Force distribution and multiscale mechanics in the mussel byssus

Supplemental Materials

Proposed Model for Hysteretic Response Under Cyclic Loading

Thread-plaque structures subjected to multi-cyclic loadings demonstrate a hysteretic effect that we aim to model through the introduction of a strain-dependent damage term in our spring-based network model [10]. Experiments subjected mussel thread-plaque structures to cyclic loading and found that no hysteresis for loadings in the first region, *i.e.* $\sigma < \bar{\sigma}_y$, consistent with a purely elastic response for small deformations. Beyond $\bar{\varepsilon}_I$, the plaque experiences significant hysteresis with different unloading and reloading behaviours, as shown in Figure S1.

To model this response, we consider a system comprising a plaque and distal thread that is subjected to the following quasi-static cyclic loading: first, it is slowly stretched to a normalized strain $\bar{\varepsilon}_1 > \bar{\varepsilon}_I$, followed by a quasi-static release to zero force. Next, the plaque is stretched from the zero-force state to a higher strain value $\bar{\varepsilon}_2 > \bar{\varepsilon}_1$ and then slowly released back to a zero-force configuration. This process continues until the detachment force is reached.

To model the hysteresis effects in the plaque, we propose the following form for the stress,

$$\bar{\sigma}(\bar{\varepsilon}, \bar{\varepsilon}_{ul}) = \phi(\bar{\varepsilon}, \bar{\varepsilon}_{ul}; b)\bar{\sigma}(\bar{\varepsilon}), \tag{S1}$$

where $\bar{\varepsilon}_{ul}$ is the maximum strain reached prior to the unloading that commences at the highest strain and

$$\phi(\bar{\varepsilon}, \bar{\varepsilon}_{ul}; b) = \begin{cases} \frac{\bar{\varepsilon}}{\bar{\varepsilon}_{ul}} \exp\big(b(\bar{\varepsilon} - \bar{\varepsilon}_{ul})\big) & \bar{\varepsilon} \leq \bar{\varepsilon}_{ul} \text{ and } \bar{\varepsilon}_1 < \bar{\varepsilon}_{ul} \\ 1 & \text{otherwise} \end{cases} \tag{S2}$$

is a damage function that accounts for the loading history in the plaque. Here, $b > 0$ is a material parameter that depends on the damage suffered by the cuticle and we emphasize that $\phi(\bar{\varepsilon}, \bar{\varepsilon}_{ul}; b) > 0$. Note that if a plaque has not been loaded beyond its yield point, $\phi(\bar{\varepsilon}_{ul} < \bar{\varepsilon}_I; b) = 1$ and $\bar{\sigma} < \bar{\sigma}_m$, implying that the material follows the main stress-strain curve that prevails when there is no unloading during the test.

Since the unloading and reloading paths are different, we define the two damage functions $\phi_{ul}(\bar{\varepsilon}, \bar{\varepsilon}_{ul}; b) = \phi(\bar{\varepsilon}, \bar{\varepsilon}_{ul}; b_{ul})$ and $\phi_{rl}(\bar{\varepsilon}, \bar{\varepsilon}_{ul}; b) = \phi(\bar{\varepsilon}, \bar{\varepsilon}_{ul}; b_{rl})$ to account for the unloading and the reloading paths, respectively. Accordingly, when loading a thread-plaque system beyond its yield point and unloading, the stress-strain curve follows an exponential behaviour that is characterized by b_{ul}. Upon reloading, the stress-strain behaviour of the plaque follows an exponential trend characterized by b_{rl}. Once the last unloading occurs, the strain $\bar{\varepsilon} = \bar{\varepsilon}_{ul}$ is reached and the function $\phi(\bar{\varepsilon}, \bar{\varepsilon}_{ul}; b_{ul}) = 1$. Consequently, $\bar{\sigma} = \bar{\sigma}_m$, and the plaque response continues to follow the main stress-strain curve given in Eq. (S1).

To illustrate the merit of the model, we compare its predictions to two sets of multi-cycling loading experiments reported in ref. [10]. Fig. (S1) depicts the model predictions (continuous curves) and the experimental data points (circular markers). The unloading and reloading parameters $b_{ul} = 5.64$ and $b_{rl} = 1.08$ are determined through a least-squares fit to the experimental data, which were chosen due to the relative similarity between the two data sets. The molecular origins and physical meaning of the damage parameters are not yet known.

In comparison to the monotonic pull-to-failure tests, in which we find robust responses under load, we find there to be much more variation in the mechanical response to cyclic loading, which leads to a greater variation in the estimated model parameters (b_{ul}, b_{rl}). Given this variation, and since we have not established a clear physical origin of the damage parameter estimates we do not show a broad range of experimental results here. Rather we show a representative case and describe the general approach to damage modelling, which we hope will spur more experimental analysis and improved model development.

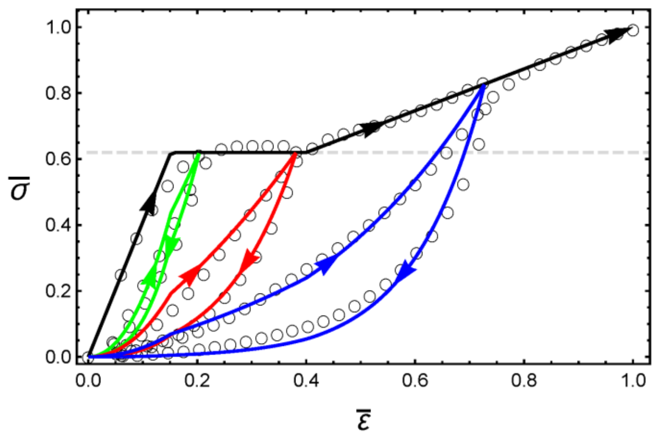

Figure S1: Comparison between the predictions of the proposed model in Eq. (S2) and the experimental results for a plaque subjected to multi-cyclic loading. The continuous curve and the circular markers correspond to the proposed model and experimental data, respectively.