

## Supplementary materials

**Article title:** A softened membrane model for PC composite box girders with corrugated steel webs under pure torsion

**Journal name:** Advances in Structural Engineering

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**Supplementary Table 1** Summary of equations employed in the proposed models.

Category of equations	Previously derived equations	Newly derived equations	Modified equations	Modification
Equilibrium equations	Eqs. (9) – (11) (Ko et al., 2013; Mo et al., 2000) Eq. (12) – (14) (Johnson and Cafolla, 1997, Shen et al., 2018) Eq. (17) (Jeng et al., 2010)		Eqs. (15) and (16)	$f_s^c$ are introduced to emphasize the initial stresses induced by prestressing in the original equations from Jeng and Hsu ( 2010)
Compatibility equations	Eqs. (19), (20) and (22) – (28) (Jeng and Hsu, 2009) Eq. (21) (Ko et al., 2013)		Eq. (18)	The sign of the variable $\bar{\epsilon}_{2s}$ is corrected in the original equations from Jeng and Hsu ( 2009)
Constitutive laws of materials	Eqs. (29) – (31) and (33) – (35) (Jeng and Hsu, 2009) Eqs. (37) and (38) (Mo et al., 2000)		Eq. (32)	the ultimate compressive strain of concrete is assumed to be $2\epsilon_0$ in the original equation from Jeng and Hsu ( 2009)
			Eq. (36)	$f_s^c$ and $\bar{\epsilon}_s^c$ are introduced to emphasize the initial stresses and strains induced by prestressing in the original equations from Jeng and Hsu ( 2009)
Hsu/Zhu ratios for torsion		Eq. (2)	Eq. (1)	the value of $(v_{21})_{shear}$ is substituted in the origin equations from Jeng and Hsu ( 2009)
Convergence criteria			Eqs. (3) and (4)	$f_s^c$ are introduced to emphasize the initial stresses induced by prestressing in the original equations from Jeng and Hsu ( 2009)
Equilibrium equations and convergence criteria including initial stress and strain			Eqs. (39) – (41)	$f_s^c$ are introduced to emphasize the initial stresses induced by prestressing in the original equations from Jeng and Hsu ( 2009)
Constitutive laws of materials including initial stress and strain	Eqs. (6), (7), (42) and (43) (Chen, 2012)	Eq. (5)		
Additional equation		Eq. (8)		

**Supplementary Table 2.** Comparison of torques from SMMT-PC, SMMT, RA-STMT and experimental results.

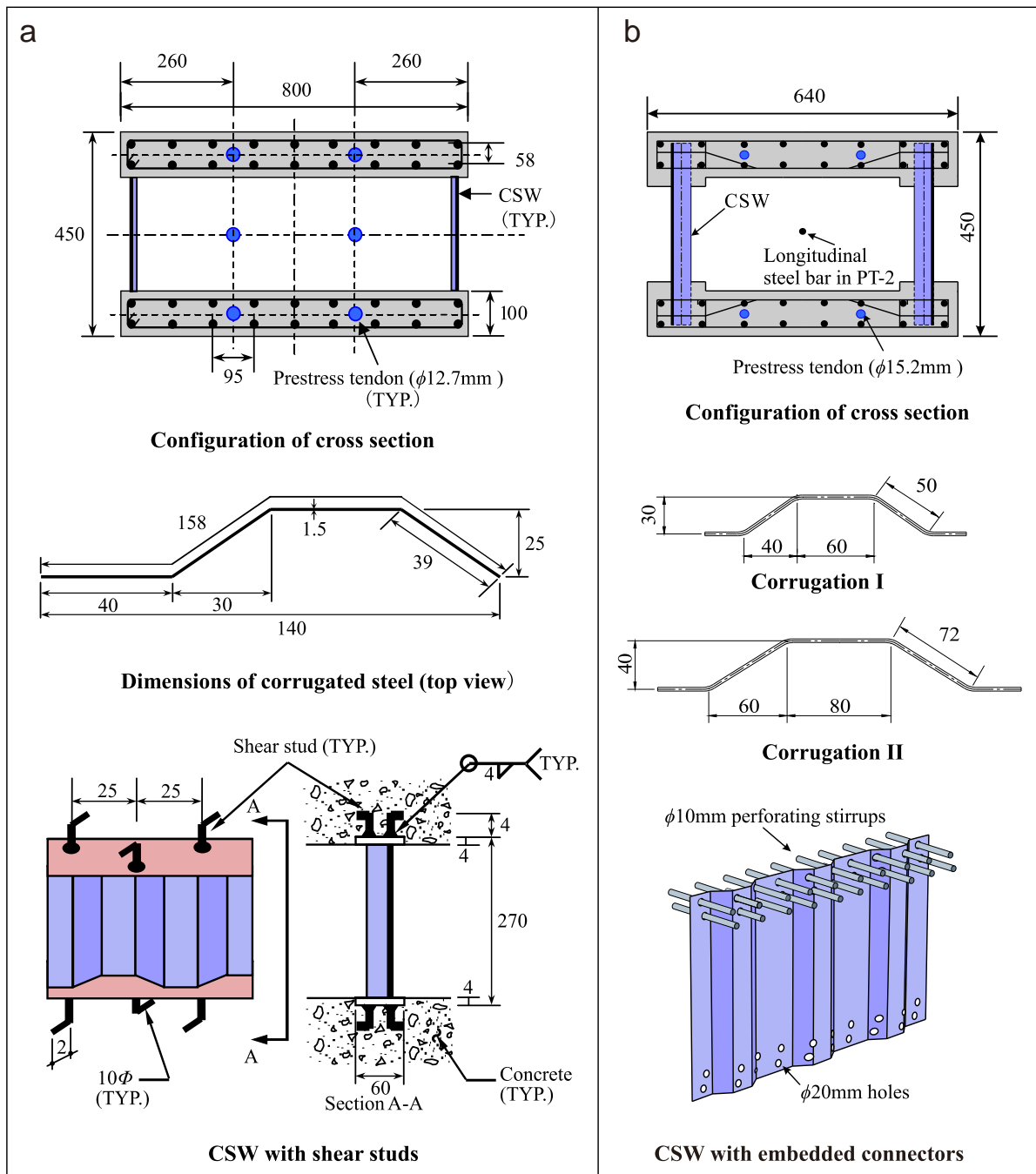
Reference	Girder	Cracking state (Concrete)							Yielding state (CSWs)							Ultimate state (Torque)						
		$T_{cr,exp}$	$T_{cr,s}$	$T_{cr,m}$	$T_{cr,p}$	$\frac{T_{cr,s}}{T_{cr,exp}}$	$\frac{T_{cr,m}}{T_{cr,exp}}$	$\frac{T_{cr,p}}{T_{cr,exp}}$	$T_{wy,exp}$	$T_{wy,s}$	$T_{wy,m}$	$T_{wy,p}$	$\frac{T_{wy,s}}{T_{wy,exp}}$	$\frac{T_{wy,m}}{T_{wy,exp}}$	$\frac{T_{wy,p}}{T_{wy,exp}}$	$T_{u,exp}$	$T_{u,s}$	$T_{u,m}$	$T_{u,p}$	$\frac{T_{u,s}}{T_{u,exp}}$	$\frac{T_{u,m}}{T_{u,exp}}$	$\frac{T_{u,p}}{T_{u,exp}}$
		(kN-m)	(kN-m)	(kN-m)	(kN-m)				(kN-m)	(kN-m)	(kN-m)	(kN-m)				(kN-m)	(kN-m)	(kN-m)	(kN-m)			
Mo et al., 2000	N5 (PE)	60.61	77.24	--	62.66	1.274	--	1.034	146.33	150.09	145.50	152.83	1.026	0.994	1.044	216.63	207.79	219.64	215.51	0.959	1.014	0.995
	N5 (NE)	58.78	77.24	--	62.66	1.314	--	1.066	159.18	150.09	145.50	152.83	0.943	0.914	0.960	214.90	207.79	219.64	215.51	0.967	1.022	1.003
	H5 (PE)	60.61	83.69	--	66.67	1.381	--	1.100	153.06	155.51	147.70	157.62	1.016	0.965	1.030	228.37	216.87	232.35	226.63	0.950	1.017	0.992
	H5 (NE)	59.39	83.69	--	66.67	1.409	--	1.123	151.22	155.51	147.70	157.62	1.028	0.977	1.042	236.33	216.87	232.35	226.63	0.918	0.983	0.959
	N8 (PE)	83.10	93.98	--	76.59	1.131	--	0.922	156.64	163.47	156.54	168.42	1.044	0.999	1.075	234.95	209.27	224.91	222.58	0.891	0.957	0.947
	N8 (NE)	80.65	93.98	--	76.59	1.165	--	0.950	155.19	163.47	156.54	168.42	1.053	1.009	1.085	243.79	209.27	224.91	222.58	0.858	0.923	0.913
	H8 (PE)	65.51	101.57	--	77.43	1.550	--	1.182	148.49	170.29	161.68	174.35	1.147	1.089	1.174	242.74	222.59	238.73	238.89	0.917	0.983	0.984
	H8 (NE)	75.92	101.57	--	77.43	1.338	--	1.020	141.43	170.29	161.68	174.35	1.204	1.143	1.233	256.53	222.59	238.73	238.89	0.868	0.931	0.931
Ding et al., 2013a	PT-1	113.63	100.91	--	97.89	0.888	--	0.861	191.14	192.96	184.54	196.82	1.010	0.965	1.030	200.00	212.27	225.20	220.98	1.061	1.126	1.105
	PT-2	115.60	100.25	--	97.78	0.867	--	0.846	196.61	196.29	186.28	199.94	0.998	0.947	1.017	222.67	218.77	228.67	226.79	0.982	1.027	1.019
	PT-3	113.46	98.25	--	96.75	0.866	--	0.853	164.05	185.87	177.51	189.63	1.133	1.082	1.156	224.36	205.00	218.65	213.71	0.914	0.975	0.953
	PT-4	113.20	87.19	--	91.35	0.770	--	0.807	157.39	150.32	144.96	154.84	0.955	0.921	0.984	180.92	161.30	181.66	170.24	0.892	1.004	0.941
Mean value					1.163	--	0.980						1.046	1.001	1.069					0.931	0.997	0.978
Standard deviation					0.258	--	0.124						0.078	0.070	0.081					0.057	0.053	0.051

Note: *cr*: cracking state; *y*: yielding state; *u*: ultimate state; *exp*: experimental result; *s*: SMMT; *m*: RA-STMT; *p*: proposed model (SMMT-PC); PE: the positive envelope of torque-twist hysteretic loop; NE: the negative envelope of torque-twist hysteretic loop, "--": cannot be predicted.

**Supplementary Table 3.** Comparison of twists from SMMT-PC, SMMT, RA-STMT and experimental results.

Reference	Girder	Cracking state (Concrete)						Yielding state (CSWs)						Ultimate state (Torque)								
		$\theta_{cr, exp}$ (°m)	$\theta_{cr, s}$ (°m)	$\theta_{cr, m}$ (°m)	$\theta_{cr, p}$ (°m)	$\frac{\theta_{cr, s}}{\theta_{cr, exp}}$	$\frac{\theta_{cr, m}}{\theta_{cr, exp}}$	$\frac{\theta_{cr, p}}{\theta_{cr, exp}}$	$\theta_{wy, exp}$ (°m)	$\theta_{wy, s}$ (°m)	$\theta_{wy, m}$ (°m)	$\theta_{wy, p}$ (°m)	$\frac{\theta_{wy, s}}{\theta_{wy, exp}}$	$\frac{\theta_{wy, m}}{\theta_{wy, exp}}$	$\frac{\theta_{wy, p}}{\theta_{wy, exp}}$	$\theta_{u, exp}$ (°m)	$\theta_{u, s}$ (°m)	$\theta_{u, m}$ (°m)	$\theta_{u, p}$ (°m)	$\frac{\theta_{u, s}}{\theta_{u, exp}}$	$\frac{\theta_{u, m}}{\theta_{u, exp}}$	$\frac{\theta_{u, p}}{\theta_{u, exp}}$
Mo et al., 2000	N5 (PE)	0.196	0.057	--	0.045	0.291	--	0.230	0.626	0.616	0.650	0.612	0.984	1.038	0.978	2.564	2.502	2.501	2.566	0.976	0.975	1.001
	N5 (NE)	0.209	0.057	--	0.045	0.273	--	0.215	0.777	0.616	0.650	0.612	0.793	0.837	0.788	2.545	2.502	2.501	2.566	0.983	0.983	1.008
	H5 (PE)	0.222	0.051	--	0.038	0.230	--	0.171	0.646	0.609	0.612	0.603	0.943	0.947	0.933	2.323	2.122	2.785	2.363	0.913	1.199	1.017
	H5 (NE)	0.228	0.051	--	0.038	0.224	--	0.167	0.666	0.609	0.612	0.603	0.914	0.919	0.905	2.512	2.122	2.785	2.363	0.845	1.109	0.941
	N8 (PE)	0.281	0.069	--	0.052	0.246	--	0.185	0.643	0.627	0.649	0.617	0.975	1.009	0.960	2.290	2.357	2.235	2.441	1.029	0.976	1.066
	N8 (NE)	0.274	0.069	--	0.052	0.252	--	0.190	0.639	0.627	0.649	0.617	0.981	1.016	0.966	2.297	2.357	2.235	2.441	1.026	0.973	1.063
	H8 (PE)	0.222	0.059	--	0.037	0.266	--	0.167	0.623	0.618	0.629	0.608	0.992	1.010	0.976	2.227	2.102	2.728	2.507	0.944	1.225	1.126
	H8 (NE)	0.274	0.059	--	0.037	0.215	--	0.135	0.620	0.618	0.629	0.608	0.997	1.015	0.981	2.500	2.102	2.728	2.507	0.841	1.091	1.003
Ding et al., 2013a	PT-1	0.228	0.136	--	0.180	0.596	--	0.789	1.314	0.729	0.753	0.718	0.555	0.573	0.546	1.643	1.859	1.741	1.777	1.131	1.060	1.082
	PT-2	0.269	0.115	--	0.165	0.428	--	0.613	1.059	0.726	0.738	0.718	0.686	0.697	0.678	1.846	1.908	1.853	1.773	1.034	1.004	0.960
	PT-3	0.100	0.125	--	0.182	1.250	--	1.820	0.250	0.723	0.749	0.712	2.892	2.996	2.848	1.155	1.832	1.731	1.792	1.586	1.499	1.552
	PT-4	0.192	0.139	--	0.249	0.724	--	1.297	0.469	0.748	0.774	0.736	1.595	1.650	1.569	1.069	1.553	1.767	1.451	1.453	1.653	1.357
Mean value					0.416	--	0.498					1.109	1.142	1.094					1.063	1.145	1.098	
Standard deviation					0.309	--	0.547					0.614	0.638	0.603					0.230	0.221	0.179	

Note: *cr*: cracking state; *y*: yielding state; *u*: ultimate state; *exp*: experimental result; *s*: SMMT ; *m*: RA-STMT; *p*: proposed model (SMMT-PC); PE: the positive envelope of torque-twist hysteretic loop; NE: the negative envelope of torque-twist hysteretic loop.



**Supplementary Figure 1.** Dimensions and configurations of specimens (unit: mm): (a) Mo et al. (2000); (b) Ding et al. (2013).