_p)p_t^*$ to give us the New Keynesian Phillips Curve

$$\pi_t^p = \beta E_t \left\{ \pi_{t+1}^p \right\} - \Theta_p(\mu_t^p - x_t^p)$$

A.1.3 Labor Market Clearing

For skill type $s \in \{H, L\}$

$$\begin{aligned}
N_t^s &= \frac{1}{\lambda}\int_0^1 \int_0^\lambda N_t^s(i,j)djdi \\
&= \frac{1}{\lambda}\int_0^1 N_t^s(i) \int_0^\lambda \frac{N_t^s(i,j)}{N_t^s(i)}djdi \\
&= \Delta_t^s \quad \frac{1}{\lambda}\int_0^1 N_t(i)\frac{N_t^s(i)}{N_t(i)}di \\
&= \Delta_t^s \quad \frac{1}{\lambda}(A_t^s)^{\eta-1}\Big(\frac{W_t^s}{W_t}\Big)^{-\eta}\int_0^1 N_t(i)di \\
&= \Delta_t^s \quad \frac{1}{\lambda}(A_t^s)^{\eta-1}\Big(\frac{W_t^s}{W_t}\Big)^{-\eta}\int_0^1 \Big(\frac{Y_t(i)}{A_t}\Big)^{\frac{1}{1-\alpha}}di \\
&= \Delta_t^s \quad \Delta_t^p \quad \frac{1}{\lambda}(A_t^s)^{\eta-1}\Big(\frac{W_t^s}{W_t}\Big)^{-\eta}\Big(\frac{Y_t}{A_t}\Big)^{\frac{1}{1-\alpha}}
\end{aligned}$$

where $\Delta_t^s = \int_0^\lambda \left(\frac{W_t^s(j)}{W_t^s}\right)^{-\epsilon_w^s} dj$ and $\Delta_t^p = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon_t^p}{1-\alpha}}$.

A.2 Relaxing hand-to-mouth assumption

When the hand-to-mouth assumption is relaxed there is a single representative household with high-skill and low-skill workers. There is perfect risk sharing among workers in the household such that consumption level for all workers are equal.

The period utility for household is given by

$$U(C_t, N_t^H(j), N_t^L(j); Z_t) = \Big(\frac{{C_t}^{1-\sigma}}{1-\sigma} - \int_0^\lambda \frac{{N_t^H(j)}^{1+\gamma^H}}{1+\gamma^H}dj - \int_\lambda^1 \frac{{N_t^L(j)}^{1+\gamma^L}}{1+\gamma^L}dj\Big)Z_t$$

The consumption-savings decision of household lead to a single Euler equation given by

$$c_t = \mathbb{E}_t\{c_{t+1}\} - \frac{1}{\sigma}\Big(i_t - \rho - E_t\{\pi^p_{t+1}\} - (1-\rho_z)z_t\Big)$$

The parameter estimates for this alternate version of the model donot show significant differences in the results

Parameter		**Prior distribution**			**Posterior distribution**	
		Dist.	Mean	S.D.	Mean	[0.05, 0.95]
Substitution HS-LS	η	N	1.5	0.25	2.04	[1.84, 2.26]
Inverse Frisch HS	γ^H	N	2.5	0.5	3.30	[3.08, 3.52]
Inverse Frisch LS	γ^L	N	1.5	0.25	1.45	[1.34, 1.56]
Wage elasticity HS	ϵ_w^H	G	5.00	0.5	2.36	[2.17, 2.55]
Wage elasticity LS	ϵ_w^L	G	5.00	0.5	7.62	[6.97, 8.27]
Interest smoothing	ρ_R	U	0.50	0.083	0.64	[0.60, 0.68]
MP reaction price inflation	ϕ_p	G	1	0.25	2.52	[2.27, 2.77]
MP reaction output gap	ϕ_y	G	0.20	0.10	0.23	[0.19, 0.27]
MP reaction HS wage inflation	ϕ_w^H	G	0.20	0.10	0.84	[0.49, 1.16]
MP reaction LS wage inflation	ϕ_w^L	G	0.20	0.10	0.26	[0.06, 0.47]

B. APPENDIX: Optimal Monetary Policy with Skill Heterogeneity

B.1 Efficient Allocation

Optimal price and wage setting in the absence of nominal rigidities imply

$$\frac{W_t^H}{P_t} = -\frac{U_{n,t}^H}{U_{c,t}^H}\mathcal{M}_w^H \quad \text{and} \quad \frac{W_t^L}{P_t} = -\frac{U_{n,t}^L}{U_{c,t}^L}\mathcal{M}_w^L \tag{1}$$

and

$$P_t = \mathcal{M}_p \frac{(1-\tau)W_t}{(1-\alpha)A_t {N_t}^{-\alpha}} \tag{2}$$

Using (1) in wage index

$$\begin{aligned}
\frac{W_t}{P_t} &= MPN_t\Big[\lambda\Big(\mathcal{M}_w^H\Big)^{1-\eta} + (1-\lambda)\Big(\mathcal{M}_w^L\Big)^{1-\eta}\Big]^{\frac{1}{1-\eta}} \\
&= (1-\alpha)A_t {N_t}^{-\alpha}\left(\lambda\Big(\frac{N_t^H}{N_t}\Big)^{-\frac{1}{\eta}} + (1-\lambda)\Big(\frac{N_t^L}{N_t}\Big)^{-\frac{1}{\eta}}\right)\Big[\lambda\Big(\mathcal{M}_w^H\Big)^{1-\eta} + (1-\lambda)\Big(\mathcal{M}_w^L\Big)^{1-\eta}\Big]^{\frac{1}{1-\eta}} \\
&= (1-\alpha)A_t {N_t}^{-\alpha}\Phi_N\mathcal{M}_w
\end{aligned}$$

where $\mathcal{M}_w \equiv \Big[\lambda\Big(\mathcal{M}_w^H\Big)^{1-\eta} + (1-\lambda)\Big(\mathcal{M}_w^L\Big)^{1-\eta}\Big]^{\frac{1}{1-\eta}}$ is the composite wage mark-up in the economy and $\Phi_N \equiv \left(\lambda\Big(\frac{N_t^H}{N_t}\Big)^{-\frac{1}{\eta}} + (1-\lambda)\Big(\frac{N_t^L}{N_t}\Big)^{-\frac{1}{\eta}}\right)$ is the distortion due to heterogeneous labor markets.

Using this in (2) we can solve for τ as

$$\tau = 1 - \frac{1}{\mathcal{M}_p\mathcal{M}_w\Phi_N}$$

B.2 Second-order approximation to obtain welfare function

Log-linearization of the labor market clearing condition gives

$$\hat{n}_t^H = -\eta\hat{w}_t^H + \frac{\hat{y}_t - a_t}{1-\alpha} + \delta_t^H + \delta_t^p \tag{3}$$

$$\hat{n}_t^L = -\eta \hat{w}_t^L + \frac{\hat{y}_t - a_t}{1-\alpha} + \delta_t^L + \delta_t^p \tag{4}$$

where $\delta_t^s = \ln(\Delta_t^s)$ are the dispersion terms.

Using low-skill budget constraint and aggregate output equation we get two equations for consumption in terms of aggregate output and price and wage dispersions.

$$\hat{c}_t^H = \Big(\frac{1}{\Gamma_c^H} - \frac{1-\Gamma_c^H}{\Gamma_c^H}\frac{\nu}{1-\alpha}\Big)(\hat{y}_t - a_t) - \frac{1-\Gamma_c^H}{\Gamma_c^H}\nu\delta_t^p - \frac{1-\Gamma_c^H}{\Gamma_c^H}\nu\delta_t^L + \frac{1}{\Gamma_c^H}a_t \tag{5}$$

$$\hat{c}_t^L = \frac{\nu}{1-\alpha}(\hat{y}_t - a_t) + \nu\delta_t^p + \nu\delta_t^L \tag{6}$$

Second-order approximation to consumer's welfare (around steady-state utility)

$$\begin{aligned}
\mathbb{W} &= -\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(U_t - U) \\
&= -\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big(\lambda(U_t^H - U^H) + (1-\lambda)(U_t^L - U^L)\Big) \\
&= -\lambda\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(U_t^H - U^H) - (1-\lambda)\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t(U_t^L - U^L)
\end{aligned}$$

Thus the welfare function for the economy can be written as a weighted average of second-order approximation to high and low-skill utility respectively.

Second-order approximation to high-skill household utility is

$$\begin{aligned}
U_t^H - U^H \simeq\ & U_c^H C^H\Big(\frac{C_t^H - C^H}{C^H}\Big) + U_n^H N^H\int_0^\lambda\Big(\frac{N_t^H(j) - N^H}{N^H}\Big)dj \\
&+ \frac{1}{2}U_{cc}^H(C^H)^2\Big(\frac{C_t^H - C^H}{C^H}\Big)^2 + \frac{1}{2}U_{nn}^H(N^H)^2\int_0^\lambda\Big(\frac{N_t^H(j) - N^H}{N^H}\Big)^2 dj \\
&+ U_c^H C^H\Big(\frac{C_t^H - C^H}{C^H}\Big)\Big(\frac{Z_t - Z}{Z}\Big) \\
&+ U_n^H N^H\Big(\frac{Z_t - Z}{Z}\Big)\int_0^\lambda\Big(\frac{N_t^H(j) - N^H}{N^H}\Big)dj + t.i.p.
\end{aligned}$$

$$U_t^H - U^H \simeq U_c^H C^H\Big((1+\hat{z}_t)\hat{c}_t^H + \frac{1-\sigma}{2}(\hat{c}_t^H)^2\Big) + U_n^H N^H\Big((1+\hat{z}_t)\lambda\hat{n}_t^H + \frac{1+\gamma^H}{2}\lambda(\hat{n}_t^H)^2 + \gamma^H\epsilon_w^H\delta_t^H\Big) + t.i.p$$

Using equations (3) and (5) and the Pareto-optimality condition

$$\frac{U_t^H - U^H}{U_c^H C^H} \simeq \hat{y}_t\Big(\frac{1}{\Gamma_c^H} - \frac{1-\Gamma_c^H}{\Gamma_c^H}\frac{\nu}{1-\alpha} - \Phi_N N^{-\alpha}\frac{\lambda N^H}{C^H}\Big)$$
$$+ (\hat{y}_t)^2\Big(\frac{1-\sigma}{2}\big(\frac{1}{\Gamma_c^H} - \frac{1-\Gamma_c^H}{\Gamma_c^H}\frac{\nu}{1-\alpha}\big)^2 - \Phi_N N^{-\alpha}\frac{\lambda N^H}{C^H}\frac{1+\gamma^H}{2(1-\alpha)}\Big)$$
$$-\delta_t^p\Big(\frac{1-\Gamma_c^H}{\Gamma_c^H}\nu + \Phi_N N^{-\alpha}\frac{\lambda N^H}{C^H}(1-\alpha)\Big) + \delta_t^H \Phi_N N^{-\alpha}\frac{N^H}{C^H}(1-\alpha)(\lambda+\gamma^H\epsilon_w^H) + \delta_t^L\frac{1-\Gamma_c^H}{\Gamma_c^H}\nu + t.i.p$$

Following a similar procedure for low-skill utility, the second-order approximation gives us

$$\frac{U_t^L - U^L}{U_c^L C^L} \simeq \hat{y}_t\Big(\frac{\nu}{1-\alpha} - \Phi_N N^{-\alpha}(1-\lambda)\frac{N^L}{C^L}\Big) + (\hat{y}_t)^2\Big(\frac{1-\sigma}{2}\frac{\nu^2}{(1-\alpha)^2} - \Phi_N N^{-\alpha}(1-\lambda)\frac{N^L}{C^L}\frac{1+\gamma^L}{2(1-\alpha)}\Big)$$
$$-\delta_t^p\Big(\Phi_N N^{-\alpha}(1-\lambda)\frac{N^L}{C^L}(1-\alpha) - \nu\Big) - \delta_t^L\Big(\Phi_N N^{-\alpha}\frac{N^L}{C^L}(1-\alpha)(1-\lambda+\gamma^L\epsilon_w^L) - \nu\Big) + t.i.p.$$

Note

$$\sum_{t=0}^{\infty}\beta^t\delta_t^p = \frac{1}{2}\frac{\epsilon_p}{(1-\alpha)\Theta_p}\sum_{t=0}^{\infty}\beta^t(\pi_t^p)^2$$
$$\sum_{t=0}^{\infty}\beta^t\delta_t^s = \frac{1}{2}\Big(\frac{\epsilon_w^s}{1+\gamma^s\epsilon_w^s}\Big)\frac{1}{\Theta_w^s}\sum_{t=0}^{\infty}\beta^t(\pi_t^{sw})^2 \qquad s \in \{H, L\}$$

Combining the high and low-skill utility approximations and using the above equations to replace the price and wage dispersion terms, the welfare function can be written as

$$\mathbb{W} = -\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[\lambda(C^H)^{1-\sigma}\Big(\Psi_p^H(\pi_t^p)^2 + \Psi_{Hw}^H(\pi_t^{Hw})^2 + \Psi_{Lw}^H(\pi_t^{Lw})^2 - \Psi_y^H\hat{y}_t - \Psi_{yy}^H\hat{y}_t^2\Big)$$
$$(1-\lambda)(C^L)^{1-\sigma}\Big(\Psi_p^L(\pi_t^p)^2 + \Psi_{Lw}^L(\pi_t^{Lw})^2 - \Psi_y^L\hat{y}_t - \Psi_{yy}^L\hat{y}_t^2\Big)\Big]$$

where the relative weights are as follows

$$\Psi_p^H = \Big(\frac{1-\Gamma_c^H}{\Gamma_c^H}\nu + \Phi_N N^{-\alpha}\frac{\lambda N^H}{C^H}(1-\alpha)\Big)\frac{\epsilon_p}{1-\alpha}\frac{1}{\Theta_p}$$

$$\Psi_{Hw}^H = \Phi_N N^{-\alpha}\frac{N^H}{C^H}(1-\alpha)(\lambda+\gamma^H\epsilon_w^H)\frac{\epsilon_w^H}{1+\gamma^H\epsilon_w^H}\frac{1}{\Theta_w^H}$$

$$\Psi^H_{Lw} = \frac{1-\Gamma^H_c}{\Gamma^H_c}\nu\frac{\epsilon^L_w}{1+\gamma^L\epsilon^L_w}\frac{1}{\Theta^L_w}$$

$$\Psi^H_y = 2\Big(\frac{1}{\Gamma^H_c} - \frac{1-\Gamma^H_c}{\Gamma^H_c}\frac{\nu}{1-\alpha} - \Phi_N N^{-\alpha}\frac{\lambda N^H}{C^H}\Big)$$

$$\Psi^H_{yy} = (1-\sigma)\Big(\frac{1}{\Gamma^H_c} - \frac{1-\Gamma^H_c}{\Gamma^H_c}\frac{\nu}{1-\alpha}\Big)^2 - \Phi_N N^{-\alpha}\frac{\lambda N^H}{C^H}\frac{1+\gamma^H}{1-\alpha}$$

$$\Psi^L_p = \Big(\Phi_N N^{-\alpha}(1-\lambda)\frac{N^L}{C^L}(1-\alpha) - \nu\Big)\frac{\epsilon_p}{1-\alpha}\frac{1}{\Theta_p}$$

$$\Psi^L_{Lw} = \Big(\Phi_N N^{-\alpha}\frac{N^L}{C^L}(1-\alpha)(1-\lambda+\gamma^L\epsilon^L_w) - \nu\Big)\frac{\epsilon^L_w}{1+\gamma^L\epsilon^L_w}\frac{1}{\Theta^L_w}$$

$$\Psi^L_y = 2\Big(\frac{\nu}{1-\alpha} - \Phi_N N^{-\alpha}(1-\lambda)\frac{N^L}{C^L}\Big)$$

$$\Psi^L_{yy} = (1-\sigma)\frac{\nu^2}{(1-\alpha)^2} - \Phi_N N^{-\alpha}(1-\lambda)\frac{N^L}{C^L}\frac{1+\gamma^L}{1-\alpha}$$

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Recitation Instructor, Principles of Economics (evaluation: 4.2/5) Spring 2015

Recitation Instructor, Principles of Economics (evaluation: 4.1/5) Fall 2014

Teaching Assistant, Macroeconomics (Undergraduate) Fall 2018

Teaching Assistant, Econometrics (Masters) Summer 2018

Teaching Assistant, Macroeconomics (Masters) Spring 2018, 2017

Teaching Assistant, Macroeconomics (PhD) Fall 2017, 2016

Teaching Assistant, Game Theory (PhD, Masters) Spring 2016

Teaching Assistant, Microeconomics (Undergraduate) Spring 2016

Conference and Seminar Presentations

Southern Economic Association (forthcoming) November 2018

Midwest Macroeconomics Meeting November 2018

Federal Reserve Bank of Kansas City August 2018

Midwest Economics Association March 2018

Missouri Valley Economic Association October 2017

Federal Reserve Bank of Chicago August 2017

Krannert Ph.D. Research Symposium November 2016, 2017

Grants and Fellowships

AEA CSWEP Fellowship, **Federal Reserve Bank of Kansas City** 2018

AEA CSWEP Fellowship, **Federal Reserve Bank of Chicago** 2017

Purdue Graduate Student Government Travel Grant 2017

Awards

Certificate for Outstanding Teaching (Principles of Microeconomics) Summer 2016

Certificate for Outstanding Recitation Teaching (Principles of Economics) Spring 2015

Certificate for Outstanding Recitation Teaching (Principles of Economics) Fall 2014

Skills

Programming: Matlab, Dynare, STATA, LaTeX, Mathematica, SPSS, Eviews

Languages: English (near native), Bengali (native), Hindi (native)