

## **Supplemental Material: Computational simulation**

### **Architecture of the formalized model**

The formalized model used for the simulations below built on models which have been used to study the neural activation dynamics of perceptual decisions in the past (compare Hock, Schönner, & Giese, 2003; Noest, Ee, Nijs, & Wezel, 2007). Recently, we showed that it can also be applied to delay discounting, where it successfully predicted within and across trial effects (Scherbaum et al., 2016).

The model comprised of two neural units with a non-linear activation function. These two units inhibit each other (see Figure A1, left) which opens a natural mapping of these two units to the architecture of more complex models, i.e. parallel constraint satisfaction networks (e.g. Glöckner & Betsch, 2008b). The two units in the model here could be seen as the two options in a parallel constraint satisfaction network that are fed by informative cues from lower network levels. By interactive activation and competition both systems lead to a settled decision state in the end of the decision process.

The activation of both units can be mapped as a two-dimensional state space in which each units' activation spans one dimension. The mutual inhibition of the two units results in two potential stable states of activation. These two stable states can, in turn, be mapped to the attractor valleys (see Figure A1 middle and right) of the model sketched in the main part of the text (see Figure A2): In the SS state, the SS unit is active and the LL unit is inhibited while in the LL state, the LL unit is active and the SS unit is inhibited.

Similar to the depth of the attractors in the conceptual model explained in the main part of the article, the input to the units is defined by their attractiveness, i.e. their subjective value as could be derived from classical discounting models, e.g. the hyperbolic discounting model (e.g. Green et al., 1994).

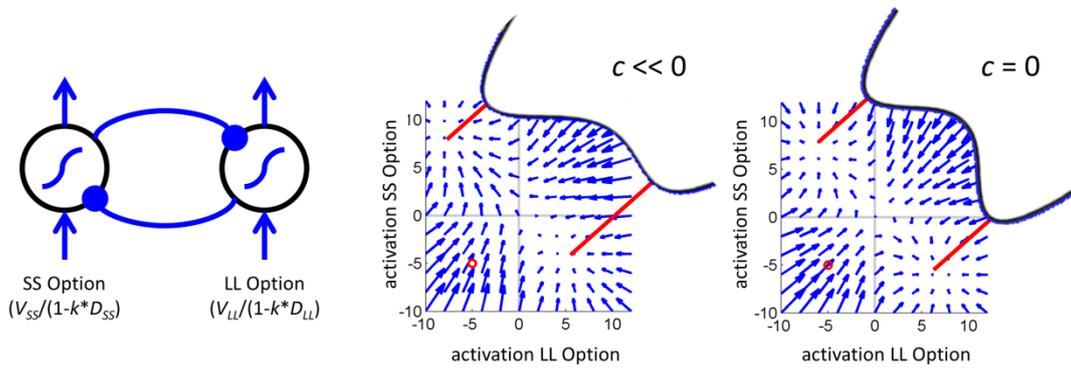


Figure A1. Left: The formalized version of the attractor model is based on the non-linear neural activation dynamics of two competing neural units representing the two choice options. The input to the two units reflects the subjective value of each option. The subjective value can be derived from classical discounting models, e.g., the hyperbolic discounting model. Middle and right: The activation of the two units mapped onto a two-dimensional state space in which each unit's activation spans one dimension. Arrows in the vector field indicate the potential trajectories of this two-dimensional system under equal input. The mutual inhibition results in two potential stable activation states (indicated by dots compared to arrows in the vector-field). The dynamics of this two-dimensional system can be mapped to the one-dimensional model consisting of two attractors, which we explain in greater detail in the introduction. Middle: A more attractive SS option biases the two-dimensional system's activation state towards a winning SS option. This is equivalent to the one-dimensional system with a control parameter  $c < 0$ . Right: Two equally attractive options yield no activation bias in the two dimensional system. This is equivalent to the one-dimensional system with a control parameter  $c = 0$ .

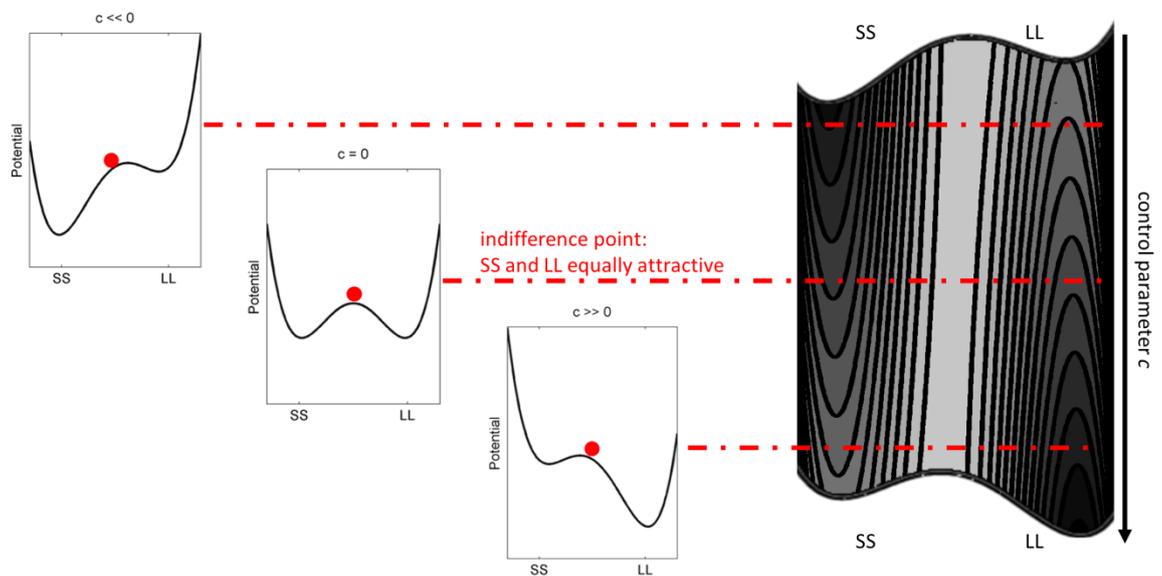


Figure A2. Applying the model of Tuller and colleagues (1994) to delay discounting: two attractors represent the possible choice states for the soon but small (SS) and the late but large (LL) option. Depths of the attractors define attractiveness of both choice states. The system's state (initial system state marked by red circle) tends to move to the bottom of the potential wells. Note that the single parameter  $c$  controls the states' stability and could be interpreted as the relative difference in subjective attractiveness of the options for a participant. For  $c = 0$ , the SS and the LL option are equally attractive, which is called the indifference point (the grey-scaled graph show deeper attractors with darker shades of grey).

## Mathematical model description

The dynamics of the model are defined by two coupled differential equations (one for each neural unit) representing non-linear neural activation dynamics (Hock et al., 2003; Noest et al., 2007; see also Amari, 1977).

$$\tau \dot{u}_{SS} = -u_{SS} + h + w_r \cdot \sigma(u_{SS}) + w_i \cdot \sigma(u_{LL}) + I_{SS}$$

$$\tau \dot{u}_{LL} = -u_{LL} + h + w_r \cdot \sigma(u_{LL}) + w_i \cdot \sigma(u_{SS}) + I_{LL}$$

Here,  $\tau$  denotes the timescale (defining the step size of the Euler solution),  $h$  denotes the resting level,  $w_i$  the (inhibitory) coupling strength of the two equations, and  $w_r$  the recurrent feedback,  $I_{SS}$  and  $I_{LL}$  the input representing the attractiveness of the two options, and  $\sigma$  a sigmoid non-linearity, mirroring non-linear neural population dynamics:

$$\sigma(u) = \frac{1}{(1 + e^{-\beta \cdot (u - a)})}$$

Hence, interactions between the two units only happen to the extent that the activation  $u$  exceeds a soft threshold (Cohen, Servan-Schreiber, & McClelland, 1992; Erlhagen & Schöner, 2002). Note that the  $\beta$ -parameter is also called the gain-parameter, modulating the discreteness of neural activation states (for hypotheses concerning  $\beta$  and aging, see e.g. Li, Lindenberger, & Sikström, 2001). For a list of the chosen parameter values, see Table 1. Importantly, these parameters are identical to the ones from our recent application of this model to delay discounting (Scherbaum et al., 2016).

Table 1. Basic parameters of the model

Parameter	Value
$\tau$	10
$h$	-5
$w_i$	-7
$w_r$	6
$a$	0
$\beta$	1

The dynamics of the system are governed by the inputs to the two units, that is  $I_{SS}$  and  $I_{LL}$  (the difference  $I_{SS}-I_{LL}$  could be taken as equivalent to the control parameter  $c$  in the one-dimensional attractor model). To drive the units near to the activation threshold, these inputs consisted of a base Input  $I_{base}$  and the subjective value of the respective option (determined by hyperbolic discounting), so that

$$I_{SS} = I + V_{SS}/(1 - k * D_{SS}) \text{ and } I_{LL} = I + V_{LL}/(1 - k * D_{LL}).$$

As in the recent application of the model  $I_{base}$  was set to 6. The additional subjective value signal ranged between [0, 1]. The discounting parameter  $k$  was set to the median of  $k$  as measured in the experiment in both conditions, resulting in  $k = 0.042$ .

### Simulation design

We simulated the independent variables from the experiment and manipulated the attractiveness of the two options with the same values and delays as used in the experiment. We orthogonally varied the time interval between the options (2, 4, 6, 10, 20, and 50 days), the delay of the SS option (0 and 10 days), the value of the LL option (90 and 95 €) and the value difference to the SS option (LL - 5, 15, 25, 35, 45, 65, 85 €). This resulted in 168

different combinations of option values and times. Each combination was presented twice, one time as SS presented first and LL presented second, and one time as LL presented first and SS presented second. Overall, this resulted in 336 simulated trials.

### **Simulation procedure**

The two coupled differential equations constitute a neural system with two units inhibiting each other so that only one unit can win the competition and elicit the final choice. This competition unfolds over time. We simulated the behavior of the system by numerical integration with each trial having a maximum length of 200 time steps. Results were obtained using Matlab 2010a running on a Windows 8 platform.

In each trial, we presented to the network at time  $t = 1$  the initially presented option, e.g. the SS option, and the average value of the second option, e.g. the LL option. This input was switched off  $t = 20$ . At  $t = 50$ , both options, the SS and the LL option, were presented to the network and a choice was performed after at least 10 time steps when the network output reached a threshold of 0.85. The simulation was reset after each trial, so that each choice was simulated independently of previous trials.

### **Simulation Results**

The qualitative model predicted firstly that presenting the SS option first induces a bias to choose the SS option – higher discounting – and presenting the LL option first induces a bias to choose the LL option – lower discounting. We calculated the probability of choosing the LL option for the two conditions. Since we had only one sample per combination of options' properties, we smoothed the LL option across intervals by a Gaussian of 5 samples to simulate the smoothing by noise across participants data (which does not change qualitative differences between curves). The results in Figure A3, left, clearly indicate the bias as predicted by the qualitative model and as found in the empirical data.

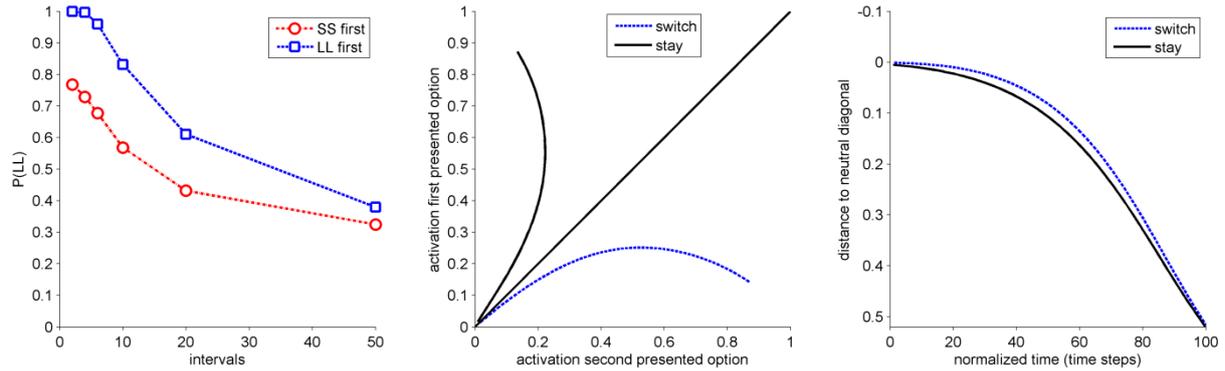


Figure A3. Left: Delay discounting of the model represented by the probability to choose the LL option as a function of the interval (in days) between the SS and the LL option. Initial presentation of the SS option led to more discounting than initial presentation of the LL option. Middle: Average movements of the system in activation state space as defined by the activation of the initially presented option or the second option. To derive simulated mouse movements, the area between the activation trajectory and the neutral diagonal was calculated for both, trials in which the initially presented option was chosen (stay trials) and trials in which the second presented option was chosen (switch trials). Right: Simulated mouse movements over time. Stay decisions were more direct than switch decisions.

The qualitative model predicted secondly that stay decisions are easier/more direct than switch decisions. To this end, we took the movement of the system in the SS-LL activation state space (Figure A3, middle) and calculated the distance of this activation trajectory from the neutral diagonal in the state space. This distance represents the system's current tendency to choose one option over the other one: the lower the distance, the more the system is in a neutral state. For example, if the SS option shows an activation of 0.2, and the LL option an activation of 0.1, this procedure yields a value of 0.071 to the neutral activation diagonal (from 0,0 to 1,1) in favour of the SS option. In contrast, if the SS option shows an activation of 1, and the LL option an activation of 0.1, this procedure yields a value of 0.63 to the neutral activation diagonal. This measure could be directly compared to the mouse movement on the X-axis. The results in Figure A3, right, show the predicted pattern of more direct movements in the state space for stay trials compared to switch trials.

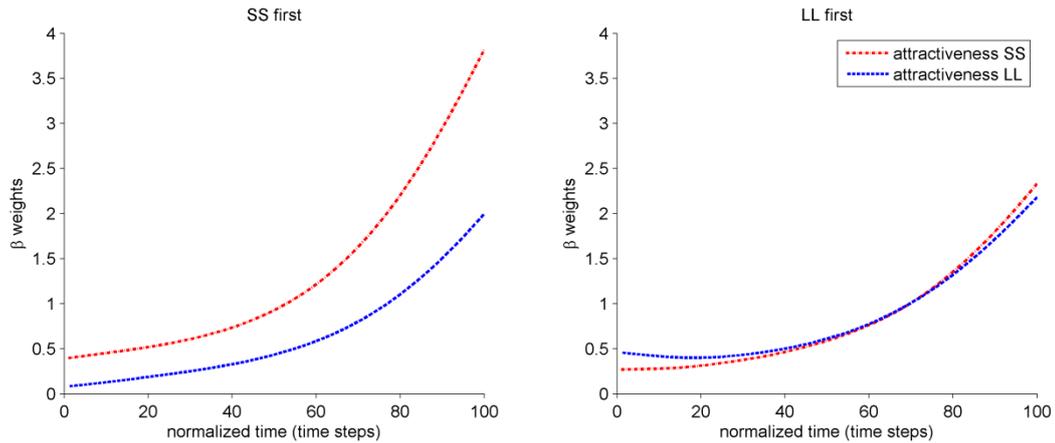


Figure A4. Continuous multiple regression of simulated mouse movements for the two conditions of temporal order. The time-continuous beta-weights representing the attractiveness of the two options show distinct temporal patterns for the two orders of presentation. Left: beta-weights for movements after initial presentation of the SS option show an early influence of the SS option's attractiveness. Right: beta-weights for movements after initial presentation of the LL option show an early influence of the LL option's attractiveness.

The qualitative model predicted thirdly that information from the first option should have an influence on the system's movement from the very start, whereas information from the second option should only have an impact later in time. Again, we took the distance to the neutral diagonal in the activation state space as a measure analogue to real mouse movements on the X-axis. We time normalized this signal from the onset of the combined stimulus presentation to the response time of the model to 100 time steps as we did for mouse movements. Then we performed the same regression analysis as on mouse movements.

Figure A44 shows the results of this regression analysis. As in the experiment, an initial presentation of the SS option (Figure A44, left) leads to a strong initial influence of the attractiveness of the SS option. In contrast, initial presentation of the LL option (Figure A44, right) yields an advantage for the LL option. As in the experimental data, the pattern is stronger in the SS first condition than in the LL first condition.

This asymmetry seems remarkable as the attractor model would in principle predict symmetric behaviour. We hence reasoned that if the model itself is symmetric, asymmetric results could only be caused by the specific input patterns to the model – the specific delay and value combinations for SS and LL options. To check for such asymmetries, we analysed

the subjective values for all SS and LL options (as explained above, SV were calculated by the  $k$  value of participants according to the formulas  $SV = V/(1 - k * D)$ ). These subjective values indeed showed an asymmetry: the SV of SS options ranged from 3.52 € to 90 € while the SV of LL options ranged from 25.56 € to 87.64 €.

To check whether this asymmetric range of subjective values explained the asymmetry in the results, we added a sixth interval between the SS and the LL option, 300 days, yielding now 6 intervals between the SS and the LL option (2, 4, 6, 10, 20, 50, and 300 days). This changed the range of SV of the LL options to 6.41 € to 87.64 €, and hence better aligned the SV of the LL options with the SV of the SS options. We reran the simulation with the only change in the additional trials for the new interval.

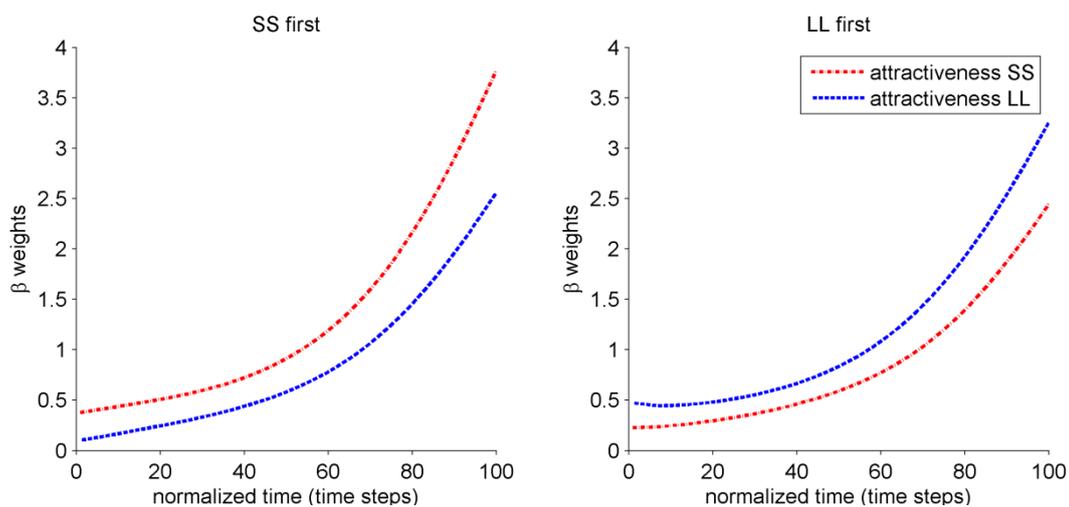


Figure A5. Continuous multiple regression of simulated mouse movements for the additional run of the simulation with an extra time interval of 300 days between the SS and the LL option. The time-continuous beta-weights representing the attractiveness of the two options show distinct temporal patterns for the two orders of presentation. Left: beta-weights for movements after initial presentation of the SS option show an early influence of the SS option's attractiveness. Right: beta-weights for movements after initial presentation of the LL option show an early influence of the LL option's attractiveness.

The results (Figure A5) show that the mouse movements now approach a more symmetric pattern. Hence, we conclude from this simulation that asymmetry in the simulation and presumably also in the empirical data stems from the asymmetric distribution of subjective values of the offered SS and LL options.

## Conclusion

Taking together, computational simulation corroborated the qualitative predictions from the one-dimensional attractor model. The model was not only able to mimic the outcome oriented measures, i.e., discounting, but also to mimic the dynamic measures, i.e., mouse movements and time-varying influence of subjective value. Notably, we used the same parameters to configure the model as we did in our previous application of the model to delay discounting.

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