

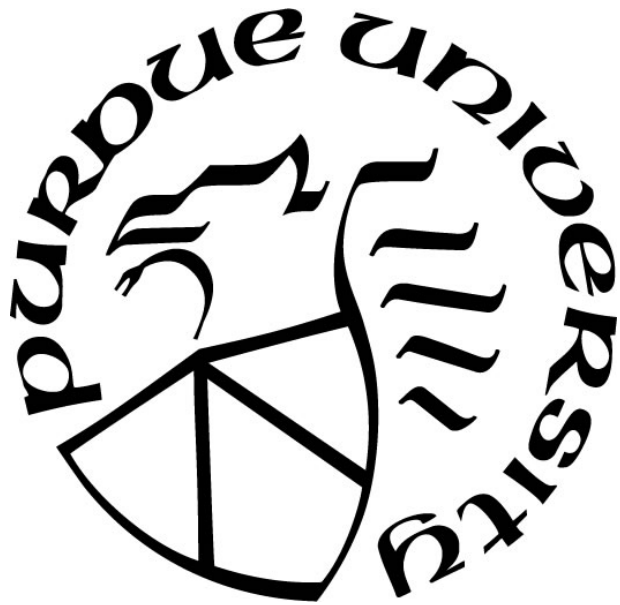
**TEACHERS' RESPONSES TO CHILDREN'S MISTAKES IN
KINDERGARTEN MATHEMATICS CLASSROOMS**

by
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A Dissertation

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Dedicated to my mom, Lingxiang Liu

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ABSTRACT

Making mistakes is an inevitable part of mathematics learning and an essential aspect of teaching. The current study examined the patterns and key variations in teachers' responses to children's mistakes in kindergarten mathematics classrooms. I developed a coding scheme using both inductive and deductive approaches to document the discourse across a series of video-recorded mathematics lessons from a sample of 24 public school kindergarten teachers. Based on previous classroom observational studies, I first outlined four significant dimensions of teachers' mistake-handling practices: (1) instructional support: the instructional strategies teachers use to elaborate on students' mistakes and incorporate their mistakes into ongoing mathematics instructions, (2) emotional reactions: the valence of teachers' affective reactions to students' mistakes (i.e., positive, negative, or neutral), (3) locus of responsibility: the individual who is responsible for correcting the mistake, and (4) the nature of mistakes: teachers' explicit communication about the causes, consequences, and value of making mistakes. Then, I followed an open-coding process to document emergent sub-categories related to each dimension. The study revealed distinct patterns of teachers' mistake-related practices for the four major dimensions and the complexity of the teacher-child interaction surrounding mistakes. In addition, results indicated that teachers' positive emotional reactions toward children's mistakes were associated with their high-quality instructional support and adaptive statements regarding the nature of mistakes. The current study contributes to the understanding of adaptive strategies teachers could use to address children's mistakes in mathematics classrooms.

INTRODUCTION

Addressing children's mistakes in mathematics classrooms is a fundamental aspect of teachers' daily instructional practices. Even though teachers' responses to mistakes are crucial for promoting students' engagement with mathematical content and their conceptual understanding, mathematics instruction is characterized by a narrow focus on correct answers (Boaler & Dweck, 2016; Turner & Meyer, 2009). The National Council of Teachers of Mathematics (NCTM, 2014) stressed that teachers should support students' productive mathematical struggles and use students' mistakes as a significant resource to facilitate their mathematical inquiry and understanding, instead of focusing primarily on seeking correct solutions to mathematics problems. In addition, a risk-free classroom climate in which students feel safe to make mistakes could help students develop a positive disposition toward mathematics learning (Clifford, 1991; Lampert, 1990; Tulis, Steuer, & Dresel, 2016).

Given the significant role of teacher-student interactions surrounding mistakes, there is a critical need to document the range of strategies that teachers use to address students' mistakes in mathematics classrooms. However, only a small body of studies have focused on this issue in actual classroom settings, and even fewer studies have addressed this issue in the early grades. This is surprising because it is in the early grades that young children begin to build their formal mathematics learning. Making mistakes during this process is inevitable as children encounter, learn, and reason using novel mathematics concepts. Accordingly, young children's classroom experiences surrounding mistakes have crucial implications for both their understanding of the role of mistakes in learning and their attitudes toward mathematics (Booth, Lange, Koedinger, & Newton, 2013; Durkin & Rittle-Johnson, 2012; Kapur, 2014). Therefore, the current study addresses this need by focusing on teacher-student exchanges in response to children's mistakes in kindergarten mathematics classrooms.

Students' mistakes and misconceptions during the learning process can be valuable for both teachers and students (Boaler, 2006; Borasi, 1994). For students, their attitudes toward mistakes, such as whether they consider making mistakes as a natural and anticipated aspect of learning or as an indicator of incompetence or failure, influence their academic performance, motivational beliefs, and emotional experiences during mathematics learning (Steuer, Rosentritt-Brunn, & Dresel, 2013). If students perceive making mistakes as an indicator of failure or a

threat to their competence, they tend to experience relatively high levels of negative affect and are more likely to withdraw effort or avoid challenging tasks, in order to protect their self-worth (Turner, Thorpe, & Meyer, 1998). On the contrary, students who perceive mistakes as a natural and constructive part of learning are more likely to use adaptive strategies to handle their mistakes. That is, they are likely to persist longer on the tasks, exert more effort, re-evaluate and reflect on their errors and learning strategies, and approach the tasks or problems with different strategies (Tulis, Steuer, & Dresel, 2018). For these reasons, students with more positive attitudes toward mistakes tend to perform better on academic tasks (Keith & Frese, 2005; Leighton, Tang, & Guo, 2018; Van Dyck, Frese, Baer, & Sonnentag, 2005), endorse more adaptive motivational beliefs (Dickhäuser, Buch, & Dickhäuser, 2011; Grassinger & Dresel, 2017), and experience higher levels of enjoyment and lower levels of anxiety (Tulis & Ainley, 2011; Turner et al., 2002) compared to students who perceive mistakes as a threat to their competence.

For teachers, mistakes (a) signal students' underlying misconceptions and confusion, (b) highlight the need for adjustments to instruction to bridge knowledge gaps, and (c) invite productive inquiry and discussion to deepen students' mathematical understanding (Heinze, 2005; Santagata, 2005; Schleppenbach, Flevares, Sims, & Perry, 2007; Tulis, 2013). Borasi (1994) suggested that teachers should consider and use students' mistakes as "springboards for inquiry" (p. 172). The goal of mathematics instruction should go beyond correcting, reducing, and preventing students' mistakes. Instead, both teachers and students should take advantage of the mistakes by initiating meaningful discussion, inquiry, and exploration surrounding relevant mistakes or misconceptions, in order to promote students' engagement and mathematical understanding. Moreover, an error-tolerant and emotionally supportive classroom context, where teachers convey the learning potential of mistakes and encourage students to reflect on and learn from their mistakes, could promote students' engagement and interests in learning mathematics (Steuer et al., 2013). Conversely, teachers' ignoring students' mistakes, showing disappointment or impatience about students' mistakes, or considering errors as indicators of failure or lack of ability, could hamper students' academic performance and competence beliefs, and result in students' negative emotions and behaviors, including anxiety, frustration, avoidance of challenging tasks, and use self-handicapping strategies (Ryan, Gheen, & Midgley, 1998; Tulis et al., 2018; Turner et al., 1998).

Given the significant effects of students' classroom experiences surrounding mistakes, the objectives of the current study are to: (a) provide in-depth descriptive information (e.g., occurrence, frequency, representative examples of different types of teacher responses) about teachers' responses to children's mistakes in kindergarten mathematics classrooms through systematic classroom observation, and (b) identify the key aspects and patterns of teachers' responses. Results of this study aim to contribute to the understanding of teachers' mistake-related practices in mathematics classrooms and the discussion of the adaptive strategies teachers could use in mathematics classrooms to address children's mistakes, especially in the early grades of school.

LITERATURE REVIEW

Previous Studies on Teachers' Responses to Mistakes

As the topic of teachers' mistake-handling practices received increased attention in recent years, a small but growing body of classroom observational research has focused on how teachers addressed students' mistakes in natural classroom settings. I identified seven observational studies that specifically focused on teachers' responses to students' mistakes in classroom settings. I synthesized the major themes or categories that emerged from the qualitative observational evidence, with the intention to reveal the key variations rooted in teachers' practices to address mistakes. Table 1 shows the summary of coding schemes from the seven observational studies, including the author(s), context of study, major coding themes, and corresponding sub-categories.

Table 1. Summary of Coding Scheme from Previous Observational Studies

Author(s)	Context	Major coding categories	Coding categories
Borasi, 1994	Grade 9 11 observations (10 - 40 minutes) of mathematics lessons in the U.S.	Stance of learning (i.e., degree of open-endedness of mistake-related instruction) Levels of student involvement	<ul style="list-style-type: none"> • Remediation; • Discovery; • Open-ended inquiry. • The inquiry stimulated by the error is mostly conducted by the instructor and later shared with the students. • The students engage actively in an error activity organized by the instructor. • The error activity is initiated and developed by the students themselves, with some (or no) participation on the part of the instructor.
Santagata, 2004	Grade 8 30 Italian and 30 U.S. mathematics lessons	Mitigation (i.e., saying the student was close to the right answer, or excusing the	Indicators, for example: <ul style="list-style-type: none"> • Expressions like “you are close” ...; • Hesitations; • Absence of acknowledgement of mistake; • Positive affect.

		student from responsibility)	
		Neutral	<ul style="list-style-type: none"> Teachers neither soften the severity of their corrective action nor expressed negative affect.
		Aggravation (i.e., criticizing the student for making the error)	<p>Indicators, for example:</p> <ul style="list-style-type: none"> Intonation; Voice quality; Use of augmentatives; Irony, parody and sarcasm.
Santagata, 2005	Grade 8 30 Italian and 30 U.S. mathematics lessons	Who is given the task of correcting the mistake?	<ul style="list-style-type: none"> Teacher; Student who made the mistake; Other student(s); Student's initiative/identification.
		What strategies do teachers use to respond to students' mistakes?	<ul style="list-style-type: none"> Give correction; Give a hint to same student; Repeat the question to same student; Ask why; Give a hint to other students; Redirect the question (to other students or whole class); Only recognize the correct answer; Ask the class; Student initiative.
Schleppenbach et al. 2007	Grade 1, 4, & 5 15 mathematics lessons from first grade, and 29 mathematics lessons from fourth and fifth grade 10 first grade and 14 fifth grade teachers from China, and 5 first grade and 12 fourth and fifth grade teachers from the U.S.	Follow errors with teachers' statements: CLASS 1 responses.	<ul style="list-style-type: none"> Telling students that the answer is wrong; Giving the correct answer; Ignoring the error; Providing explanation or direction; Students spontaneously correcting themselves;
		Follow errors with teachers' questions: CLASS 2 responses.	<ul style="list-style-type: none"> Re-asking the question; Clarifying the question; Asking for an addition to the answer; Asking for certainty or agreement; Redirecting the question; Asking for student explanation.

Bray, 2011	Grade 3	Intentional focus on flawed solutions	<ul style="list-style-type: none"> • Purposefully incorporate students' incorrect answers in class discussion; • Limited recognition of the flawed solutions.
	16 observations of mathematics lessons	Addressing student errors in conceptually supportive ways	<ul style="list-style-type: none"> • Conceptual understanding; • Knowledge of procedures.
	4 teachers	Mobilization of a community of learners to address errors.	<ul style="list-style-type: none"> • An inquiry/argument classroom culture; • A strategy-reporting classroom culture.
Tulis, 2013	Grade 5 - 13	Maladaptive	<ul style="list-style-type: none"> • Ignoring mistake; • Criticizing student; • Redirecting the question to another student; • Humiliating/laughing; • Disappointment/Hopelessness;
	16 mathematics lessons, 17 German lessons, and 15 economics lessons from German schools	Neutral	<ul style="list-style-type: none"> • Correction by the teacher;
		Adaptive	<ul style="list-style-type: none"> • Discussion with the whole class; • Correction by the student; • Waiting; • Emphasizing the learning potential; • Impeding negative reactions from class.
Matteucci et al. 2015	Grade 1, 3, & 5.	Type of teachers' responses	<ul style="list-style-type: none"> • Give correction; • Hint to the same student; • Repeat the question to the same student; • Ask for explanation; • Give a hint to other students; • Redirect the question; • Pick the right answer; • Ask the class to correct; • Student initiative; • Blame the student who made the error; • Highlight the error in the answer; • Student corrects him/herself immediately; • Give feedback about the right part of the answer;

	<ul style="list-style-type: none"> • Suggest techniques to avoid the error; • Stop the student as soon as the error appears.
Affective stance	<ul style="list-style-type: none"> • Positive or mitigation; • Neutral; • Negative or aggravation.

Five of the observational studies followed an open-coding approach to document teacher-student interactions surrounding mistakes in mathematics classrooms. The other two studies, both recent (Matteucci et al., 2015; Tulis, 2013) adopted a deductive approach by first developing the coding themes based on previous studies, and then categorizing the observed instances into coding themes. As summarized in Table 1, four important dimensions of teachers' mistake-handling practices during instruction emerged: (a) *instructional support*, which described how teachers elaborated on students' mistakes and used students' mistakes to promote mathematical understanding; (b) teachers' *emotional reactions* to students' mistakes; (c) the *locus of responsibility*, which referred to the individual who received the opportunity to correct the mistakes; and (d) *the nature of mistakes*, which involved teachers' explicit communication about the causes, consequences, and value of making mistakes. I discuss these dimensions next.

Instructional support. The first dimension of teachers' mistake-handling practices involves elaborating on students' mistakes and incorporating the mistakes into instruction, in order to facilitate further mathematical inquiry and understanding. Prior studies characterized these adaptive instructional strategies as the intentional use of students' mistakes to facilitate understanding and the high open-endedness of mistake-related instruction. Examples include initiating class discussions about the mistakes, prompting students to explain their thoughts, identifying the nature of students' mistakes, and providing mistake-related conceptual instruction. The adaptive instructional strategies to address students' mistakes were positively associated with students' mathematical understanding, as well as showed students that mistakes could be constructive for their mathematics learning (Borasi, 1994; Steuer & Dresel, 2015). By contrast, the maladaptive instructional strategies were characterized as the teacher's absence of addressing students' mistakes or immediately correcting mistakes without further elaboration or discussions surrounding the mistakes.

Emotional reactions. The second dimension or category of practices involves teachers' emotional responses to students' mistakes and includes positive reactions (e.g., compliment on students' effort, affirming the correct part in mistakes), negative reactions (e.g., showing disappointment, criticizing student), and neutral reactions (i.e., the absence of both positive and negative reactions). Santagata (2004) provided evidence on all three types of teachers' emotional reactions to students' mistakes and documented different patterns of emotional reactions across the U.S. and Italian teachers. In particular, Italian teachers were more likely to "aggravate" students' mistakes by criticizing or showing disappointment when students made mistakes, whereas U.S. teachers tended to "mitigate" the mistakes by showing tolerance to mistakes, acknowledging students' effort, and only addressing the correct answers. Tulis (2013) and Matteucci et al. (2015) also incorporated the affective dimension of teachers' mistake-management practices into their coding schemes and revealed teachers' different affective stances regarding students' mistakes.

Locus of responsibility. The third dimension is the locus of responsibility to correct the mistakes (i.e., who received the opportunities to correct the mistakes: the same child who made the mistake, another student, or the teacher); it emerged from both Santagata's (2005) and Borasi's (1994) studies. Both researchers emphasized the importance of students' own active involvement in correcting and reflecting on their mistakes. In Borasi's (1994) study, students' narratives about their experiences surrounding mistakes in mathematics classrooms also indicated that students perceived that they benefitted more when teachers allowed them to correct their own mistakes and to discover and reflect on the nature of their mistakes. Tulis (2013) further observed that students reacted differently depending on whether their mistakes were corrected by themselves, the teacher, or their peers. She found that students tended to show negative affect when their mistakes were redirected to their peers, whereas they expressed positive affect when they received the opportunity to correct their own answers. Based on the results, Tulis (2013) pointed out the potentially negative effects of asking a different student to correct the answer. On one hand, the confusion or misconception of the student who made the mistake might not be appropriately addressed or resolved. On the other hand, this strategy might implicitly convey social comparison information, by implying that the student(s) who corrected the mistake performed better than the student(s) who made the mistake. Therefore, students

could be more actively engaged in the problem-solving process and identify and resolve their misconceptions better when correcting their own mistakes.

The nature of mistakes. Researchers (Schleppenbach et al., 2007; Tulis, 2013) addressed how teachers explicitly communicate the nature of mistakes. However, only Tulis (2013) added one coding category of how teachers treat mistakes as learning opportunities (e.g., teachers' encouragement of students' effort, teachers' emphasis on the learning potential of mistakes). Though Tulis (2013) observed that only approximately 3.0% of the instances involved teachers' communication regarding the nature of mistakes, she argued that the explicit mistake-related statements could have significant effects on students' attitudes and reactions toward mistakes. If teachers could support students' perceptions that mistakes are a natural and constructive part of learning from which students can benefit, their students tend to have lower levels of negative affective reactions (e.g. anxiety, frustration, and fear) and are less likely to consider mistakes as a failure or a threat to their self-perception of competence (Tulis et al., 2016).

Associations with Characteristics of Teachers and Lessons

As previous studies identified different strategies that teachers used to address students' mistakes, researchers raised questions regarding whether and how these different patterns of teachers' mistake-handling practices were associated with the characteristics of the teachers and lessons. First, during the past decades, mathematics education has gone through an important shift from traditional view of mathematics teaching to a reform-oriented teaching that emphasizes mathematical inquiry, discussion, and collaboration (Cobb, Boufi, McClain, & Whitenack, 1997). The reform-oriented classroom norms highlight the need to support students' mathematics struggles and use students' mistakes as a valuable resource to facilitate their conceptual understanding, rather than focusing on the accuracy of students' answers (NCTM, 2014). Due to the major changes in mathematics education, teachers who started teaching a few decades ago and teachers who just started their teaching career may receive different professional training regarding how to address students' mistakes in classrooms. In addition, Bray's (2011) study revealed that teachers' own knowledge about the relevant mathematical concepts had significant effect on how they address students' mistakes in classrooms. If the teachers had a deep understanding of the key mathematical concepts, they were more likely to

identify the nature of students' misconception and provide informative feedback about students' mistakes. Therefore, in this study, I examine whether the teachers' mistake-handling practices were associated with teachers' professional background, including their years of teaching experience and educational qualifications.

Second, Borasi (1994) argued that when teachers and students were working on different mathematical concept or topics, the form or nature of students' mistakes might differ. Accordingly, teachers might adopt different strategies to respond to students' mistakes. However, limited empirical evidence were available on the association between teachers' mistake-handling practices and characteristics of the mathematics lessons. Therefore, in the present study, I focus on two major mathematical topics in kindergarten: number sense and computation and examine whether teachers' responses to children's mistakes were related to the characteristics of the mathematics lessons, including the related mathematical topics and the semester of the lesson.

Present Study

Given the significant role of teacher-student interactions surrounding mistakes, more research is needed to deepen understanding of the effective strategies that teachers use to address students' mistakes in classrooms and the potential effect of teachers' mistake-handling practices on students' learning experiences. However, there is dearth of evidence on this issue, especially in early grades. As summarized in Table 1, only the Schleppenbach et al. (2007) study involved first grade mathematics lessons. Since the major purpose of the study was the comparison between Chinese and U.S. teachers' instructional strategies, the study provided limited information and discussion about teachers' mistake-handling behaviors with specific attention to early grades. Therefore, the current study aims to address the literature gap by focusing on young children in kindergarten, because children's early experiences surrounding mistakes and failures play a critical role in the development of their motivational beliefs, and in turn have a long-lasting effect on their mathematics learning and future academic and career choices (Booth et al., 2013; Durkin, et al., 2012; Wigfield & Eccles, 2002).

Furthermore, these previous studies (e.g., Santagata, 2005; Tulis, 2013) primarily focused on teachers' first verbal response or non-verbal reaction to students' mistakes. They adopted that procedure because, as Santagata (2005) argued, "the first response the teacher gave shaped the

activity in fundamental ways.” (p. 498). However, it is common that teachers make multiple types of responses within a single mistake-related interaction episode. For example, when a child makes a mistake, the teacher may first show disappointment and then ask the child to explain his or her thinking and initiate a discussion around the mistake. In this case, the teacher’s first response is the negative reaction as showing disappointment, which is associated with the emotional reactions dimension. The latter responses, including both prompting the child’s further explanation and initiation of class discussion, would be identified as high-quality responses on the instructional support dimension because the teacher provided further elaboration on the mistakes. On one hand, it is difficult to decide whether the effects of the negative emotional reaction are stronger than the later high-quality instructional responses, only because it is the first response. On the other hand, in this example, the first reaction only documents the affective dimension of the teacher’s responses, while the latter responses reflect the instructional dimension. If the study only focused on the first response, the important information about how this teacher used children’s mistakes in mathematics instruction could be overlooked. Additionally, it would be difficult to examine whether teachers’ responses on different dimensions are associated. Therefore, the classroom observations in the current study document the back-and-forth exchanges between teachers and children surrounding mistakes until the mistake was resolved, in order to gain a more comprehensive understanding of teachers’ mistake-handling practices and to further examine whether teachers’ responses on different dimensions (i.e., instructional support, emotional reactions, locus of responsibilities, and the nature of mistakes) are associated with specific characteristics of teachers and mathematics lessons.

Overall, based on the previous studies (e.g., Bray, 2011; Santagata, 2005; Schleppenbach et al., 2007; Tulis, 2013), four major questions guided the classroom observations with specific attention to teachers’ mistake-handling practices: (a) How did teachers elaborate on children’s mistakes and incorporate the mistakes into ongoing mathematics instructions? (b) What was the valence of teachers’ affective reaction to children’s mistakes (i.e., positive, negative, or neutral)? (c) Who had the responsibility to correct the mistakes? (d) How did teachers explicitly communicate the causes, consequences, and values of making mistakes? The current study aims to examine (a) the patterns and key variations in the four major dimensions of teachers’ mistake-handling practices (i.e., instructional support, emotional reactions, locus of responsibility, the

nature of mistakes) in kindergarten mathematics classrooms; (b) the associations among the four dimensions of teachers' mistake-handling practices; and (c) the associations between teachers' mistake-handling practices with the characteristics of teachers (i.e., teaching experience and educational background) and lessons (i.e., related mathematics content or topics, the semester of the lesson).

METHODS

I conducted video-based classroom observations to identify the patterns of teachers' mistake-handling practices during mathematics instruction. The current investigation aims to provide in-depth descriptive information and representative examples to illustrate the major variations in how teachers respond to children's mistakes.

Sample

Teachers. Data for this investigation were drawn from a secondary project developed within the large-scale Comparing Measures of Effective Teaching (COMET) project (Patrick, Mantzicopoulos, & French, 2014). There were 24 kindergarten teachers, 23 Caucasian females and 1 Caucasian male. The participants' teaching experience ranged from 1 to 21 years ($M = 10$ years). Among the 24 teachers, 13 teachers had a Bachelor's degree, 6 had a Master's degree, and 5 had some graduate level education. Consent forms were received from both teachers and children's parents at the beginning of the school year.

Schools. The teachers were from eight elementary schools within seven school districts in a single Mid-western state. The school sizes ranged from 319 to 643 children ($M = 425$). The distributions of children's ethnicity and free or reduced-cost lunch status varied considerably among the eight schools: 19.6% to 93.1% of children were White; 3.8% to 76.6% were Hispanic; 0.5% to 52.6% were Black; and 0.0% to 4.1% were Asian; 48.9% to 84.1% of children received free or reduced cost lunch. Schools' state-assigned report card grade ranged from A to D.

Children. There were 355 children in the 24 kindergarten classrooms; 184 (51.8%) boys and 171 (48.2%) girls, and 196 (55.2%) received free or reduced cost lunch. Children were diverse in ethnicity; 230 (64.8%) were Caucasian, 76 (21.4%) were Hispanic, 32 (9%) were African American, and 17 (4.8%) were classified as Other.

Procedure

Video-recorded mathematics lessons. There were 96 mathematics lessons, comprised of 4 lessons (2 from each of Fall and Spring semesters) from each of the 24 teachers. Lessons ranged from 17 minutes, 41 seconds to 58 minutes, 33 seconds ($M = 30$ minutes, 11 seconds). I selected these lessons from a larger pool of 20 lessons per teacher (10 from each semester,

recorded in separate weeks). In order to provide a clear comparison of teachers' instructional feedback and strategies, I focused on lessons about two central mathematics topics in kindergarten: number sense and computation. I selected two videos from each of the mathematics topic for each teacher (only one teacher had three lessons on the topic of computation and one lesson on the topic of number sense because this teacher did not have enough number sense lessons that fit the criteria of selecting lessons in this study). The criteria for selecting lessons, in addition to balancing the semesters sampled, were that the videos: (a) have clear visual and audio information, (b) address either number sense or computation, (c) contain rich evidence about teacher-child interactions surrounding mathematics learning (e.g., lessons in which children were working on worksheets on their own quietly or lessons comprised only of teacher-dominant lectures were excluded), and (d) included instances of children's mathematical mistakes (e.g., lessons in which children made no mistakes were excluded).

Coders' background and training. Two coders were engaged in observing the video-recorded mathematics lessons. Both coders worked as research assistants on large-scale project (Patrick et al., 2014, from which the data were drawn). They received extensive training on conducting classroom observations, and were certified to use multiple observation protocols to conduct systematic observations and to rate kindergarten instruction, including the Classroom Assessment Scoring System (CLASS, K-3; Pianta et al., 2008), Framework for Teaching (FfT; Danielson, 2013), and Mathematical Quality of Instruction (MQI; Hill, 2014). Both coders have more than 200 hours of experience in observing video-recorded kindergarten mathematics instruction from 82 different classrooms and were also experienced in establishing inter-rater agreement and resolving coding disagreement.

Qualitative Data Analysis

Unit of analysis. To focus on how teachers responded to children's mistakes, the mistake-related interaction episodes, including teachers' question or elicitation, children's incorrect answer, and follow-up conversations, were located and extracted from each video-recorded mathematics lesson and adopted as the unit of analysis. Consistent with the previous studies (e.g., Santagata, 2005; Tulis, 2013), the identification of mistake-related interaction episodes was triggered by a child making a mistake or a teacher providing corrective feedback or activities. The two coders transcribed the corresponding teacher-child interaction, including

teachers' questions or elicitation, children's mistakes, and the follow-up feedback loop. Both teachers' verbal feedback (e.g., "*This is not correct*", "*Let's try this again*") and non-verbal reactions (e.g., smiling to the child, warm and calm voice, harsh voice, shaking head) were documented in the transcription. The ending point of the mistake-related interaction was signaled by either the teacher moving to the next problem, topic or activity, or the teacher beginning to work with another child. Accordingly, the transcription of mistake-related interactions included back-and-forth feedback loops between teachers and children, rather than only documenting teachers' first response to children's mistakes.

First Cycle Coding. Consistent with Miles, Huberman, and Saldaña (2014) and Saldaña (2009), the qualitative coding process is organized around two stages: first cycle coding and second cycle coding. Figure 1 shows the overall coding process. The first cycle coding was conducted both deductively and inductively. The coding process was guided by the four pre-determined major dimensions that were distilled from literature presented previously, including teachers' instructional support, emotional reactions, locus of responsibility, and the nature of mistakes. Then I followed an open-coding process to document initial observational evidence and emergent sub-categories that were related to each major dimension in order to further develop the coding scheme with corresponding codes on sub-categories. The first cycle coding was conducted with 2 lessons (1 fall and 1 spring) from each of 10 randomly selected teachers.

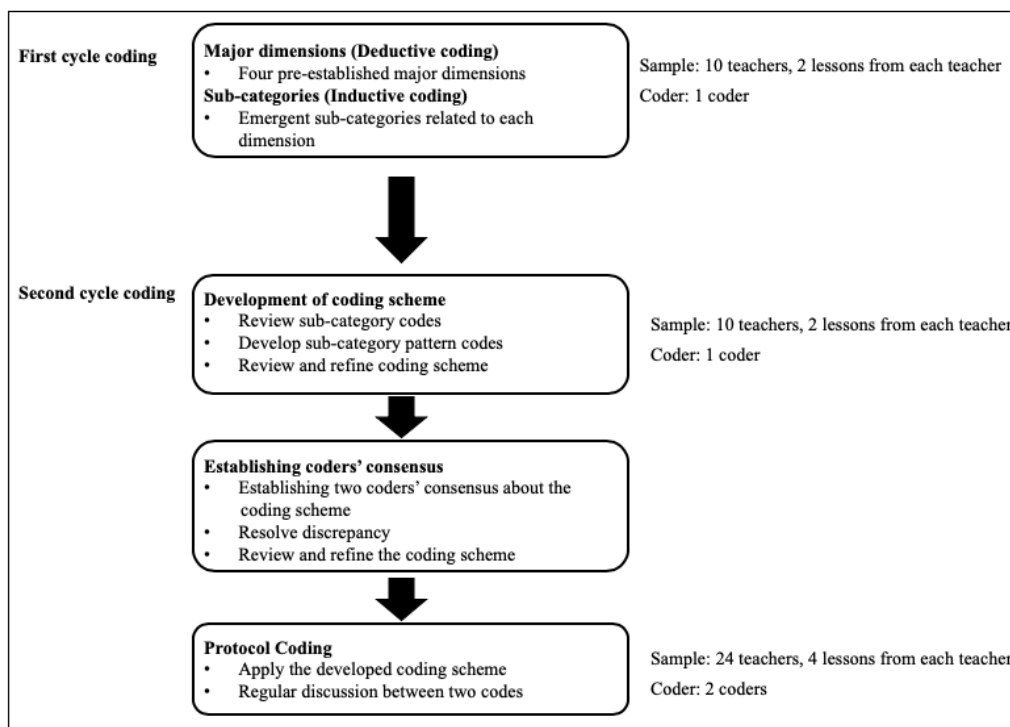


Figure 1. Overall qualitative coding process

Second Cycle Coding. In the second cycle coding, I first re-configured the observed sub-category codes and developed a systematic coding scheme to document teachers' mistake-handling practices. The major purposes of developing a systematic coding scheme were to focus observers' attention on certain aspects of teacher practices and to condense the extensive classroom observational data into patterns or themes in a way to consolidate the interpretations of the qualitative data. After the first cycle coding with 20 lessons, a total of 72 mistake-related interaction episodes were located and transcribed from. For each episode, I first highlighted the evidence that related to each of the four dimensions and provided a brief descriptive label as the sub-category code for teachers' responses. Next, I reviewed the transcript and the descriptive labels within each major dimension, with specific attention to the reoccurring patterns and the diversity of sub-categories under each major dimension, and then made decisions about the specific coding scheme for each dimension. Table 2 presents the developed coding scheme. Note that it was possible that teachers' one comment or reaction could be coded in more than one dimensions. For example, teachers' comment "*I love the way you can solve your problem without giving up*" was coded as both Emotional Reactions and the Nature of Mistakes. Finally,

two coders applied the developed coding scheme to code and analyze teacher’s responses to children’s mistakes across the 96 lessons (24 teachers and four lessons each). Besides the coding for teachers’ responses, the mathematical topics (i.e., number sense, computation) involved in each mistake-related interaction episode were documented at the same time.

Table 2. Coding Scheme for Mistake-related Interaction Episodes in Mathematics Classrooms

Major dimensions	Sub-categories	Indicators
Instructional support	Not present	<ul style="list-style-type: none"> • Ignore the mistake • Only address the correct answer • Provide irrelevant feedback (behavioral or classroom management feedback)
	Low-quality	<ul style="list-style-type: none"> • Provide corrective feedback directly • Repeat the question or direction again • Point out the answer is incorrect but do not provide correct answer
	Mid-quality	<ul style="list-style-type: none"> • Provide hints or explanation of question • Provide justification of the correct answer • Provide clarification of the incorrect answer • Provide one example or strategy
	High-quality	<ul style="list-style-type: none"> • Initiate open-ended class discussion • Prompt children to explain their own thinking • Provide more than one examples or strategies
Emotional reactions	Positive	<ul style="list-style-type: none"> • Compliment on effort • Encouragement of contribution and effort • Affirmation or recognition of the correct part of the answer or children’s contribution • Impeding other children from interrupting or laughing at the child who made mistakes
	Negative	<ul style="list-style-type: none"> • Expressing disappointment through comments, gestures, face expressions, or voice tones • Expressing impatience through comments, gestures, face expressions, or voice tones • Ignoring other children laughing at the child
	Neutral	Neither positive nor negative reactions
Locus of responsibility	<ul style="list-style-type: none"> • Teacher • Same child who made the mistake • Another child • Whole class 	
The nature of mistakes	Not present	Teacher made no explicit statements regarding the nature of mistakes.
	Present	Explicit statements regarding the causes, consequences, value or expectations of making mistakes.

Inter-rater reliability. Before engaging in the second-cycle of coding, the two coders reviewed the coding scheme together, discussed discrepancies, and established shared understandings on each dimension. During the second-cycle of coding, 10% of the mathematics lessons ($n = 10$) were coded by both coders. For the 44 mistake-related episodes located in the 10 lessons, the two coders' agreement on identifying mistake-related episodes was 95.5%. Disagreements on two mistake-related episodes were both related to the situation when the teacher dismissed one child's mistake among the choral responses. The two coders had bi-weekly meetings to discuss coding discrepancies and to reach agreement on the final codes. The Cohen's Kappa of the inter-rater agreement on the three dimensions were: 0.96 for Instructional Support, 0.86 for Emotional Reactions, and 0.93 for Locus of Responsibility, indicated excellent inter-rater reliability (Landis & Koch, 1977).

Quantitative Data Analysis

The quantitative data analyses aimed to examine: (a) whether the frequencies of multiple sub-categories of teachers' responses within one dimension varied (e.g., whether the frequencies of positive, neutral, negative reactions differed), through one-way repeated measures ANOVAs; (b) the associations among the dimensions of teachers' mistake-handling practices by conducting chi-square tests of independence; (c) associations between teachers' responses to children's mistakes and their teaching experience using Pearson correlation coefficients; (d) associations between teachers' educational qualification and their responses to children's mistakes through one-way ANOVA; and (e) associations between teacher responses and mathematical topics (i.e., number sense or computation) and semester (i.e., fall or spring) by conducting chi-square tests of independence.

RESULTS

Overall, a total of 280 mistake-related interaction episodes were identified and transcribed from the 96 mathematics lessons of 24 teachers. An average of 12 mistake-related interaction episodes were identified from each teacher (range was 5-29 per teacher and 1-22 per lesson). Table 3 presents the frequencies and typical examples of different types of teachers' responses to children's mistakes on the four major dimensions.

Instructional Support

Descriptive data. With respect to the four categories (i.e., *High-quality*, *Mid-quality*, *Low-quality*, and *Not Present*) of teachers' instructional support when children made mistakes, the most frequent responses were of low-quality (53.2%), signaling that the mistake was immediately corrected without further explanation or the mistake was pointed out, but the correct answer was not provided. Mid-quality responses were observed in 36.4% of the episodes. High quality and "*Not Present*" responses were identified in 5.0% and 5.4% of the episodes, respectively. A one-way repeated measures ANOVA was conducted to examine whether the numbers of the four types of responses within each teacher varied, and results revealed significant differences: Wilk's Lambda = 0.27, $F(3,21) = 18.85$, $p < 0.001$. Post-hoc analysis indicated that the frequencies of high-quality responses were significantly lower than the frequencies of low-quality ($p < 0.001$) and mid-quality responses ($p < 0.001$), whereas the frequencies of not-present responses were also lower than low-quality ($p < 0.001$) and mid-quality responses ($p < 0.001$). In general, in more than half of the mistake-related episodes (58.6%), teachers only provided perfunctory responses (*Low-quality* responses; e.g., "*No. That is not correct.*" "*It's not two. The correct answer is three.*") or dismissed children's mistakes (*Not present* responses). Of note, in 41.4% of the episodes, teachers' responses went beyond simply telling children that their answer was correct or not (i.e., *Mid-quality* and *High-quality* responses). Rather, teachers took time to provide clarifications or explanations regarding children's mistakes.

Table 3. Number, Percent, and Examples of Each Type of Teachers' Responses to Children's Mistakes

Major dimensions	Sub-categories	# and % of codes ^a	# of teachers ^b	Typical examples
Instructional support	High-quality	14 (5.0%)	7/24	<p>Teacher wrote number nine and four on the board, then asked Anne to write $>$ or $<$.</p> <p>Anne wrote $9 < 4$.</p> <p>Teacher: <i>Oh, is that right? The alligator needs to eat the bigger number or the smaller number?</i></p> <p>Anne corrected the answer and wrote $9 > 4$.</p> <p>Teacher (to the whole class): <i>Here comes the question. She put the alligator over the left side, facing the nine. But if we put number four over here ... (she writes $4 > 9$) does that change the alligator mouth?</i></p> <p>Most children (chorus): <i>Yes.</i></p> <p>Teacher: <i>Will the alligator still look like this ($4 > 9$)?</i></p> <p>All children: <i>No!</i></p> <p>Teacher: <i>No? Why?</i></p> <p>Bob: <i>Because four is smaller, and alligator should eat the bigger number.</i></p> <p>Teacher: <i>Yes, so the alligator should face that way (pointing to the nine), right? (Teacher wrote $4 < 9$) So, it doesn't matter which side you put the number four, as long as you use the correct sign.</i></p>
	Mid-quality	102 (36.4%)	24/24	<p>Children were working on the problem "$6-0=?$" on their worksheet. Teacher looked at Jennie's paper.</p> <p>Teacher: <i>Jennie, now you have six, are you taking anything away?</i></p> <p>Jennie: <i>No.</i></p> <p>Teacher: <i>So how many do you have left?</i></p> <p>Jennie: <i>Zero!</i></p> <p>Teacher: <i>No, you have how many? You have six, and you are not taking anything away. So how many do you have?</i></p> <p>Jennie did not respond.</p>

Major dimensions	Sub-categories	# and % of codes ^a	# of teachers ^b	Typical examples
				<p>Teacher: <i>That's hard when we work with zeros. You have six left. Right? You didn't take any away. Look, Jennie, look at my fingers. (showing six fingers). I'm taking away zero, and how many do I have?</i></p> <p>Jennie: <i>six.</i></p> <p>Teacher: <i>Right, I didn't take any away.</i></p>
	Low-quality	149 (53.2%)	24/24	<p>Teacher: <i>Who can give me the equation? Five, and then? Haley?</i></p> <p>Haley: <i>Em...Em... Plus.</i></p> <p>Teacher: <i>Not plus, Sarah?</i></p> <p>Sarah: <i>Five equals.</i></p> <p>Teacher: <i>Not equals, Kate?</i></p> <p>Kate: <i>Five minus two equals three.</i></p> <p>Teacher: <i>Kate got it right. The number sentence is five minus two equals three.</i></p>
	Not present	15 (5.4%)	6/24	<p>Teacher: <i>Show me a way to make the number nine with your fingers. Everybody should be showing me something.</i></p> <p>Around five children showed different numbers, such as seven, five, and eight. Teacher looked around the classroom to check children's finger patterns, and then directly moved to the next problem without any comments.</p>
Emotional support	Positive	23 (8.7%)	11/24	<p>Teacher showed the problem "8-2" on the board.</p> <p>Teacher: <i>Who can come up and write their answer? Luke?</i></p> <p>Luke wrote "8-2=7" on the board.</p> <p>Teacher: <i>I need you to go back and check your answer.</i></p> <p>Luke went back to his seat and checked the circles he drew on his paper. Other children in the class raised their hand high.</p> <p>Teacher (to other children): <i>Wait, freeze. He's going back to check his answer.</i></p> <p>Teacher waited for 10 seconds. Luke went back to the board and wrote "8-2=6."</p> <p>Teacher: <i>Awesome!!! Give me five! I love the way you can solve your problem without giving up.</i></p>
	Negative	58 (21.9%)	16/24	<p>The child pointed to the smaller number.</p> <p>Teacher: <i>What are you doing? Stop! You need to decide which number is greater!</i></p>

Major dimensions	Sub-categories	# and % of codes ^a	# of teachers ^b	Typical examples
	Neutral	184 (69.4%)	24/24	<p>Teacher: <i>Write me a three.</i></p> <p>Children wrote the number on their own white board. Teacher looked around to check.</p> <p>Teacher: <i>Mike, your three is backwards.</i></p> <p>Children practice writing $>$ or $<$ on their own white boards.</p> <p>Teacher (to child who did not write the correct sign): <i>What is the number? Which one is greater?</i></p>
Locus of responsibility	Same child	65 (24.5%)	23/24	<p>Teacher showed a card with number five in a ten-frame</p> <p>Teacher: <i>Sam, how many does this card have?</i></p> <p>Sam: <i>Six?</i></p> <p>Teacher (pointing to the ten-frame on the card): <i>So, it has three and two, and that's the way to make... (pause)?</i></p> <p>Sam: <i>Five!</i></p>
	Another child	38 (14.3%)	19/24	<p>Teacher: <i>Mimi, what does equal mean?</i></p> <p>Mimi: <i>Three?</i></p> <p>Teacher: <i>No! Alejandro, what does equal mean?</i></p> <p>Alejandro: <i>It means the same.</i></p>
	Teacher	116 (43.8%)	24/24	<p>Teacher showed five dots in one ten-frame on the board.</p> <p>Teacher: <i>What is this number?</i></p> <p>Some children (chorus): <i>Five!</i></p> <p>Some children (chorus): <i>Four!</i></p> <p>Teacher (pointed to one dot in the ten-frame): <i>There is one dot down here, so it's five.</i></p>
	Class	46 (17.4%)	18/24	<p>Teacher: <i>My family has four people, and we went to the park with two more people. How many people will be in the car?</i></p> <p>Children showed answers with their fingers.</p> <p>Teacher: <i>Amy knows, Henry knows... James, try again.</i></p> <p>Teacher (to the class): <i>Are we adding or subtracting?</i></p> <p>Most kids (chorus): <i>Adding.</i></p>

Major dimensions	Sub-categories	# and % of codes ^a	# of teachers ^b	Typical examples
				Teacher (to the class): <i>How many together?</i> Most kids (chorus): <i>Six.</i>
The nature of mistakes		20 (7.1%)		Teacher wrote the number 30 on the board. Teacher: <i>How many rows of 10 do I need for this number?</i> Elly: <i>Two.</i> Teacher: <i>Oh? Why do I need two tens?</i> Elly: <i>Three!</i> Teacher: <i>Oh, why did you change your answer?</i> Elly: <i>Because I look up the number and it's three.</i> Teacher: <i>Ok, so she changed her answer. That's good. That's what mathematicians do. She changed from two to three, because she looked up there (pointing to the board), didn't you?</i>

^a Frequencies and percentages of each sub-category within each major dimension. ^b Numbers of teachers that we observed a certain subcategory

All 24 teachers used a mix of both low-quality responses and mid-quality responses across their mathematics lessons, whereas high-quality responses were observed from only seven teachers. Approximately half of the high-quality responses (46.2%) were observed from one teacher. Based on the observation data, this teacher's instructional routines involved: (a) frequently asking open-ended questions to prompt children to explain their thinking or describe their problem-solving strategies (e.g., "*How do you know this is the biggest number?*" "*Why do you think there are two?*" "*Why do you disagree?*"); and (b) encouraging children to make comments on peers' answer and pressing children to use the statement "*I disagree/agree because.*" For example, when a child disagreed on his peer's answer, the teacher said "*So you disagree? Tell us why. Say I disagree because.*" Then the child responded, "*I disagree because you started with ten (showing 10 fingers), and you counted six, one, two, three, four, five, six (folding one finger while counting one number). So, you should have four left.*" As a result of teachers' repeated encouragement, children in this class become familiar with using the statement "*I disagree/agree because ...*" to comment on a peer's answer, as well as when explaining their own rationale for the answers.

Differences between mid- and high-quality responses. Two key elements differentiated teachers' high-quality and mid-quality responses: (a) teachers asking open-ended vs. close-ended follow-up questions and (b) teachers intentionally sustaining conversations around children's mistakes, vs. teachers giving up when children hesitated to answer the follow-up questions. First, for example, when a child made a mistake on a subtraction problem, one teacher started with an open-ended question "*What (method) do you use to figure it out? Can you show us what you did?*", whereas another teacher asked close-ended questions in a similar situation, "*How many counters do you see here? Then you need to cross out how many? Then how many are not crossed out?*" Accordingly, in the first scenario, the child was prompted to show the process of how he arrived at the answer with different methods (i.e., finger patterns and number lines) and then corrected his mistakes, whereas in the second scenario, the child simply followed the teacher's direction to count.

Second, among the instances of mid-quality responses, several teachers also tried to elicit children's explanations (e.g., "*Tell us what you did.*"). However, these teachers did not press when children paused or hesitated, thus they retained the dominant role of explaining the correct or incorrect answers. By contrast, when the child paused or was hesitating to respond, the teacher

who frequently responded to children's mistakes with high-quality instructional strategies tended to wait for the child or ask follow-up questions to elicit the child's response (e.g., "*Or, can you tell us what you did first?*"). These strategies facilitated children's engagement in the back-and-forth exchanges around their mistakes. Another type of high-quality response, which was not identified in previous studies, was that teachers provided multiple strategies or examples to help children solve the problem correctly. For example, when a child made a mistake in an addition problem, the teacher explicitly discussed and tried multiple solution methods, such as using finger patterns, drawing counters to represent the numbers, and using a number line.

Ignoring mistakes and instructional activity routine. The *Not Present* instances (i.e., ignoring mistakes or only addressing the correct answers) were related closely to the format of instructional activities (e.g., whole class lecture, group activity, independent learning). Among the 280 identified responses, there were 15 instances from 6 teachers (ranging from 1-7 instances from each of the 6 teachers) when the teacher ignored a child's mistake or only addressed the correct answer. All instances occurred when the teacher asked children to say or present their answers at the same time. In this situation, although multiple children gave incorrect answers, the teacher did not address the mistakes and directly moved on to the next problem. Moreover, around half of the "*Not Present*" responses came from one teacher. Based on the observation evidence, this teacher established the activity routine of children saying or presenting their answers all together and moving on to the next question at a fast pace. Accordingly, under this situation, this teacher was not able to address children's mistakes.

Emotional Reactions

Regarding the dimension of Emotional Reactions, 70% of teachers' responses were neutral (i.e., the absence of positive or negative reactions) that neither intensified nor mitigated children's negative emotional experience regarding making mistakes. Beyond the neutral responses, there were more negative responses (58 instances, 21.9%; ranging from 0-14 per teacher, $M = 2.0$ per teacher) than positive responses (23 instances, 8.7%; ranged from 0-4 instances per teacher, $M = 1.0$ per teacher). Results from one-way repeated measures ANOVA indicated a significant difference in the frequencies of three types of emotional reaction: Wilk's Lambda = 0.21, $F(2,22) = 41.12$, $p < 0.001$. Post-hoc analysis showed teachers tended to provide more neutral responses than positive ($p < 0.001$) or negative responses ($p < 0.001$).

However, the frequencies of positive and negative responses were not significantly different ($p = 0.108$).

None of the teachers provided only neutral responses to children's mistakes, and all teachers to some extent showed their positive or negative emotional stance regarding children's mistakes across their mathematics lessons. Only five teachers provided both positive and negative responses to children's mistakes across their lessons. Most of the teachers (22, 91.7%) provided more neutral responses than either positive or negative responses. The mean ratio of positive to neutral responses was 1.8 to 10, whereas the mean ratio of negative to neutral responses was approximately 3.0 to 10. Moreover, one teacher provided positive responses more frequently than neutral (ratio of positive to neutral responses was 4 to 3) or negative responses (ratio of positive to negative responses was 2 to 1); this was the same teacher who provided the greatest number of high-quality instructional responses to children's mistakes across the four mathematics lessons. Another teacher tended to show negative reactions more often (i.e., 48.3% of responses were negative; 44.8% were neutral; and 6.9% were positive).

Most negative emotional responses were captured in teachers' non-verbal reactions, such as voice tones (e.g., harsh tone, raised voice, sarcastic voice), facial expressions (e.g., rolling eyes, frown, grimace), and actions (e.g., shrug, wave fingers or arms) that appeared to reflect disappointment or impatience. In addition to the non-verbal negative reactions, there were nine instances (15.5% of all negative reaction instances) from four teachers, in which teachers provided negative verbal feedback. The negative verbal responses were associated with two major reasons: (a) the child made a mistake (four instances; e.g., "*Impossible! Count it again.*" "*No! Absolutely not!*" "*Stop. I cannot solve it! Because you forgot so many steps.*") and (b) why the child made the mistake (five instances; e.g., "*That is wrong! You are jumping ahead of me. You are not listening.*", "*Hey, this is work. It's not a fun game.*").

Most of the negative reactions happened immediately after a child made a mistake. By contrast, there were no instances when teachers provided encouragement or praised the child immediately after his/her mistake, which might be due to the incorrect nature of the children's answers. All of the positive responses were provided during or after the following back-and-forth exchanges, especially after the mistakes were corrected. Then teachers comforted the child (e.g., "*That's all right. It's a little tricky.*"), acknowledged the correct part of the child's answer (e.g., "*It's ok. You are very close.*") or praised the child when the child corrected the mistake on his or

her own (e.g., *“Awesome! Give me five! I love the way you can solve your problem without giving up.”*). Moreover, all positive emotional reactions were expressed through verbal comments, instead of non-verbal reactions. In addition, teachers’ different emotional stances also reflected how they handled the situations when other children in the class teased the child who made a mistake or tried to interrupt when the child was trying to correct the answer. Some teachers showed positive reactions by impeding other children’s interruptions (e.g., *“Let her brain do the thinking”*, *“Let’s pick someone who is respectfully disagreeing.”*), whereas other teachers ignored other children’s negative reactions toward the child who made a mistake.

Locus of Responsibility

Regarding the dimension of locus of responsibility, teachers themselves corrected children’s mistakes in 43.8% of the episodes, compared to the same child correcting his or her own mistake 24.5% of the time. The teacher directed the mistake to the whole class in 17.4% of cases and redirected the question to another child in 14.3% of cases. Results from one-way repeated measures ANOVA indicated a significant difference in the frequencies of Locus of Responsibility categories: Wilk’s Lambda = 0.60, $F(3,21) = 4.75$, $p = 0.011$. Post-hoc analysis revealed that children’s mistakes were more frequently corrected by the teacher than by another child in the classroom ($p = 0.004$, Bonferroni-adjusted $p = 0.008$). In addition, a close analysis of the mistake-related episodes within each teacher revealed there was no teacher who showed a clear pattern of locus of responsibility responses. Most teachers (22, 91.7%) used a mix of three or four ways to assign the locus of responsibility across their mathematics lessons.

The Nature of Mistakes

Teachers’ explicit communication regarding the nature of mistakes were identified in 20 of the 280 mistake-related interaction episodes, across 13 of the 24 teachers (ranging from 1-3 instances per teacher). In 13 instances, the teacher addressed the causes of children’s mistakes and attributed their mistakes to: (a) not paying attention or not following directions (9 instances; e.g., *“You are very smart. But you are not paying attention at all.”* *“See? You are not listening.”*) and (b) the difficulty level of the problem (3 instances; e.g., *“That’s alright. This is a little tricky.”*). In another instance, the teacher assumed children were purposefully giving incorrect

answers to attract the teacher's attention and get the opportunity to explain their thought ("No! Hands down! 'Cause all of you just want to change your answer (to incorrect answers), so you can talk. That is not how this math works.").

Beyond the statements linked to the causes of making mistakes, some teachers also expressed that there were no negative consequences for making mistakes, as long as children could correct their mistakes: "Ok, so she changed her answer. That's good. That's what mathematicians do," and "It's ok. I didn't expect you to be masters at this yet. We are just exploring." There was no occasion when a teacher explicitly pointed out the learning potential of mistakes. Of note, as shown in the previous example, teachers' emotional responses were closely associated with their explicit statements regarding the nature of mistakes. The teacher's statement of "Ok, so she changed her answer. That's good. That's what mathematicians do," not only showed a positive valence, but also conveyed an adaptive motivational belief regarding mistakes. When teachers perceived that children's mistakes were due to not paying attention or not following directions, teachers' reactions tended to be negative. By contrast, when teachers considered the difficulty level was the major reason of children's mistakes, they tended to comfort the children that "It's ok" to make mistakes or acknowledge their efforts by saying "Nice try."

Associations among the Major Dimensions

I conducted chi-square tests of independence to examine the associations among Instructional Support, Emotional Reactions, and Locus of Responsibility. Results indicated that there was a statistically significant association between Instructional Support and Emotional Reactions ($X^2(4) = 12.37, p = 0.015$), and between Emotional Reactions and Locus of Responsibility ($X^2(6) = 12.82, p = 0.046$). The association between Instructional Support and Locus of Responsibility was not statistically significant ($X^2(6) = 12.48, p = 0.052$). Then I performed post hoc comparisons in order to investigate whether certain types of teachers' responses differed from other types of responses. Results revealed that there was no statistically significant differences in the proportions of specific combinations of response dimensions. However, differences between two types of teachers' responses were marginally significant, when using the Bonferroni-adjusted p -value. First, when teachers provided high-quality instructional strategies to address children's mistakes, they were more likely to show positive

emotional reactions (Bonferroni-adjusted $p = 0.007 > 0.006$). Second, among the neutral emotional responses, teachers were less likely to ask children to correct their own mistakes (Bonferroni-adjusted $p = 0.007 > 0.006$).

Associations of Teachers' Responses with Characteristics of Teachers and Lessons

Teaching experience. I first examined the correlation between teachers' mistake-handling practices, as indicated by the percentages of different categories of teachers' responses (e.g., the percentage of high-quality responses across all the mistake-related interaction episodes identified from one teacher), and their years of teaching experience. Teachers' experience was not significantly associated with the percentages of any specific types of response to children's mistakes.

Teaching qualifications. I next conducted one-way ANOVAs to examine whether teachers' educational qualifications were related to a specific type of response. The percentages of two types of teachers' responses, positive emotional reactions and redirecting the mistake to the whole class, varied based on teachers' qualification. However, further post hoc analyses did not indicate significant differences within a certain pair of comparison.

Lessons. I conducted chi-square tests of independence to examine the associations between types of teachers' responses with both the mathematics topics (i.e., number sense, computation) and the semester of the lesson (i.e., fall or spring). Results indicated that there was no statistically significant association between the mathematics topic and types of teachers' responses: Instructional Support ($X^2(2) = 0.97, p = 0.617$), Emotional Reactions ($X^2(2) = 1.05, p = 0.592$), and Locus of Responsibility ($X^2(3) = 4.27, p = 0.234$). The associations between the semester of the lesson and types of teachers' responses were also not statistically significant: Instructional Support ($X^2(2) = 2.19, p = 0.335$), Emotional Reactions ($X^2(2) = 3.61, p = 0.165$), and Locus of Responsibility ($X^2(3) = 5.90, p = 0.117$).

DISCUSSION AND CONCLUSIONS

Discussion

The current study examined the patterns and key variations on four dimensions of teachers' mistake-handling practices in kindergarten mathematics classrooms, including teachers' instructional support, emotional reactions, locus of responsibility, and the nature of mistakes. This study contributed in-depth descriptive evidence to inform how teachers could use practical and effective strategies to address students' mistakes in classroom settings. The findings in the current study regarding the range of strategies teachers used to respond to children's mistakes were generally consistent with the results in previous studies with older students (e.g., Schleppenbach et al., 2007; Tulis, 2013). However, there were also important differences.

Complexity of the feedback loop. Different from previous studies that primarily focused on teachers' first response to students' mistakes (e.g., Santagata, 2005; Tulis, 2013), the current study documented the back-and-forth exchanges surrounding children's mistakes, in order to gain a more comprehensive understanding of teachers' mistake-handling practices in classroom settings. Results revealed the complexity of the teacher-child interaction surrounding mistakes and the necessity of examining the whole feedback loop. For example, in one of the mistake-related episodes, when one child made a mistake, the teacher first showed a disappointed facial expression and then asked open-ended questions to prompt the child to explain his thinking (e.g., "*Tell me why you think this group has more. How did you know that?*") and correct the mistake by himself. In this case, if the study only focuses on the teacher's first response, only the teacher's immediate and negative response would be documented on the *Emotional Reactions* dimension, whereas the following high-quality response for the *Instructional Support* dimension would be ignored. Therefore, focusing on teachers' first response might overlook important evidence on different dimensions of teachers' mistake-handling practices. In addition, the findings indicated that for the dimension of *Emotional Reactions*, most of the negative reactions were observed immediately after children's mistake, whereas most of the positive reactions were captured during the later part of the feedback loop. Accordingly, if the study only examined the teacher's initial response, rather than the full interactional sequence related to the mistake, the

negative emotional reactions would be more likely to be documented than the positive reactions, and thus lead to bias in the results.

Instructional support. Bray's (2011) study with third grade students emphasized conceptual correction as a high-quality response, however there was no observed instance of teachers referring to the nature of children's misconceptions in the present study. Teachers in these kindergarten mathematics classrooms focused primarily on re-demonstrating the problem-solving procedures or discussing relevant examples and strategies. The inconsistencies in the results between the current study and previous studies with older students highlight the need for researchers to be attentive to issues involving students' developmental stages or grade levels.

Also, the findings revealed that the major difference in teachers' responses to mistakes was located in the quality of teachers' elaboration on children's mistakes during the mathematics instruction. The variations of teacher responses to children's mistakes are consistent with the previous observational studies (e.g., Borasi, 1994; Schleppenbach et al., 2007) with older students. Teachers' responses ranged from not elaborating on or addressing the mistakes at all (e.g., ignoring mistakes), providing the correct answer immediately, providing brief hints or explanation about the correct or incorrect answer, to initiating classroom discussion or open-ended inquiries about the incorrect answers. The high-quality instructional responses to children's mistakes were characterized with intensive back-and-forth conversation surrounding children's mistakes. In addition, our findings suggested that in approximately 60% of the instances, teachers either corrected children's mistakes without further explanation or did not address the mistake. This finding was consistent with some researchers' argument that in mathematics classrooms, teachers tend to focus more on task completion and the accuracy of students' answers, instead of valuing students' mistakes and developing mistakes into mathematical inquiries (Hiebert & Wearne, 1993; Race & Powell, 2000; Turner et al., 2002; Turner & Meyer, 2004). Previous studies also found that during mathematics lessons, teachers more frequently ignored students' mistakes, showed more instances of negative reactions toward mistakes, and communicate the learning potential of mistakes less frequently, compared to instructional practices in other content areas (Matteucci et al., 2015; Tulis, 2013). Therefore, it is essential to explore the adaptive strategies teachers use to respond to students' mistakes, especially in mathematics classrooms.

I identified two classrooms with distinctive patterns of teacher responses. In one classroom a large proportion of the teacher's instructional responses were high quality, whereas in the other classroom the teacher tended to ignore children's mistakes. In both classrooms, teachers' responses to children's mistakes were closely associated with their learning activity routines. In the first classroom, the teacher established the routine of asking a specific child to answer the question. Then, when the child made a mistake, she asked the same child to explain his or her thinking or asked another child whether and why they agree or disagree with the initial answer. The teacher consistently asked the child to clarify his/her rationale or strategies regarding the mistakes. As a result, some children in this class could spontaneously explain their thinking when answering questions. On the contrary, in the second classroom, the mathematics learning activities were characterized by a combination of fast pacing, moving through questions quickly, and asking the whole class to say or present the answers together. Consequently, it was difficult for the teacher to notice children's mistakes and elaborating on their mistakes would slow the pace. This finding highlighted the importance of constructing learning activity routines that are both efficient and also effective in increasing children's engagement in mathematics learning. In addition, the examples also showed that establishing effective learning activity routines requires teachers' persistence. When the teacher intentionally sustained the discussion around children's mistakes and consistently prompted children to explain their thinking, kindergarteners could become familiar with the process and were able to reflect on their answers spontaneously.

Emotional reactions. Teachers' negative emotional reactions primarily happened immediately after children made mistakes, whereas their positive responses were provided in the later part of the feedback loop. In addition, teachers' negative responses were mostly delivered through non-verbal reactions (84.5% of all negative reaction instances), whereas positive responses were expressed with verbal comments. The major differences between teachers' positive and negative emotional reactions suggested that teachers' immediate and non-verbal negative responses might be instinctive reactions elicited because children's answer were different from teachers' expectations. Meanwhile, the verbal positive responses rarely happened spontaneously and required teachers' specific intention of encouraging or comforting children. Therefore, for teachers to demonstrate higher levels of tolerance and fewer negative reactions to

mistakes, it is critical to both promote teachers' own adaptive beliefs regarding mistakes and raise awareness of the learning potential of mistakes.

Teachers' emotional reactions were associated with the other three dimensions (i.e., instructional support, locus of responsibility, the nature of mistakes). Teachers who adopted high-quality instructional strategies to address children's mistakes also tended to provide positive feedback when children made mistakes. When teachers made positive statements about the nature of mistakes, they were more likely to show positive emotional reactions, and vice versa. Accordingly, teachers' responses in one mistake-related episode tended to show a consistently adaptive or maladaptive pattern across four major dimensions. Certain teachers also presented a consistent pattern of responses across their lessons. I argue it is possible that teachers' mistake-related practices were in line with whether they perceive mistakes as a natural and anticipated aspect of learning or as an indicator of incompetence or failure. Accordingly, teachers' attitudes toward mistakes could be associated with the patterns of how they responded to mistakes.

Locus of responsibility. Teachers' most frequent practice was to correct children's mistakes themselves, whereas redirecting the question to another child in the class was the least frequently-used strategy. This result is not consistent with studies of middle school students (Santagata, 2005; Tulis, 2013), where teachers' most frequent response was to redirect the question to another child. Moreover, redirecting the question to the whole class was not documented in previous studies, however, in the current study there were 46 instances when the teacher redirected the question to the whole class and asked children to say the correct answer together.

The nature of mistakes. Across all 96 mathematics lessons, there was no instance of a teacher explicitly addressing the learning potential of mistakes. Most positive statements regarding the nature of mistakes emphasized the acceptability of making mistakes (e.g., it is ok to make mistakes). However, teachers did not talk with their students about the role of mistakes in learning (e.g., mistakes can be good for learning because...). Different from some researchers' argument that mistakes could "make your brain grow" (Boaler & Dweck, 2016, p. 11), all teachers' positive acknowledgement only focused on the process of children correcting their own mistakes but without sharing with the children the rationale for doing so. Previous empirical evidence from experimental studies also supported that learning from mistakes could be beneficial for students' mathematical understanding and lead to better performance, especially

when teachers conveyed the learning potentials of mistakes and modeled how to learn from the incorrect examples. Findings in the present study further highlighted the need for promoting teachers' awareness of the learning potential of mistakes, so that teachers could intentionally use more high-quality and positive strategies to address students' mistakes during mathematics instruction (Booth, Lange, Koedinger, & Newton, 2013; Durkin & Rittle-Johnson, 2012; Granberg, 2016; Loibl & Rummel, 2014).

Educational Implications

Addressing students' mistakes is an important part of teachers' day-to-day instructional practices. The current investigation on teachers' mistake-handling practices in actual classroom settings has significant implications for teacher education. First, the findings in the present study highlight the importance for teachers to establish effective and efficient instructional routines. Whether and how teachers elaborate on children's mistakes were closely related to their instructional routines. If a teacher's instructional routine involves children saying or presenting the answer at the same time and pursuing a fast pace, it is difficult for this teacher to pay attention to students' mistakes. As a result, this teacher may not be aware of the lack of clarity in the instructions or directions he or she delivered to the students and overlook students' major misconceptions. By contrast, based on the observations of teachers' high-quality instructional strategies to address children's mistakes, the instructional routines of asking open-ended questions and prompting children to explain their thinking could also be efficient. It requires teachers' persistence in following effective instructional routines, and the routines should be clearly communicated and consistently reinforced.

Second, the findings emphasize the potential effect of teachers' own beliefs regarding mistakes. Results in the current study indicated that there were no instances when the teacher immediately reacted to children's mistakes in a positive way or spontaneously highlighted the learning potential of children's mistakes. Most of the positive emotional reactions or positive statements regarding the nature of mistakes were captured during or after the feedback loop. The findings suggested that the adaptive patterns of teachers' mistake-handling practices require teachers' own awareness of the learning potential of mistakes and their specific intent to use mistakes as resource to support children's mathematical understanding. Schleppebach et al (2007) also emphasized that teachers' beliefs regarding mistakes shaped their practices to

respond to mistakes. Therefore, it is important to help the teachers construct a positive disposition toward mistakes, and thus teachers would accept mistakes as a natural and constructive part of learning and teaching and use adaptive instructional strategies to address students' mistakes in their classrooms.

Limitations and Future Directions

The findings of the current study are limited in three aspects. First, because the current study primarily focused on teachers' responses, I selected video-recorded mathematics lessons with clear visual and audio information on teacher-child interactions surrounding mathematics learnings. Accordingly, some lessons in which children were engaged in group or individual activities, and the video only recorded the learning process of a small number of children, were not selected because it was difficult to document teachers' interaction with the children who were not in the recording. As a result, the majority of the mistake-related interaction episodes were identified under the whole class activity settings. Future studies could examine whether the patterns of teachers' mistake-handling practices are consistent in specific group or independent learning activities.

Second, in order to provide a clear comparison of teachers' instructional feedback and strategies, the current study primarily focused on the mathematics lessons on two topics (i.e., number sense, computation). There were no differences in teachers' responses to mistakes across the two mathematics topics. However, geometry and measurement are other foundational areas of mathematics content in the early grades (Cross, Woods, & Schweingruber, 2009), and it remains unknown whether teachers' mistake-handling practices vary across these topics. Future studies could address this issue.

Third, findings of this study suggested that teachers' mistake-handling practices were not associated with teacher experience or qualifications or the semester. Further studies could contribute to the investigation of potential factors that are associated with how teachers respond to children's mistakes in classroom settings. Considerations for possible research include their content knowledge, pedagogical content knowledge, and self-efficacy for mathematics teaching in the early grades, in addition to their beliefs about mindsets and the role of mistakes for children's learning.

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