

Supplementary Material 1: Solid mechanics formulation

Variational principle for solid tissue

In order to prevent volumetric locking a three field Hu-Washizu formulation has been used. The functional depends on the state of deformation (ϕ), the volume change independently of the motion (\bar{J}) and the pressure (p) that is a Lagrange multiplier enforcing the condition that $J = \bar{J}$. The framework is split into deviatoric and volumetric components

$$\Pi_{HW}(\phi, \bar{J}, p) = \int_V \hat{\Psi}(\mathbf{C}) dV + \int_V U(\bar{J}) dV + \int_V p(J - \bar{J}) dV - \Pi_{ext}(\phi) \quad (1)$$

where \mathbf{C} is the right Cauchy deformation tensor.

The stationary conditions of Equation (1) with respect to \bar{J}, ϕ and p will yield the virtual work principle and the constitutive and kinematic relationships associated with the volumetric behavior:

$$\begin{cases} D\Pi_{HW}(\phi, \bar{J}, p)[\delta \mathbf{v}] = \int_V \boldsymbol{\sigma} : \delta \mathbf{d} dv - \delta W_{ext}(\phi, \delta \mathbf{v}) = 0 \\ D\Pi_{HW}(\phi, \bar{J}, p)[\delta \bar{J}] = \int_V \left(\frac{dU}{d\bar{J}} - p \right) \delta \bar{J} dV = 0 \\ D\Pi_{HW}(\phi, \bar{J}, p)[\delta p] = \int_V (J - \bar{J}) \delta p dV = 0 \end{cases} \quad (2)$$

where J is the pointwise jacobian such that $J = \frac{dv}{dV}$. As $\delta \bar{J}$ and δp can be any arbitrary functions, Equation (2) gives $\bar{J} = J$ and $p = \frac{dU}{d\bar{J}}$.

Cauchy deformation and stress tensors

The Right Cauchy Deformation Tensor is calculated as

$$\mathbf{C} = \mathbf{F}^T \mathbf{F} \quad (3)$$

The Cauchy Stress Tensor is computed as

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_{vol} + \bar{\boldsymbol{\sigma}}_a + \bar{\boldsymbol{\sigma}}_p \quad (4)$$

where:

$$\boldsymbol{\sigma}_{vol} = \kappa \frac{J-1}{J} \mathbf{I} \quad (5)$$

$$\bar{\boldsymbol{\sigma}}_a = \frac{2}{J} \frac{d\bar{\Psi}_a}{d\bar{I}_4} \text{dev}(\mathbf{a} \otimes \mathbf{a}) \quad (6)$$

$$\bar{\boldsymbol{\sigma}}_p = \frac{\mu_p}{J} \text{dev} \bar{\mathbf{b}} + \frac{2}{J} \frac{d\bar{\Psi}_p}{d\bar{I}_4} \text{dev}(\mathbf{a} \otimes \mathbf{a}) \quad (7)$$

with:

$$\frac{d\bar{\Psi}_a}{d\bar{I}_4} = \mu_a \bar{L}_0 (n_{AMP} + n_{AM}) \frac{\lambda - 1 - \bar{u}_{fs}}{2\bar{I}_4} \quad (8)$$

$$\frac{d\bar{\Psi}_p}{d\bar{I}_4} = c_{p1} (\bar{I}_4 - 1) \exp [c_{p2} (\bar{I}_4 - 1)^2] \quad (9)$$