

Appendix: Proof of Theorem 1

When the treatment effect is homogeneous across subgroups, $T \rightarrow 0$ when $N_j \rightarrow \infty$ because the Chi-squared test statistic of homogeneity is consistent. As $b > 0$, it follows that $b \times \log(T) \rightarrow -\infty$, and thus $\sigma^2 = \exp\{a + b \times \log(T)\} \rightarrow 0$. In other words, the CBHM achieves full information borrowing, as with the pooled analysis. Similarly, when the treatment effect is heterogeneous across subgroups, $T \rightarrow \infty$ when $N_j \rightarrow \infty$. Thus $\sigma^2 = \exp\{a + b \times \log(T)\} \rightarrow \infty$, which means that no information will be borrowed across subgroups, as with the independent analysis