Inter-Competitor Licensing and Product Innovation

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WEB APPENDIX

In this Web Appendix, we provide complementary analyses to the paper titled "Inter-Competitor Licensing and Product Innovation". In particular, in Part A, we derive the parameter conditions for the main model such that both firms have non-negative profit in the market. In Part B, we analyze the model when assuming that one of the entrant's quality choices is higher and the other is lower than the incumbent's quality. In Part C, we analyze the model with a different game sequence where the incumbent decides the royalty licensing fee after the entrant's decides its quality. In Part D, we analyze the model when the incumbent endogenously decides its quality with the anticipation of its competitor's entry.

A. Derivations for sufficient parameter condition such that both firms coexist in the market for the main model

According to the proof of Proposition 1, under no licensing contract, the incumbent's and the entrant's prices are $p_i^* = \frac{q_i(q_e-q_i+c_e)+2q_ec_i}{4q_e-q_i}$ and $p_e^* = \frac{q_e(2q_e-2q_i+2c_e+c_i)}{4q_e-q_i}$. To ensure positive demand for each firm, we need $\frac{p_i}{q_i} < \frac{p_e-p_i}{q_e-q_i} < 1$. Plugging in the optimal prices p_e^* and p_i^* for $\frac{p_e-p_i}{q_e-q_i} < 1$ leads to $\frac{q_e-c_e}{q_i-c_i} > \frac{q_e}{2q_e-q_i}$; Plugging in the optimal prices p_e^* and p_i^* for $\frac{p_i}{q_i} < \frac{p_e-p_i}{q_e-q_i}$ leads to $\frac{q_e-c_e}{q_i-c_i} < \frac{2q_e-q_i}{q_i}$. To summarize, to ensure positive demand for each firm, it is required $\frac{q_e}{2q_e-q_i} < \frac{q_e-c_e}{q_i-c_i} < \frac{2q_e-q_i}{q_i}$.

To ensure positive profit margin for each firm, we need $p_i > c_i$ and $p_e > c_e$. Plugging in the optimal prices p_e^* and p_i^* for $p_i > c_i$ leads to $\frac{q_e-c_e}{q_i-c_i} < \frac{2q_e-q_i}{q_i}$; Plugging in the optimal prices p_e^* and p_i^*

for $p_e > c_e$ leads to $\frac{q_e-c_e}{q_i-c_i} > \frac{q_e}{2q_e-q_i}$. To summarize, to ensure positive profit margin for each firm, it is required $\frac{q_e}{2q_e-q_i} < \frac{q_e-c_e}{q_i-c_i} < \frac{2q_e-q_i}{q_i}$.

To ensure non-negative profit for the entrant, $\pi_e^* = \frac{[q_e(2q_e-2q_i+c_i)-(2q_e-q_i)c_e]^2}{(q_e-q_i)(4q_e-q_i)^2} - F \geq 0 \Rightarrow F \leq \max\{\frac{[q_e^H(2q_e^H-2q_i+c_i)-(2q_e^H-q_i)c_e^H]^2}{(q_e^H-q_i)(4q_e^H-q_i)^2}, \frac{[q_e^L(2q_e^L-2q_i+c_i)-(2q_e^L-q_i)c_e^L]^2}{(q_e^L-q_i)(4q_e^L-q_i)^2}\}$.

Hence, in the no licensing case, the parameters need to satisfy $\frac{q_e}{2q_e-q_i} < \frac{q_e-c_e}{q_i-c_i} < \frac{2q_e-q_i}{q_i}$ and $F \leq \max\{\frac{[q_e^H(2q_e^H-2q_i+c_i)-(2q_e^H-q_i)c_e^H]^2}{(q_e^H-q_i)(4q_e^H-q_i)^2}, \frac{[q_e^L(2q_e^L-2q_i+c_i)-(2q_e^L-q_i)c_e^L]^2}{(q_e^L-q_i)(4q_e^L-q_i)^2}\}$.

Under a licensing contract, the incumbent's and the entrant's prices are $p_{ir}^* = \frac{q_i(q_{er}-q_i+c_{er}+3r)+2q_{er}c_i}{4q_{er}-q_i}$ and $p_{er}^* = \frac{q_{er}[2(q_{er}-q_i+c_{er}+r)+c_i]+q_i r}{4q_{er}-q_i}$. To ensure positive demand for each firm, it is required that $r < \min\{\frac{q_{er}(c_i+2q_{er}-2q_i)-(2q_{er}-q_i)c_{er}}{2(q_{er}-q_i)}, \frac{q_{er}[(c_{er}+q_{er}-q_i)q_i-(2q_{er}-q_i)c_i]}{(q_{er}-q_i)q_i}\}$. Note that $\frac{q_{er}(c_i+2q_{er}-2q_i)-(2q_{er}-q_i)c_{er}}{2(q_{er}-q_i)} < \frac{q_{er}[(c_{er}+q_{er}-q_i)q_i-(2q_{er}-q_i)c_i]}{(q_{er}-q_i)q_i}$ if and only if $\frac{q_{er}-c_{er}}{q_i-c_i} < \frac{c_{er}}{c_i}$. To ensure positive profit margin for each firm, it is required that $r > \frac{q_i(q_{er}-c_{er})-(2q_{er}-q_i)(q_i-c_i)}{3q_i}$ (which is reduced to $r \geq 0$ because of the condition $\frac{q_e-c_e}{q_i-c_i} < \frac{2q_e-q_i}{q_i}$ for the no licensing case) and $r < \frac{q_{er}(c_i+2q_{er}-2q_i)-(2q_{er}-q_i)c_{er}}{2(q_{er}-q_i)}$. Note that non-negative profit for the entrant is guaranteed by $\overline{r}$. The condition of $\overline{r}$ dominates $r < \frac{q_{er}(c_i+2q_{er}-2q_i)-(2q_{er}-q_i)c_{er}}{2(q_{er}-q_i)}$.

Hence, in the case with a licensing contract, the parameters need to satisfy: $\frac{q_{er}-c_{er}}{q_i-c_i} < \frac{c_{er}}{c_i}$ and $0 < r \leq \overline{r}$; or $\frac{q_{er}-c_{er}}{q_i-c_i} > \frac{c_{er}}{c_i}$ and $0 < r < \min\{\overline{r}, \frac{q_{er}[(c_{er}+q_{er}-q_i)q_i-(2q_{er}-q_i)c_i]}{(q_{er}-q_i)q_i}\}$.

Therefore, combining with the condition in the no licensing case, one set of sufficient parameter conditions is: $\frac{q_e}{2q_e-q_i} < \frac{q_e-c_e}{q_i-c_i} < \min\{\frac{2q_e-q_i}{q_i}, \frac{c_e}{c_i}\}$ and
$F \leq \max\{\frac{[q_e^H(2q_e^H-2q_i+c_i)-(2q_e^H-q_i)c_e^H]^2}{(q_e^H-q_i)(4q_e^H-q_i)^2}, \frac{[q_e^L(2q_e^L-2q_i+c_i)-(2q_e^L-q_i)c_e^L]^2}{(q_e^L-q_i)(4q_e^L-q_i)^2}\}$.

Next, we show that with this set of sufficient parameter conditions, no firm will deviate from the equilibrium characterized in the paper.

First, we show that $(p_{er}, p_{ir}) = (p_{em}, \infty)$ (i.e., the incumbent sets its price very high, let the entrant be the monopoly in the market and the incumbent's profit only comes from the licensing fee) cannot be an equilibrium, where $p_{em} = argmax_{p_{em}}(p_{em} - c_{em} - r)(1 - \frac{p_{em}}{q_{em}}) = \frac{q_{em}+c_{em}+r}{2}$. When either the entrant or the incumbent is better off with deviating from (p_{em}, ∞), then it is not an equilibrium.

The incumbent will deviate from (p_{em}, ∞) if $\pi_{ir}(p_{em}, p'_{ir}) > \pi_{ir}(p_{em}, \infty) = r(1 - \frac{p_{em}}{q_{em}}) = r\frac{q_{em}-c_{em}-r}{2q_{em}}$. Note that given $p_{er} = p_{em}$, the incumbent's best response is given by $p'_{ir} = argmax_{p_{ir}}(p_{ir} - c_i)(\frac{p_{em}-p_{ir}}{q_{em}-q_i} - \frac{p_{ir}}{q_i}) + r(1 - \frac{p_{em}-p_{ir}}{q_{em}-q_i}) = \frac{q_i(p_{em}+r)+q_{em}c_i}{2q_{em}}$, and the incumbent's corresponding profit is $\pi_{ir}(p_{em}, p'_{ir}) = \frac{4c_i^2q_{em}^2-4c_iq_{em}q_i(c_{em}+q_{em}-r)+q_i[c_{em}^2q_i+(q_{em}-r)(q_{em}q_i+8q_{em}r-9q_ir)+2c_{em}(q_{em}q_i-4q_{em}r+3q_ir)]}{16q_{em}(q_{em}-q_i)q_i}$. Note that $\pi_{ir}(p_{em}, p'_{ir}) = \pi_{ir}(p_{em}, \infty)$ if $r = q_{em} + c_{em} - \frac{2c_iq_{em}}{q_i}$ and $\pi_{ir}(p_{em}, p'_{ir}) > \pi_{ir}(p_{em}, \infty)$ otherwise. If $\frac{q_{em}-c_{em}}{q_i-c_i} < \frac{c_{em}}{c_i}$ (which comes from the set of parameter condition derived above), that is, if $\frac{q_{em}}{c_{em}} < \frac{q_i}{c_i}$, then $q_{em} + c_{em} - \frac{2c_iq_{em}}{q_i} > q_{em} - c_{em}$, which means when the licensing fee is higher than $q_{em} - c_{em}$ the incumbent will be indifferent. Hence, $\pi_{ir}(p_{em}, p'_{ir}) > \pi_{ir}(p_{em}, \infty)$. So $(p_{er}, p_{ir}) = (p_{em}, \infty)$ cannot be an equilibrium. Therefore, the incumbent setting its price extremely high and letting the entrant be the monopoly cannot be an equilibrium.

Second, we show that given p_i^*, the entrant will not be better off with deviating to price-out the incumbent. If the entrant sets its price $p'_e = p_i^*\frac{q_e}{q_i}$, then the entrant can price out the incumbent and the entrant's profit is $\pi'_e = (p_i^*\frac{q_e}{q_i} - c_e)(1 - \frac{p_i^*}{q_i}) = \frac{\{2c_iq_e^2+q_i[q_e(q_e-q_i)-c_e(3q_e-q_i)]\}[q_i(3q_e-c_e)-2q_ec_i]}{q_i(4q_e-q_i)^2}$. The entrant will only deviate from (p_i^*, p_e^*) to (p_i^*, p'_e) when making higher profit, i.e., when $\pi'_e > \pi_e^*$. Note that $\pi'_e = \pi_e^*$ if $c_i = \frac{q_i(q_e-q_i+c_e)}{2q_e-q_i}$ and $\pi'_e < \pi_e^*$ otherwise. Hence, for given p_i^*, the entrant will not be better off with deviating to price-out the incumbent.

Last, we show that given p_e^*, the incumbent will not be better off with deviating to price-out the entrant. If the incumbent sets its price $p'_i = p_e^* - q_e + q_i$, then the incumbent can price out the entrant and the incumbent's profit is $\pi'_i = (p'_i - c_i)(1 - \frac{p'_i}{q_i}) = \frac{q_e(2q_e+q_i-2c_e-c_i)(2q_ec_e+3q_eq_i+q_ic_i-2q_e^2-q_i^2-3c_iq_e)}{q_i(4q_e-q_i)^2}$. The incumbent will only deviate from (p_i^*, p_e^*) to (p'_i, p_e^*) when making higher profit, i.e., when $\pi'_i > \pi_i^*$.

Note that $\pi_i' = \pi_i^*$ if $c_i = q_i(2 - \frac{c_e}{q_e}) - (2q_e - 2c_e)$ and $\pi_i' < \pi_i^*$ otherwise. Hence, for given p_e^*, the incumbent will not be better off with deviating to price-out the entrant.

Therefore, the set of parameter conditions $\frac{q_e}{2q_e - q_i} < \frac{q_e - c_e}{q_i - c_i} < \min\{\frac{2q_e - q_i}{q_i}, \frac{c_e}{c_i}\}$ and $F \leq \max\{\frac{[q_e^H(2q_e^H - 2q_i + c_i) - (2q_e^H - q_i)c_e^H]^2}{(q_e^H - q_i)(4q_e^H - q_i)^2}, \frac{[q_e^L(2q_e^L - 2q_i + c_i) - (2q_e^L - q_i)c_e^L]^2}{(q_e^L - q_i)(4q_e^L - q_i)^2}\}$ is sufficient to ensure two firms co-exist in the market.

B. Analysis for the model with $q_e^H > q_i > q_e^L$ and $c_e^H > c_i > c_e^L$

In this part of the Web Appendix, we allow the entrant's quality choices and associated costs are such that $q_e^H > q_i > q_e^L$ and $c_e^H > c_i > c_e^L$. In the following, we first analyze the case when the entrant develops the non-core technology on its own, i.e., without a licensing contract; then we analyze the case when there is a licensing contract between the two competitors; last, we compare the entrant's optimal quality in the two cases to examine the effect of licensing on the entrant's product innovation.

Without a licensing contract, if the entrant chooses $(q_e, c_e) = (q_e^H, c_e^H)$, then $D_e^H = 1 - \frac{p_e^H - p_i^H}{q_e^H - q_i}$ and $D_i^H = \frac{p_e^H - p_i^H}{q_e^H - q_i} - \frac{p_i^H}{q_i}$. The entrant's and the incumbent's profit functions are $\pi_e^H = (p_e^H - c_e^H)D_e^H - F$ and $\pi_i^H = (p_i^H - c_i)D_i^H$, respectively. We solve the game based on backward induction. Since π_e^H is a concave function of p_e^H and π_i^H is a concave function of p_i^H, simultaneously solving the first order conditions $\frac{d\pi_e^H}{dp_e^H} = 0$ and $\frac{d\pi_i^H}{dp_i^H} = 0$ gives: $p_e^{H*} = \frac{q_e^H(2q_e^H - 2q_i + 2c_e^H + c_i)}{4q_e^H - q_i}$ and $p_i^{H*} = \frac{q_i(q_e^H - q_i + c_e^H) + 2q_e^H c_i}{4q_e^H - q_i}$. Two firms' profits are: $\pi_e^{H*} = \frac{[q_e^H(2q_e^H - 2q_i + c_i) - (2q_e^H - q_i)c_e^H]^2}{(q_e^H - q_i)(4q_e^H - q_i)^2} - F$ and $\pi_i^{H*} = \frac{q_e^H[q_i(q_e^H - q_i + c_e^H) - (2q_e^H - q_i)c_i]^2}{q_i(q_e^H - q_i)(4q_e^H - q_i)^2}$.

Alternatively, if the entrant chooses $(q_e, c_e) = (q_e^L, c_e^L)$, then $D_e^L = \frac{p_i^L - p_e^L}{q_i - q_e^L} - \frac{p_e^L}{q_e^L}$ and $D_i^L = 1 - \frac{p_i^L - p_e^L}{q_i - q_e^L}$. The entrant's and the incumbent's profit functions are $\pi_e^L = (p_e^L - c_e^L)D_e^L - F$ and $\pi_i^L = (p_i^L - c_i)D_i^L$, respectively. Since π_e^L is a concave function of p_e^L and π_i^L is a concave function of p_i^L, simultaneously solving the first order conditions $\frac{d\pi_e^L}{dp_e^L}=0$ and $\frac{d\pi_i^L}{dp_i^L}=0$ gives: $p_e^{L*}=\frac{q_e^L(q_i-q_e^L+c_i)+2q_ic_e^L}{4q_i-q_e^L}$, $p_i^{L*}=\frac{q_i(2q_i-2q_e^L+2c_i+c_e^L)}{4q_i-q_e^L}$. Two firms' profits are: $\pi_e^{L*}=\frac{q_i[q_e^L(q_i-q_e^L+c_i)-(2q_i-q_e^L)c_e^L]^2}{q_e^L(q_i-q_e^L)(4q_i-q_e^L)^2}-F$ and $\pi_i^{L*}=\frac{[q_i(2q_i-2q_e^L+c_e^L)-(2q_ei-q_e^L)c_i]^2}{(q_i-q_e^L)(4q_i-q_e^L)^2}$.

Hence, in the case of no licensing contract, the entrant's optimal quality decision is $q_e^*=q_e^H$ if $\pi_e^{H*}\geq\pi_e^{L*}$, that is, if $c_e^H\leq c_F=\frac{q_e^H(2q_e^H-2q_i+c_i)-R[q_e^L(q_i-q_e^L+c_i)-(2q_i-q_e^L)c_e^L]}{2q_e^H-q_i}$ where $R=\sqrt{\frac{q_i(q_e^H-q_i)}{q_e^L(q_i-q_e^L)}}\frac{4q_e^H-q_i}{4q_i-q_e^L}$; $q_e^*=q_e^L$ otherwise.

In the case of a licensing contract with royalty fee $r>0$, if the entrant chooses $(q_{er},c_{er})=(q_e^H,c_e^H)$, then $D_{er}^H=1-\frac{p_{er}^H-p_{ir}^H}{q_e^H-q_i}$ and $D_{ir}^H=\frac{p_{er}^H-p_{ir}^H}{q_e^H-q_i}-\frac{p_{ir}^H}{q_i}$. The entrant's and the incumbent's profit functions are $\pi_{er}^H=(p_{er}^H-c_e^H)D_{er}^H-r\times D_{er}^H$ and $\pi_{ir}^H=(p_{ir}^H-c_i)D_{ir}^H+r\times D_{er}^H$, respectively. Simultaneously solving the first order conditions $\frac{d\pi_{er}^H}{dp_{er}^H}=0$ and $\frac{d\pi_{ir}^H}{dp_{ir}^H}=0$ gives: $p_{ir}^{H*}=\frac{q_i(q_e^H-q_i+c_e^H+3r)}{4q_e^H-q_i}$ and $p_{er}^{H*}=\frac{q_e^H[2(q_e^H-q_i+c_e^H+r)+c_i]+q_ir}{4q_e^H-q_i}$. Two firms' profits are: $\pi_{er}^{H*}(r)=\frac{[q_e^H(2q_e^H-2q_i+c_i)-c_e^H(2q_e^H-q_i)-r(2q_e^H-2q_i)]^2}{(q_e^H-q_i)(4q_e^H-q_i)^2}$ and $\pi_{ir}^{H*}(r)=\frac{q_e^H[q_i(q_e^H-q_i+c_e^H)-c_i(2q_e^H-q_i)]^2+q_i(q_e^H-q_i)[q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i]r-(8q_e^H+q_i)(q_e^H-q_i)q_ir^2}{q_i(q_e^H-q_i)(4q_e^H-q_i)^2}$.

Alternatively, if the entrant chooses $(q_{er},c_{er})=(q_e^L,c_e^L)$, then $D_{er}^L=\frac{p_{ir}^L-p_{er}^L}{q_i-q_e^L}-\frac{p_{er}^L}{q_e^L}$ and $D_{ir}^L=1-\frac{p_{ir}^L-p_{er}^L}{q_i-q_e^L}$. The entrant's and the incumbent's profit functions are $\pi_{er}^L=(p_{er}^L-c_e^L)D_{er}^L-r\times D_{er}^L$ and $\pi_{ir}^L=(p_{ir}^L-c_i)D_{ir}^L+r\times D_{er}^L$, respectively. Simultaneously solving the first order conditions $\frac{d\pi_{er}^L}{dp_{er}^L}=0$ and $\frac{d\pi_{ir}^L}{dp_{ir}^L}=0$ gives: $p_{er}^{L*}=\frac{q_e^L(q_i-q_e^L+c_i)+2q_ic_e^L+(2q_i+q_e^L)r}{4q_i-q_e^L}$ and $p_{ir}^*(q_e^L)=\frac{q_i(2q_i-2q_e^L+2c_i+c_e^L+3r}{4q_i-q_e^L}$. Two firms' profits are: $\pi_{er}^{L*}(r)=\frac{q_i[q_e^L(q_i-q_e^L+c_i)-(2q_i-q_e^L)c_e^L-(2q_i-2q_e^L)r]^2}{q_e^L(q_i-q_e^L)(4q_i-q_e^L)^2}$ and
$\pi_{ir}^{L*}(r)=\frac{[q_i(2q_i-2q_e^L+c_e^L)-(2q_i-q_e^L)c_i]^2+(q_i-q_e^L)[(q_i-c_i)(q_e^L)^2+8(q_e^L-c_e^L)q_i^2]r-q_i(q_i-q_e^L)(8q_i+q_e^L)r^2}{(q_i-q_e^L)(4q_i-q_e^L)^2}$.

Hence, given a licensing contract with royalty fee $r>0$, the entrant's optimal quality decision is $q_{er}^*=q_e^H$ if $\pi_{er}^{H*}(r)\geq\pi_{er}^{L*}(r)$, that is, $c_e^H\leq c_r=c_F+2r\frac{R(q_i-q_e^L)-(q_e^H-q_i)}{2q_e^H-q_i}$; and $q_{er}^*=q_e^L$ otherwise. Therefore, if $R(q_i-q_e^L)-(q_e^H-q_i)>0$, then $c_r>c_F$ and licensing can increase the entrant's optimal quality. If $R(q_i-q_e^L)-(q_e^H-q_i)<0$, then $c_r<c_F$ and licensing can decrease the entrant's optimal

quality. Next, we show the parameter conditions under which licensing can increase or decrease the entrant's optimal quality.

Let $\alpha \equiv \frac{q_e^H}{q_i}$ and $\beta \equiv \frac{q_e^L}{q_i}$ where $\alpha > 1 > \beta > 0$. Then $R(q_i - q_e^L) - (q_e^H - q_i) > 0$ is equivalent to $\frac{4\alpha-1}{4-\beta} > \sqrt{\frac{\beta(\alpha-1)}{1-\beta}}$. Solving the inequality $\frac{4\alpha-1}{4-\beta} > \sqrt{\frac{\beta(\alpha-1)}{1-\beta}}$ leads to the following solution: When $\beta < \frac{2}{3}(4 - \frac{32}{(27\sqrt{73}-143)^{1/3}} + (27\sqrt{73} - 143)^{1/3}) \approx 0.82658$ and $\alpha > 1$; or when $\beta > 0.82658$ and $\alpha > \frac{(4-\beta)^2}{64}[\sqrt{\frac{\beta}{1-\beta}} + \sqrt{\frac{\beta}{1-\beta} - \frac{48}{(4-\beta)^2}}]^2 + 1$ or $\alpha < \frac{(4-\beta)^2}{64}[\sqrt{\frac{\beta}{1-\beta}} - \sqrt{\frac{\beta}{1-\beta} - \frac{48}{(4-\beta)^2}}]^2 + 1$, then $\frac{4q_e^H - q_i}{4q_i - q_e^L} > \sqrt{\frac{q_e^L(q_e^H - q_i)}{q_i(q_i - q_e^L)}}$ holds (i.e., $c_r > c_F$ holds) and licensing can increase the entrant's optimal quality.

Accordingly, when $\beta > 0.82658$ and $\frac{(4-\beta)^2}{64}[\sqrt{\frac{\beta}{1-\beta}} - \sqrt{\frac{\beta}{1-\beta} - \frac{48}{(4-\beta)^2}}]^2 + 1 < \alpha < \frac{(4-\beta)^2}{64}[\sqrt{\frac{\beta}{1-\beta}} + \sqrt{\frac{\beta}{1-\beta} - \frac{48}{(4-\beta)^2}}]^2 + 1$, then $\frac{4\alpha-1}{4-\beta} < \sqrt{\frac{\beta(\alpha-1)}{1-\beta}}$ holds, which means that $\frac{4q_e^H - q_i}{4q_i - q_e^L} < \sqrt{\frac{q_e^L(q_e^H - q_i)}{q_i(q_i - q_e^L)}}$ holds (i.e., $c_r < c_F$ holds) and licensing can decrease the entrant's optimal quality.

Given the entrant's optimal response of its quality decision, the incumbent optimally decides its royalty fee by maximizing its resulting profit function subject to the constraint that the contract is mutually acceptable. We follow the same procedure of the derivation of $\bar{r}$ in the proof of Proposition 2 to derive the upper bound of royalty fee such that the entrant is (weakly) better off with accepting the licensing contract than developing its own non-core technology. Specifically, when $c_r > c_F$, $\bar{r}$ is given by solving the following problem:

$$\begin{cases} \pi_{er}^{H*}(r) \geq \pi_e^{H*}, & \text{if } c_e^L < c_e^H \leq c_F; \\ \pi_{er}^{H*}(r) \geq \pi_e^{L*}, & \text{if } c_F < c_e^H \leq c_r; \\ \pi_{er}^{L*}(r) \geq \pi_e^{L*}, & \text{if } c_e^H > c_r. \end{cases} \tag{W1}$$

Hence, the entrant will accept the royalty licensing contract if $r \leq \bar{r}$, where

$$\bar{r} = \begin{cases} \bar{r}^{HH} \equiv \frac{q_e^H(2q_e^H - 2q_i + c_i) - (2q_e^H - q_i)c_e^H - (4q_e^H - q_i)\sqrt{(q_e^H - q_i)\pi_e^{H*}}}{2q_e^H - 2q_i}, & \text{if } c_e^L < c_e^H \leq c_F; \\ \bar{r}^{LH} \equiv \frac{q_e^H(2q_e^H - 2q_i + c_i) - (2q_e^H - q_i)c_e^H - (4q_e^H - q_i)\sqrt{(q_e^H - q_i)\pi_e^{L*}}}{2q_e^H - 2q_i}, & \text{if } c_F < c_e^H \leq c_r; \\ \bar{r}^{LL} \equiv \frac{q_e^L(q_i - q_e^L + c_i) - (2q_i - q_e^L)c_e^L - (4q_i - q_e^L)\sqrt{q_e^L(q_i - q_e^L)\pi_e^{L*}/q_i}}{2q_i - 2q_e^L}, & \text{if } c_e^H > c_r. \end{cases} \tag{W2}$$

When $c_r < c_F$, $\bar{r}$ is given by solving the following problem:

$$\begin{cases} \pi_{er}^{H*}(r) \geq \pi_e^{H*}, & \text{if } c_e^L < c_e^H \leq c_r; \\ \pi_{er}^{L*}(r) \geq \pi_e^{H*}, & \text{if } c_r < c_e^H \leq c_F; \\ \pi_{er}^{L*}(r) \geq \pi_e^{L*}, & \text{if } c_e^H > c_F. \end{cases} \tag{W3}$$

Hence, the entrant will accept the royalty licensing contract if $r \leq \bar{r}$, where

$$\bar{r} = \begin{cases} \bar{r}^{HH}, & \text{if } c_e^L < c_e^H \leq c_r; \\ \bar{r}^{HL} \equiv \frac{q_e^L(q_i - q_e^L + c_i) - (2q_i - q_e^L)c_e^L - (4q_i - q_e^L)\sqrt{q_e^L(q_i - q_e^L)\pi_e^{H*}/q_i}}{2q_i - 2q_e^L}, & \text{if } c_r < c_e^H \leq c_F; \\ \bar{r}^{LL}, & \text{if } c_e^H > c_F. \end{cases} \tag{W4}$$

Following the same procedure as in the proof of Proposition 5, we next derive the incumbent's optimal royalty fee r^*.

Anticipating the entrant's best response on quality decision, the incumbent's resulting profit function is

$$\pi_{ir} = \begin{cases} \pi_{ir}^H(r) \equiv \pi_{ir}^*(q_e^H, c_e^H, r), & \text{if } c_e^H \leq c_r; \\ \pi_{ir}^L(r) \equiv \pi_{ir}^*(q_e^L, c_e^L, r), & \text{if } c_e^H > c_r; \end{cases} \tag{W5}$$

where both $\pi_{ir}^H(r)$ and $\pi_{ir}^L(r)$ are concave in r.

When $c_r > c_F$, then the incumbent maximizes its profit in (W5) subject to the constraint $0 < r \leq \bar{r}$, where $\bar{r}$ is defined in (W2), and obtains its optimal royalty fee r^*:

$$r^* = \begin{cases} \min\{\frac{8q_e^H(q_e^H - c_e^H) + q_i(q_i - c_i)}{2(8q_e^H + q_i)}, \overline{r}^{HH}\}, & \text{if } c_e^H \leq c_F; \\ r^{LH*} \equiv \max\{r_c, \min\{\frac{8q_e^H(q_e^H - c_e^H) + q_i(q_i - c_i)}{2(8q_e^H + q_i)}, \overline{r}^{LH}\}\}, & \text{if } c_F < c_e^H \leq \overline{c}_r \;\&\; \pi_{ir}^H(r^{LH*}) \geq \pi_{ir}^L(r^{LL*}); \\ r^{LL*} \equiv \min\{\frac{8q_i^2(q_e^L - c_e^L) + (q_e^L)^2(q_i - c_i)}{2q_i(8q_i + q_e^L)}, r_c\}, & \text{if } c_F < c_e^H \leq \overline{c}_r \;\&\; \pi_{ir}^H(r^{LH*}) < \pi_{ir}^L(r^{LL*}); \\ \min\{\frac{8q_i^2(q_e^L - c_e^L) + (q_e^L)^2(q_i - c_i)}{2q_i(8q_i + q_e^L)}, \overline{r}^{LL}\}, & \text{if } c_e^H > \overline{c}_r; \end{cases} \tag{W6}$$

where $r_c \equiv \frac{(2q_e^H - q_i)(c_e^H - c_F)}{R(2q_i - 2q_e^L) - (2q_e^H - 2q_i)}$ is the alternative form of writing $c_e^H = c_r$, and $\overline{c}_r \equiv c_r|_{r=\overline{r}^{LL}}$. Note that, if $c_F < c_e^H \leq \overline{c}_r$ & $\pi_{ir}^H(r^{LH*}) \geq \pi_{ir}^L(r^{LL*})$, the incumbent's optimal royalty fee is r^{LH*} and $q_{er}^* = q_e^H > q_e^L = q_e^*$.

When $c_r < c_F$, then the incumbent maximizes its profit in (W5) subject to the constraint $0 < r \leq \bar{r}$, where $\bar{r}$ is defined in (W4), and obtains its optimal royalty fee r^*:

$$r^* = \begin{cases} \min\{\frac{8q_e^H(q_e^H - c_e^H) + q_i(q_i - c_i)}{2(8q_e^H + q_i)}, \overline{r}^{HH}\}, & \text{if } c_e^H \leq \underline{c}_r; \\ r^{HL*} \equiv \max\{r_c, \min\{\frac{8q_i^2(q_e^L - c_e^L) + (q_e^L)^2(q_i - c_i)}{2q_i(8q_i + q_e^L)}, \overline{r}^{HL}\}\}, & \text{if } \underline{c}_r < c_e^H \leq c_F \;\&\; \pi_{ir}^L(r^{HL*}) \geq \pi_{ir}^H(r^{HH*}); \\ r^{HH*} \equiv \min\{\frac{8q_e^H(q_e^H - c_e^H) + q_i(q_i - c_i)}{2(8q_e^H + q_i)}, r_c\}, & \text{if } \underline{c}_r < c_e^H \leq c_F \;\&\; \pi_{ir}^L(r^{HL*}) < \pi_{ir}^H(r^{HH*}); \\ \min\{\frac{8q_i^2(q_e^L - c_e^L) + (q_e^L)^2(q_i - c_i)}{2q_i(8q_i + q_e^L)}, \overline{r}^{LL}\}, & \text{if } c_e^H > c_F; \end{cases} \tag{W7}$$

where $r_c \equiv \frac{(2q_e^H - q_i)(c_F - c_e^H)}{(2q_e^H - 2q_i) - R(2q_i - 2q_e^L)}$ is the alternative form of writing $c_e^H = c_r$, and $\underline{c}_r$ is the c_e^H such that $c_r = \overline{r}^{HL}$. Note that, if $\underline{c}_r < c_e^H \leq c_F$ & $\pi_{ir}^L(r^{HL*}) \geq \pi_{ir}^H(r^{HH*})$, the incumbent's optimal royalty fee is r^{HL*} and $q_{er}^* = q_e^L < q_e^H = q_e^*$.

C. Analysis for the model with a different game sequence

In this part of the Web Appendix, the game sequence is defined as follows: first, the entrant decides its product quality; second, the incumbent sets its royalty licensing fee; third, the entrant decides to accept or not accept the contract; last, two firms set prices. Except this game sequence, the other aspects of model setup are the same as in the main model. In particular, $q_e^H > q_e^L > q_i$, $c_e^H > c_e^L > c_i$, and the entrant incurs a fixed cost F when developing the non-core technology on its own. In the following, we first solve the game where the entrant develops the non-core technology on its own, i.e., without a licensing contract; then we solve the game where there is a licensing contract between the two competitors; last, we compare the entrant's optimal quality in the two cases to examine the effect of licensing on the entrant's product innovation. The game solving is based on backward induction.

When the entrant develops the non-core technology on its own, the entrant's optimal product quality, two firms' optimal prices and their corresponding profits are the same as in the benchmark case in the main model. That is, $q_e^* = q_e^H$ with the incumbent's and entrant's optimal profits π_i^{H*} and π_e^{H*} if $c_e^H \leq c_F$ and $q_e^* = q_e^L$ with the incumbent's and entrant's optimal profits π_i^{L*} and π_e^{L*} otherwise.

When the entrant anticipates an acceptable licensing contract from the incumbent, there are two sub-games based on the entrant's possible quality decision: $q_{er} = q_e^H$ or $q_{er} = q_e^L$.

Sub-game 1: $q_{er} = q_e^H$

Given p_{er}^H, p_{ir}^H, $q_{er} = q_e^H$, q_i, and r^H, the incumbent's and the entrant's profit functions are $\pi_{ir}^H = (p_{ir}^H - c_i)(\frac{p_{er}^H - p_{ir}^H}{q_e^H - q_i} - \frac{p_{ir}^H}{q_i}) + r^H(1 - \frac{p_{er}^H - p_{ir}^H}{q_e^H - q_i})$ and $\pi_{er}^H = (p_{er}^H - c_e^H - r^H)(1 - \frac{p_{er}^H - p_{ir}^H}{q_e^H - q_i})$, respectively. Solving the first order conditions $\frac{d\pi_{ir}^H}{dp_{ir}^H} = 0$ and $\frac{d\pi_{er}^H}{dp_{er}^H} = 0$ simultaneously gives the optimal prices:

$p_{ir}^{H*} = \frac{q_i(q_e^H - q_i + c_e^H + 3r^H)}{4q_e^H - q_i}$ and $p_{er}^{H*} = \frac{q_e^H[2(q_e^H - q_i + c_e^H + r^H) + c_i] + q_i r^H}{4q_e^H - q_i}$. The corresponding profits are:

$\pi_{ir}^H(r^H) = \frac{q_e^H[q_i(q_e^H - q_i + c_e^H) - c_i(2q_e^H - q_i)]^2 + q_i(q_e^H - q_i)[q_i^2 + 8q_e^H(q_e^H - c_e^H) - c_i q_i]r^H - (8q_e^H + q_i)(q_e^H - q_i)q_i(r^H)^2}{q_i(q_e^H - q_i)(4q_e^H - q_i)^2}$ and

$\pi_{er}^H(r^H) = \frac{[q_e^H(2q_e^H - 2q_i + c_i) - c_e^H(2q_e^H - q_i) - r^H(2q_e^H - 2q_i)]^2}{(q_e^H - q_i)(4q_e^H - q_i)^2}$.

Next, we solve for the incumbent's optimal licensing fee in this sub-game, which we denote as r^{H*}. Specifically, $r^{H*} = argmax_{r^H} \pi_{ir}^H(r^H)$ s.t. $\pi_{er}^H(r^H) \geq \pi_e^{H*}$, $\pi_{ir}^H(r^H) \geq \pi_i^{H*}$. Constraint $\pi_{er}^H(r^H) \geq \pi_e^{H*}$ is reduced to $r^H \leq \frac{q_e^H(2q_e^H - 2q_i + c_i) - c_e^H(2q_e^H - q_i) - (4q_e^H - q_i)\sqrt{(q_e^H - q_i)\pi_e^{H*}}}{2q_e^H - 2q_i} = \bar{r}^{HH}$; and $\pi_{ir}^H(r^H) \geq \pi_i^{H*}$ is reduced to $r^H \geq 0$. Therefore, given $q_{er} = q_e^H$, the optimal licensing fee is $r^{H*} = \min\{\bar{r}^{HH}, \frac{q_i^2 + 8q_e^H(q_e^H - c_e^H) - c_i q_i}{2(8q_e^H + q_i)}\}$. And the entrant's corresponding profit is

$\pi_{er}^{H*} = \frac{[q_e^H(2q_e^H - 2q_i + c_i) - c_e^H(2q_e^H - q_i) - r^{H*}(2q_e^H - 2q_i)]^2}{(q_e^H - q_i)(4q_e^H - q_i)^2}$.

Sub-game 2: $q_{er} = q_e^L$

Given p_{er}^L, p_{ir}^L, $q_{er} = q_e^L$, q_i, and r^L, the incumbent's and the entrant's profit functions are $\pi_{ir}^L = (p_{ir}^L - c_i)(\frac{p_{er}^L - p_{ir}^L}{q_e^L - q_i} - \frac{p_{ir}^L}{q_i}) + r^L(1 - \frac{p_{er}^L - p_{ir}^L}{q_e^L - q_i})$ and $\pi_{er}^L = (p_{er}^L - c_e^L - r^L)(1 - \frac{p_{er}^L - p_{ir}^L}{q_e^L - q_i})$, respectively. Solving the first order conditions $\frac{d\pi_{ir}^L}{dp_{ir}^L} = 0$ and $\frac{d\pi_{er}^L}{dp_{er}^L} = 0$ simultaneously gives the optimal prices: $p_{ir}^{L*} = \frac{q_i(q_e^L - q_i + c_e^L + 3r^L)}{4q_e^L - q_i}$ and $p_{er}^{L*} = \frac{q_e^L[2(q_e^L - q_i + c_e^L + r^L) + c_i] + q_i r^L}{4q_e^L - q_i}$. The corresponding profits are: $\pi_{ir}^L(r^L) = \frac{q_e^L[q_i(q_e^L - q_i + c_e^L) - c_i(2q_e^L - q_i)]^2 + q_i(q_e^L - q_i)[q_i^2 + 8q_e^L(q_e^L - c_e^L) - c_i q_i]r^L - (8q_e^L + q_i)(q_e^L - q_i)q_i(r^L)^2}{q_i(q_e^L - q_i)(4q_e^L - q_i)^2}$ and $\pi_{er}^L(r^L) = \frac{[q_e^L(2q_e^L - 2q_i + c_i) - c_e^L(2q_e^L - q_i) - r}{(q_e^L - q_i)(4q_e^L - q_i)^2}$

Next, we solve for the incumbent's optimal licensing fee in this sub-game, which we denote as r^{L*}. Specifically, $r^{L*} = argmax_{r^L} \pi_{ir}^L(r^L)$ s.t. $\pi_{er}^L(r^L) \geq \pi_e^{L*}$, $\pi_{ir}^L(r^L) \geq \pi_i^{L*}$. Constraint $\pi_{er}^L(r^L) \geq \pi_e^{L*}$ is reduced to $r^L \leq \frac{q_e^L(2q_e^L - 2q_i + c_i) - c_e^L(2q_e^L - q_i) - (4q_e^L - q_i)\sqrt{(q_e^L - q_i)\pi_e^{L*}}}{2q_e^L - 2q_i} = \bar{r}^{LL}$; and $\pi_{ir}^L(r^L) \geq \pi_i^{L*}$ is reduced to $r^L \geq 0$. Therefore, given $q_{er} = q_e^L$, the optimal licensing fee is $r^{L*} = \min\{\bar{r}^{LL}, \frac{q_i^2 + 8q_e^L(q_e^L - c_e^L) - c_i q_i}{2(8q_e^L + q_i)}\}$. And the entrant's corresponding profit is $\pi_{er}^{L*} = \frac{[q_e^L(2q_e^L - 2q_i + c_i) - c_e^L(2q_e^L - q_i) - r^{L*}(2q_e^L - 2q_i)]^2}{(q_e^L - q_i)(4q_e^L - q_i)^2}$.

Last, we solve for the entrant's optimal quality decision by comparing the entrant's profit $\pi_{er}^{H*} = \frac{[q_e^H(2q_e^H - 2q_i + c_i) - c_e^H(2q_e^H - q_i) - r^{H*}(2q_e^H - 2q_i)]^2}{(q_e^H - q_i)(4q_e^H - q_i)^2}$ to its profit $\pi_{er}^{L*} = \frac{[q_e^L(2q_e^L - 2q_i + c_i) - c_e^L(2q_e^L - q_i) - r^{L*}(2q_e^L - 2q_i)]^2}{(q_e^L - q_i)(4q_e^L - q_i)^2}$, where $r^{H*} = \min\{\bar{r}^{HH}, \frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}\}$ and $r^{L*} = \min\{\bar{r}^{LL}, \frac{q_i^2+8q_e^L(q_e^L-c_e^L)-c_iq_i}{2(8q_e^L+q_i)}\}$. Hence, under a licensing contract, the entrant's optimal quality decision is: $q_{er}^* = q_e^H$ if $\pi_{er}^{H*} \geq \pi_{er}^{L*}$, that is, if $c_e^H \leq \frac{q_e^H(2q_e^H-2q_i+c_i)-R[q_e^L(2q_e^L-2q_i+c_i)-c_e^L(2q_e^L-q_i)]}{2q_e^H-q_i} + \frac{R(2q_e^L-2q_i)r^{L*}-(2q_e^H-2q_i)r^{H*}}{2q_e^H-q_i} = c_F + \frac{R(2q_e^L-2q_i)r^{L*}-(2q_e^H-2q_i)r^{H*}}{2q_e^H-q_i}$ and $q_{er}^* = q_e^L$ otherwise. Next, we show that with this different game sequence, the quality-increasing effect and quality-decreasing effect of licensing can still occur under certain conditions by considering two cases: (i) at $c_e^H = c_F$, $\bar{r}^{HH} < \frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$, (ii) at $c_e^H = c_F$, $\bar{r}^{HH} > \frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$.

(i) If at $c_e^H = c_F$, $\bar{r}^{HH} < \frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$, then $r^{H*} = \bar{r}^{HH}$ for $c_e^H \leq c_F$ since $\bar{r}^{HH}$ is increasing in c_e^H and $\frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$ is decreasing in c_e^H. Note that then $r^{H*}|_{c_e^H=c_F} = \bar{r}^{HH}|_{c_e^H=c_F}$ $= \frac{R[q_e^L(2q_e^L-2q_i+c_i)-c_e^L(2q_e^L-q_i)]-(4q_e^H-q_i)\sqrt{(q_e^H-q_i)\pi_e^{H*}}}{2q_e^H-2q_i} = \frac{R[q_e^L(2q_e^L-2q_i+c_i)-c_e^L(2q_e^L-q_i)-(4q_e^L-q_i)\sqrt{(q_e^L-q_i)\pi_e^{L*}}]}{2q_e^H-2q_i}$ $= \frac{R(q_e^L-q_i)\bar{r}^{LL}}{q_e^H-q_i}$. That is, $[R(q_e^L-q_i)\bar{r}^{LL} - (q_e^H-q_i)r^{H*}]|_{c_e^H=c_F} = 0$. By definition, $r^{L*} \leq \bar{r}^{LL}$, hence, $[R(q_e^L-q_i)r^{L*} - (q_e^H-q_i)r^{H*}]|_{c_e^H=c_F} \leq [R(q_e^L-q_i)\bar{r}^{LL} - (q_e^H-q_i)r^{H*}]|_{c_e^H=c_F} = 0$. Let c_r be the value of c_e^H such that $c_e^H = c_F + \frac{R(2q_e^L-2q_i)r^{L*}-(2q_e^H-2q_i)r^{HH}}{2q_e^H-q_i}$. Then $c_r < c_F$; $q_{er}^* = q_e^H$ when $c_e^H \leq c_r$, and $q_{er}^* = q_e^L$ when $c_r < c_e^H \leq c_F$. Hence, when $c_e^H \leq c_r$, licensing does not change the entrant's optimal quality; when $c_r < c_e^H \leq c_F$, licensing decreases the entrant's optimal quality. One numerical example for the quality-decreasing effect of licensing is: $q_i = 1, c_i = 0.7, q_e^L = 1.1, c_e^L = 0.78, q_e^H = 1.5, c_e^H = 1.18, F = 0.0025$. In this example, $q_{er}^* = q_e^L < q_e^* = q_e^H$ and $r^{L*} = 0.1589$.

Since $\bar{r}^{HH}$ is increasing in c_e^H and $\frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$ is decreasing in c_e^H, there exists a unique value of c_e^H, which we denote as c_1, such that $\bar{r}^{HH} = \frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$ at c_1. Since $c_e^H = c_F$, $\bar{r}^{HH} < \frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$, then $c_1 > c_F$. Hence, when $c_F < c_e^H \leq c_1$, $q_{er}^* = q_e^L$ and licensing does not change the entrant's optimal quality. When $c_e^H > c_1$, then $\bar{r}^{HH} > \frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$, $r^{H*} = \frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$ and $\pi_{er}^{H*}(r^{H*}) = \frac{[q_e^H-c_e^H-(q_i-c_i)]^2(2q_e^H+q_i)^2}{(q_e^H-q_i)(8q_e^H+q_i)^2}$, which is decreasing in c_e^H. Hence, for any $c_e^H > c_1$, $\pi_{er}^{H*}(r^{H*}, c_e^H > c_1) < \pi_{er}^{H*}(r^{H*}, c_e^H = c_1) = \pi_{er}^{H*}(\bar{r}^{HH}, c_e^H = c_1) = \pi_e^{H*}(c_e^H = c_1) < \pi_e^{L*} \leq \pi_{er}^{L*}$. Hence, when $c_e^H > c_1$, $q_{er}^* = q_e^L$ and licensing does not affect the entrant's optimal quality decision.

To summarize, we analytically proved that if at $c_e^H = c_F$, $\bar{r}^{HH} < \frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$, then licensing leads to the entrant's same or lower optimal quality.

(ii) If at $c_e^H = c_F$, $\bar{r}^{HH} > \frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$, then $r^{H*} = \frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$ for $c_e^H \geq c_F$ since $\bar{r}^{HH}$ is increasing in c_e^H and $\frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$ is decreasing in c_e^H. At $c_e^H = c_F$, $r^{H*}|_{c_e^H=c_F} = \frac{q_i^2+8q_e^H(q_e^H-c_F)-c_iq_i}{2(8q_e^H+q_i)}$. We first show that under certain conditions, the quality-increasing effect and the quality-decreasing effect of licensing can occur, then we discuss the effect of licensing under other conditions. Note that $\bar{r}^{HH}|_{c_e^H=c_F} > \frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}|_{c_e^H=c_F}$ requires $\frac{q_e^H(2q_e^H-2q_i+c_i)-c_F(2q_e^H-q_i)}{2q_e^H-2q_i} > \frac{q_i^2+8q_e^H(q_e^H-c_F)-c_iq_i}{2(8q_e^H+q_i)}$ and $\frac{[q_e^L(2q_e^L-2q_i+c_i)-c_e^L(2q_e^L-q_i)]^2}{(q_e^L-q_i)(4q_e^L-q_i)^2} > F \geq \max\{0, F_1\}$ where $F_1 = \frac{q_i(q_i-c_i)+8q_e^H(q_e^H-c_F)}{(8q_e^H+q_i)(4q_e^H-q_i)^2}\{2q_e^H(2q_e^H - 2q_i + c_i) - 2c_F(2q_e^H - q_i) - \frac{(q_e^H-q_i)[q_i(q_i-c_i)+8q_e^H(q_e^H-c_F)]}{8q_e^H+q_i}\}$. We denote this condition as condition (a). We next consider two cases: (1) $\bar{r}^{LL} \leq \frac{q_i^2+8q_e^L(q_e^L-c_e^L)-c_iq_i}{2(8q_e^L+q_i)}$, (2) $\bar{r}^{LL} > \frac{q_i^2+8q_e^L(q_e^L-c_e^L)-c_iq_i}{2(8q_e^L+q_i)}$.

(1) If $\bar{r}^{LL} \leq \frac{q_i^2+8q_e^L(q_e^L-c_e^L)-c_iq_i}{2(8q_e^L+q_i)}$ holds, then it requires $\frac{q_e^L(2q_e^L-2q_i+c_i)-c_e^L(2q_e^L-q_i)}{2q_e^L-2q_i} \leq \frac{q_i^2+8q_e^L(q_e^L-c_e^L)-c_iq_i}{2(8q_e^L+q_i)}$; or $\frac{q_e^L(2q_e^L-2q_i+c_i)-c_e^L(2q_e^L-q_i)}{2q_e^L-2q_i} > \frac{q_i^2+8q_e^L(q_e^L-c_e^L)-c_iq_i}{2(8q_e^L+q_i)}$ and $F \leq F_2 = \frac{q_i(q_i-c_i)+8q_e^L(q_e^L-c_e^L)}{(8q_e^L+q_i)(4q_e^L-q_i)^2}\{2q_e^L(2q_e^L - 2q_i + c_i) - 2c_e^L(2q_e^L - q_i) - \frac{(q_e^L-q_i)[q_i(q_i-c_i)+8q_e^L(q_e^L-c_e^L)]}{8q_e^L+q_i}\}$. We denote this condition as condition (b). Note that only if both conditions (a) and (b) hold, this case of at $c_e^H = c_F$, $r^{H*} = \frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$ and $r^{L*} = \bar{r}^{LL}$ occurs. Then $[R(q_e^L-q_i)r^{L*} - (q_e^H-q_i)r^{H*}]|_{c_e^H=c_F} = R(q_e^L-q_i)\bar{r}^{LL} - (q_e^H-q_i)\frac{q_i^2+8q_e^H(q_e^H-c_F)-c_iq_i}{2(8q_e^H+q_i)}$. We denote $R(q_e^L-q_i)\bar{r}^{LL} - (q_e^H-q_i)\frac{q_i^2+8q_e^H(q_e^H-c_F)-c_iq_i}{2(8q_e^H+q_i)} > 0$ as condition (c). Let c_r be the value of c_e^H such that $c_e^H = c_F + \frac{R(2q_e^L-2q_i)r^{L*}-(2q_e^H-2q_i)\frac{q_i^2+8q_e^H(q_e^H-c_F)-c_iq_i}{2(8q_e^H+q_i)}}{2q_e^H-q_i}$. Then when condition (c) holds, $c_r > c_F$; $q_{er}^* = q_e^H$ when $c_F < c_e^H \leq c_r$, and $q_{er}^* = q_e^L$ when $c_e^H > c_r$. Hence, when the parameters are such that conditions $(a), (b)$ and (c) hold and $c_F < c_e^H \leq c_r$, then licensing increases the entrant's optimal quality. One numerical example for this quality-increasing effect of licensing is: $q_i = 1, c_i = 0.3, q_e^L = 1.5, c_e^L = 0.8, q_e^H = 3, c_e^H = 2.2721, F = 0.00978$. In this example, $q_{er}^* = q_e^H > q_e^* = q_e^L$ and $r^{H*} = 0.363392$. When the parameters are such that conditions

$(a),(b)$ and (c) hold and $c_e^H > c_r$, licensing does not change the entrant's optimal quality. When the parameters are such that conditions $(a),(b)$ and (c) hold and $c_e^H \leq c_F$, licensing leads to the entrant's same or lower optimal quality. When the parameters are such that conditions (a) and (b) hold but condition (c) does not hold, then licensing may decrease but cannot increase the entrant's optimal quality.

(2) If $\bar{r}^{LL} > \frac{q_i^2+8q_e^L(q_e^L-c_e^L)-c_iq_i}{2(8q_e^L+q_i)}$ holds, then it requires $\frac{q_e^L(2q_e^L-2q_i+c_i)-c_e^L(2q_e^L-q_i)}{2q_e^L-2q_i} > \frac{q_i^2+8q_e^L(q_e^L-c_e^L)-c_iq_i}{2(8q_e^L+q_i)}$ and $F > \max\{0, F_2\}$. This means that the case at c_e^H, $r^{H*} = \frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$ and $r^{L*} = \frac{q_i^2+8q_e^L(q_e^L-c_e^L)-c_iq_i}{2(8q_e^L+q_i)}$ occurs only if $\frac{q_e^H(2q_e^H-2q_i+c_i)-c_F(2q_e^H-q_i)}{2q_e^H-2q_i} > \frac{q_i^2+8q_e^H(q_e^H-c_F)-c_iq_i}{2(8q_e^H+q_i)}$, $\frac{q_e^L(2q_e^L-2q_i+c_i)-c_e^L(2q_e^L-q_i)}{2q_e^L-2q_i} > \frac{q_i^2+8q_e^L(q_e^L-c_e^L)-c_iq_i}{2(8q_e^L+q_i)}$ and $\frac{[q_e^L(2q_e^L-2q_i+c_i)-c_e^L(2q_e^L-q_i)]^2}{(q_e^L-q_i)(4q_e^L-q_i)^2} > F > \max\{0, F_1, F_2\}$ hold. We denote this condition as condition (d). Note that $[R(q_e^L-q_i)r^{L*}-(q_e^H-q_i)r^{H*}]|_{c_e^H=c_F} = R(q_e^L-q_i)\frac{q_i^2+8q_e^L(q_e^L-c_e^L)-c_iq_i}{2(8q_e^L+q_i)} - (q_e^H-q_i)\frac{q_i^2+8q_e^H(q_e^H-c_F)-c_iq_i}{2(8q_e^H+q_i)}$. We denote $R(q_e^L-q_i)\frac{q_i^2+8q_e^L(q_e^L-c_e^L)-c_iq_i}{2(8q_e^L+q_i)} - (q_e^H-q_i)\frac{q_i^2+8q_e^H(q_e^H-c_F)-c_iq_i}{2(8q_e^H+q_i)} > 0$ as condition (e). Let c_r be the value of c_e^H such that $c_e^H = c_F + \frac{R(2q_e^L-2q_i)r^{L*}-(2q_e^H-2q_i)\frac{q_i^2+8q_e^H(q_e^H-c_F)-c_iq_i}{2(8q_e^H+q_i)}}{2q_e^H-q_i}$. Then when condition (e) holds, $c_r > c_F$; $q_{er}^* = q_e^H$ when $c_F < c_e^H \leq c_r$, and $q_{er}^* = q_e^L$ when $c_e^H > c_r$. Hence, when the parameters are such that conditions (d) and (e) both hold, and $c_F < c_e^H \leq c_r$, then licensing increases the entrant's optimal quality. One numerical example for this quality-increasing effect of licensing is: $q_i = 1, c_i = 0.31, q_e^L = 1.5, c_e^L = 0.8, q_e^H = 3, c_e^H = 2.2649, F = 0.01064$. In this example, $q_{er}^* = q_e^H > q_e^* = q_e^L$ and $r^{H*} = 0.366648$. When the parameters are such that conditions (d) and (e) both hold, and $c_e^H > c_r$, licensing does not change the entrant's optimal quality. When the parameters are such that conditions (d) and (e) both hold, and $c_e^H < c_F$, licensing leads to the entrant's same or lower optimal quality. When the parameters are such that condition (d) holds, but condition (e) does not hold, then licensing may decrease but cannot increase the entrant's optimal quality.

To summarize, we showed that if at $c_e^H = c_F$, $\bar{r}^{HH} > \frac{q_i^2+8q_e^H(q_e^H-c_e^H)-c_iq_i}{2(8q_e^H+q_i)}$, then licensing leads to the entrant's same, higher, or lower optimal quality.

D. Analysis for the model with the incumbent's endogenous quality

In this part of the Web Appendix, the incumbent endogenously decides its product quality in anticipation of its competitor's entry in the market. Let the incumbent's and the entrant's marginal cost be $k_i q_i^2$ (or $k_i q_{ir}^2$ in the case of licensing), and $k_e q_e^2$ (or $k_e q_{er}^2$ in the case of licensing), respectively. Because of tractability issues, next, we introduce two numerical examples to demonstrate the effect of licensing on the entrant's optimal quality when the incumbent endogenously decides its product quality.

Example 1: When the entrant's production of its core technology has significant improvement over the incumbent's, let $k_i = 1$, $k_e = 0.5$, and $F = 0.01$. In the case of no licensing, for given q_i, q_e, p_i and p_e, the incumbent's and the entrant's demand functions are: $D_i = \frac{p_e - p_i}{q_e - q_i} - \frac{p_i}{q_i}$ and $D_e = 1 - \frac{p_e - p_i}{q_e - q_i}$, respectively, if $q_e > q_i$ and $p_e > p_i$; $D_i = 1 - \frac{p_i - p_e}{q_i - q_e}$ and $D_e = \frac{p_i - p_e}{q_i - q_e} - \frac{p_e}{q_e}$ if $q_e < q_i$ and $p_e < p_i$; $D_i = 1 - \frac{p_i}{q_i}$ and $D_e = 0$ if $q_e < q_i$ and $p_e > p_i$; and $D_i = 0$ and $D_e = 1 - \frac{p_e}{q_e}$ if $q_e > q_i$ and $p_e < p_i$. Their profit functions are $\pi_i = (p_i - q_i^2)D_i$ and $\pi_e = (p_e - 0.5q_e^2)D_e - F$. One can easily show that in this example, the entrant will optimally respond with $q_e > q_i$ and the incumbent will optimally set $q_i < q_e$ in anticipation of a more efficient competitor's entry. In the licensing case, we will only list the demand function where both firms compete in the market. We solve the game based on backward induction. First, since π_i is a concave function of p_i and π_e is a concave function of p_e, simultaneously solving the first order conditions $\frac{d\pi_i}{dp_i} = 0$ and $\frac{d\pi_e}{dp_e} = 0$ leads to the optimal prices $p_i^* = \frac{[0.125q_e^2 + q_e(0.25 + 0.5q_i) - 0.25q_i]q_i}{q_e - 0.25q_i}$ and $p_e^* = \frac{q_e[0.5q_e + 0.25q_e^2 + (0.25q_i - 0.5)q_i]}{q_e - 0.25q_i}$. Then the two firms' profits are: $\pi_i(q_e, q_i) = \frac{0.015625q_e q_i[q_e^2 - q_e(4q_i - 2) - q_i(2 - 2q_i)]^2}{(q_e - q_i)(q_e - 0.25q_i)^2}$ and $\pi_e(q_e, q_i) = \frac{0.0625q_e^2[q_e^2 - q_e(2 + 0.5q_i) - q_i(q_i - 2)]^2}{(q_e - q_i)(q_e - 0.25q_i)^2}$.

Next, we maximize the entrant's profit $\pi_e(q_e, q_i)$ over q_e to obtain its optimal quality $q_e^*(q_i)$ for given q_i. Let $q_e^*(q_i) = argmax_{q_e}\pi_e(q_e, q_i)$ (i.e., $q_e^*(q_i)$ is the solution to the first order condition $\frac{d\pi_e(q_e,q_i)}{dq_e} = 0$ that maximizes the entrant's profit). Last, we maximize the incumbent's profit $\pi_i(q_e^*(q_i), q_i)$ over q_i to obtain its optimal product quality q_i^* in anticipation of $q_e^*(q_i)$, p_i^* and p_e^*. Let $q_i^* = argmax_{q_i}\pi_i(q_e^*(q_i), q_i)$. Our numerical analysis procedure is: among the seven solutions to the first order condition $\frac{d\pi_e(q_e,q_i)}{dq_e} = 0$, we can analytically rule out three solutions that give $\pi_e(q_e, q_i) = 0$ and there are four solutions left. We denote these four solutions as $q_{e1}^*(q_i)$, $q_{e2}^*(q_i)$, $q_{e3}^*(q_i)$, and $q_{e4}^*(q_i)$. For each of these four solutions, we numerically maximize the incumbent's profit $\pi_i(q_{ej}^*(q_i), q_i)$ where $j = 1, 2, 3, 4$ to compare the maximum of each $\pi_i(q_{ej}^*(q_i), q_i)$ and obtain the optimal q_i^* and the corresponding $q_e^*(q_i^*)$ as follows: $q_i^* \approx 0.20559$, $q_e^*(q_i^*) \approx 0.71007$, $\pi_i^* \approx 0.01071$, and $\pi_e^* \approx 0.04535$.

In the case of a licensing contract with royalty fee r, for given q_{ir}, q_{er}, p_{ir} and p_{er}, the incumbent's and the entrant's demand functions are $D_{ir} = \frac{p_{er}-p_{ir}}{q_{er}-q_{ir}} - \frac{p_{ir}}{q_{ir}}$ and $D_{er} = 1 - \frac{p_{er}-p_{ir}}{q_{er}-q_{ir}}$, respectively. Their profit functions are $\pi_{ir} = (p_{ir} - q_{ir}^2)D_{ir} + r \times D_{er}$ and $\pi_{er} = (p_{er} - 0.5q_{er}^2 - r)D_{er}$. First, simultaneously solving the first order conditions $\frac{d\pi_{ir}}{dp_{ir}} = 0$ and $\frac{d\pi_{er}}{dp_{er}} = 0$ leads to the optimal prices $p_{ir}^* = \frac{[0.125q_{er}^2+q_{er}(0.25+0.5q_{ir})-0.25q_{ir}+0.75r]q_{ir}}{q_{er}-0.25q_{ir}}$ and $p_{er}^* = \frac{q_{er}[0.5q_{er}+0.25q_{er}^2+(0.25q_{ir}-0.5)q_{ir}+0.5r]+0.25rq_{ir}}{q_{er}-0.25q_{ir}}$. Let $\pi_{ir}(q_{er}, q_{ir}, r)$ and $\pi_{er}(q_{er}, q_{ir}, r)$ be the incumbent and the entrant's corresponding profit. Next, we maximize the entrant's profit $\pi_{er}(q_{er}, q_{ir}, r)$ over q_{er} to obtain its optimal quality $q_{er}^*(q_{ir}, r)$. Let $q_{er}^*(q_{ir}, r) = argmax_{q_{er}}\pi_{er}(q_{er}, q_{ir}, r)$ (i.e., one of the solutions to the first order condition $\frac{d\pi_{er}(q_{er},q_{ir},r)}{dq_{er}} = 0$ that maximizes the entrant's profit). Last, we numerically maximize the incumbent's profit $\pi_{ir}(q_{er}^*(q_{ir}, r), q_{ir}, r)$ jointly over q_{ir} and r to obtain its optimal product quality q_{ir}^* and its optimal royally fee r^* subject to the constraint that neither firm is worse off with the licensing contract, that is, $\pi_{ir}(q_{er}^*(q_{ir}, r), q_{ir}, r) \geq \pi_i^*$ and $\pi_{er}(q_{er}^*(q_{ir}, r), q_{ir}, r) \geq \pi_e^*$. Following this procedure, π_{ir}^* is achieved at $(q_{ir}^*, r^*) = (0.10349, 0.06914)$. So, $q_{er}^*(q_{ir}^*, r^*) \approx 0.73907 > q_e^*$. Therefore, with consideration of the incumbent's response in quality decision, licensing leads to the entrant's higher optimal quality.

Example 2: When the entrant's production of its core technology has incremental improvement over the incumbent's, let $k_i = 1$, $k_e = 0.98$, and $F = 0.001$. One can easily show that in this example, the entrant will optimally respond with $q_e > q_i$ and the incumbent will optimally set $q_i < q_e$ in anticipation of a more efficient competitor's entry. Hence, in the case of no licensing, for given q_i, q_e, p_i and p_e, the incumbent's and the entrant's demand functions are $D_i = \frac{p_e - p_i}{q_e - q_i} - \frac{p_i}{q_i}$ and $D_e = 1 - \frac{p_e - p_i}{q_e - q_i}$, respectively. Their profit functions are $\pi_i = (p_i - q_i^2)D_i$ and $\pi_e = (p_e - 0.98q_e^2)D_e - F$. We solve the game based on backward induction. First, simultaneously solving the first order conditions $\frac{d\pi_i}{dp_i} = 0$ and $\frac{d\pi_e}{dp_e} = 0$ leads to the optimal prices $p_i^* = \frac{[0.245q_e^2 + q_e(0.25+0.5q_i) - 0.25q_i]q_i}{q_e - 0.25q_i}$ and $p_e^* = \frac{q_e[0.5q_e + 0.49q_e^2 + (0.25q_i - 0.5)q_i]}{q_e - 0.25q_i}$. Then the two firms' profits are:

$\pi_i(q_e, q_i) = \frac{0.060025q_eq_i[q_e^2 - q_e(2.04082q_i - 1.02041) - q_i(1.02041 - 1.02041q_i)]^2}{(q_e - q_i)(q_e - 0.25q_i)^2}$ and

$\pi_e(q_e, q_i) = \frac{0.2401q_e^2[q_e^2 - q_e(1.02041 + 0.5q_i) - q_i(0.510204q_i - 1.02041)]^2}{(q_e - q_i)(q_e - 0.25q_i)^2}$. Next, we maximize the entrant's profit over q_e to obtain its optimal quality $q_e^*(q_i)$. Let $q_e^*(q_i) = argmax_{q_e}\pi_e(q_e, q_i)$ (i.e., $q_e^*(q_i)$ is the solution to the first order condition $\frac{d\pi_e(q_e,q_i)}{dq_e} = 0$ that maximizes the entrant's profit). Last, we maximize the incumbent's profit $\pi_i(q_e^*(q_i), q_i)$ over q_i to obtain its optimal product quality q_i^* in anticipation of $q_e^*(q_i)$, p_i^* and p_e^*. Then $q_i^* = argmax_{q_i}\pi_i(q_e^*(q_i), q_i)$. Following the same procedure as in Example 1, we numerically obtain the optimal solutions: $q_i^* \approx 0.26711$, $q_e^*(q_i) \approx 0.45162$, $\pi_i^* \approx 0.01269$, and $\pi_e^* \approx 0.01076$.

In the case of a licensing contract with royalty fee r, for given q_{ir}, q_{er}, p_{ir} and p_{er}, the incumbent's and the entrant's demand functions are $D_{ir} = \frac{p_{er} - p_{ir}}{q_{er} - q_{ir}} - \frac{p_{ir}}{q_{ir}}$ and $D_{er} = 1 - \frac{p_{er} - p_{ir}}{q_{er} - q_{ir}}$, respectively. Their profit functions are $\pi_{ir} = (p_{ir} - q_{ir}^2)D_{ir} + r \times D_{er}$ and $\pi_{er} = (p_{er} - 0.5q_{er}^2 - r)D_{er}$. First, simultaneously solving the first order conditions $\frac{d\pi_{ir}}{dp_{ir}} = 0$ and $\frac{d\pi_{er}}{dp_{er}} = 0$ leads to the optimal prices $p^*_{ir} = \frac{[0.245q^2_{er} + q_{er}(0.25+0.5q_{ir}) - 0.25q_{ir} + 0.75r]q_{ir}}{q_{er} - 0.25q_{ir}}$ and $p^*_{er} = \frac{q_{er}[0.5q_{er} + 0.49q^2_{er} + (0.25q_{ir} - 0.5)q_{ir} + 0.5r] + 0.25rq_{ir}}{q_{er} - 0.25q_{ir}}$. Let $\pi_{ir}(q_{er}, q_{ir}, r)$ and $\pi_{er}(q_{er}, q_{ir}, r)$ be the incumbent and the entrant's corresponding profit. Next, we maximize the entrant's profit over q_{er} to obtain its optimal quality $q^*_{er}(q_{ir}, r)$. Let $q^*_{er}(q_{ir}, r) = argmax_{q_{er}} \pi_{er}(q_{er}, q_{ir}, r)$ (i.e., one of the solutions to the first order condition $\frac{d\pi_{er}(q_{er}, q_{ir}, r)}{dq_{er}} = 0$ that maximizes the entrant's profit). Last, we numerically maximize the incumbent's profit $\pi_{ir}(q^*_{er}(q_{ir}, r), q_{ir}, r)$ jointly over q_{ir} and r to obtain its optimal product quality q^*_{ir} and its optimal royally fee r^* subject to the constraint that neither firm is worse off with the licensing contract, that is, $\pi_{ir}(q^*_{er}(q_{ir}, r), q_{ir}, r) \geq \pi^*_i$ and $\pi_{er}(q^*_{er}(q_{ir}, r), q_{ir}, r) \geq \pi^*_e$. Following this procedure, π^*_{ir} is achieved at $(q^*_{ir}, r^*) = (0.14223, 0.06688)$. So, $q^*_{er}(q^*_{ir}, r^*) \approx 0.41282 < q^*_e$. Therefore, with consideration of the incumbent's response in quality decision, licensing leads to the entrant's lower optimal quality.