Supplementary Online Appendix

Partisan Strength and Legislative Bargaining

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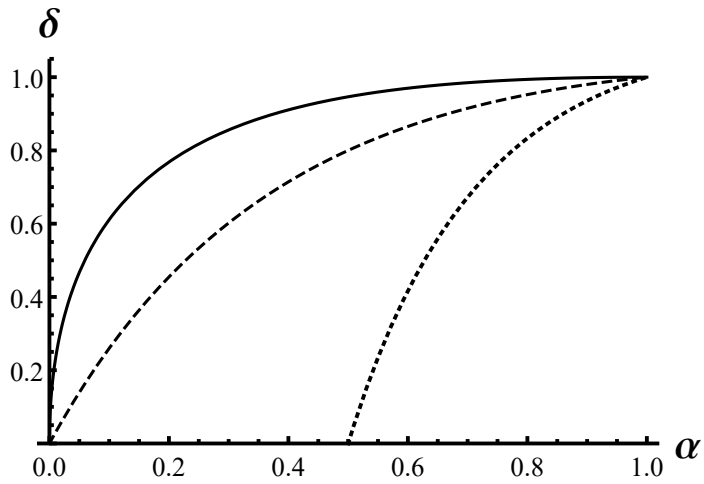
Mathematica was used to plot some of the figures and for derivations in some of the proofs. Here, we provide the *Mathematica* notebooks.

To implement our *Mathematica* derivations, we use a for α and d for δ. We let xPH denote the expression in (60), xPI denote the expression in (62), xNH denote the expression in (64), xSH denote the expression in (66), and xSIL denote the expression in (68). A partial derivative of the form $\partial f/\partial x$ is implemented by writing D[f, x]. The following notation for operators is used: $*$ for multiplication, && for conjunction, and $\|$ for disjunction. The symbol $==$ is used to denote that two expressions are identically equal.
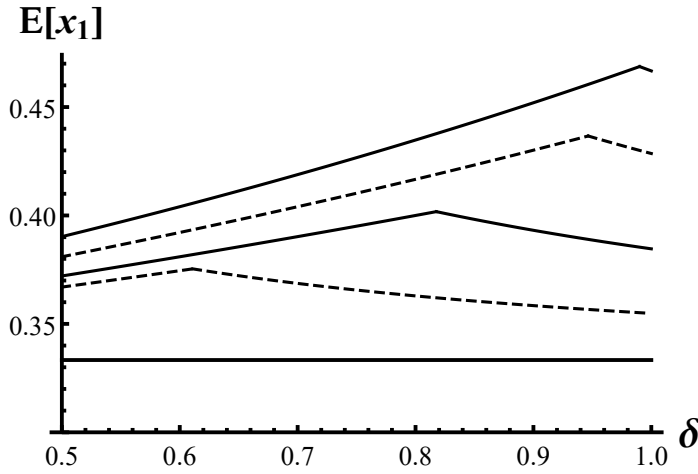
Plotting the graphs in Figure 1.

$$\text{Plot}\Big[\Big\{\frac{-3\,a - a^2}{2\,(1+a)} + \frac{1}{2}\sqrt{\frac{24\,a + 33\,a^2 + 6\,a^3 + a^4}{(1+a)^2}}\,,\ \frac{6\,a}{(1+a)\,(2+a)}\,,\ \frac{2\,(2\,a - 1)}{(1+a)\,a}\Big\},$$

`{a, 0, 1}, PlotStyle → {{Black}, {Black, Dashed}, {Thick, Black, Dotted}},`
`LabelStyle → {FontFamily → "Times New Roman"},`
`AxesLabel → {Style["α", Bold, 22], Style["δ", Bold, 22]},`
`PlotRange → {0, 1.05}, AxesStyle → Thick, TicksStyle → Larger]`

Plotting the graphs in Figure 3.

```
Plot[Evaluate@Table[Piecewise[{{ (a + d) / (d (3 + a)) ,  d ≥ (-3 a - a² + √(24 a + 33 a² + 6 a³ + a⁴)) / (2 (1 + a)) },
```
$$\text{Plot}\left[\text{Evaluate@Table}\left[\text{Piecewise}\left[\left\{\left\{\frac{(a+d)}{d\,(3+a)},\ d \geq \frac{-3\,a - a^2 + \sqrt{24\,a + 33\,a^2 + 6\,a^3 + a^4}}{2\,(1+a)}\right\},\right.\right.\right.\right.$$

$$\left\{\frac{2}{6 - d - a * d},\ \frac{6\,a}{(1+a)\,(2+a)} \leq\ d < \frac{-3\,a - a^2 + \sqrt{24\,a + 33\,a^2 + 6\,a^3 + a^4}}{2\,(1+a)}\right\},$$

$$\left.\left.\left\{\frac{2}{6 - d - a * d},\ \frac{6\,a}{(1+a)\,(2+a)} > d\right\}\right\}\right],\ \{a,\ \{0,\ 0.1,\ 0.25,\ 0.5,\ 0.75\}\}\right],\ \{d,\ 0.5,\ 1\},$$

```
 PlotStyle → {{Thick, Black}, {Black, Dashed}, {Black}, {Black, Dashed}, {Black}},
 PlotRange → {0.3, 0.475},
 LabelStyle → {FontFamily → "Times New Roman"},
 AxesLabel → {Style["δ", Bold, 22], Style["E[x₁]", Bold, 20]},
 AxesStyle → Thick, TicksStyle → Larger]
```


Verifying (47).

```
Reduce[{p == (6 * a - 7 a * d - a² * d + d² + a * d²) / (2 d (d + a * d - 2 a)),
  0 ≤ p ≤ 1, 0 < a < 1, 0 ≤ d ≤ 1}, {d, p}]
```

$$0 < a < 1\ \&\&\ \frac{-3\,a - a^2}{2\,(1+a)} + \frac{1}{2}\sqrt{\frac{24\,a + 33\,a^2 + 6\,a^3 + a^4}{(1+a)^2}}\ \leq d \leq 1\ \&\&\ p == \frac{6\,a - 7\,a\,d - a^2\,d + d^2 + a\,d^2}{-4\,a\,d + 2\,d^2 + 2\,a\,d^2}$$

Verifying (48).

```
Reduce[{p == (6 * a - 7 a * d - a² * d + d² + a * d²) / (2 d (d + a * d - 2 a)),
  p ≥ 0, 0 < a < 1, 0 ≤ d ≤ 1}, {d, p}]
```

$$0 < a < 1\ \&\&\ \frac{2\,a}{1+a} < d \leq 1\ \&\&\ p == \frac{6\,a - 7\,a\,d - a^2\,d + d^2 + a\,d^2}{-4\,a\,d + 2\,d^2 + 2\,a\,d^2}$$

Definitions for following steps.

$$p = \frac{6a - 7ad - a^2d + d^2 + ad^2}{-4ad + 2d^2 + 2ad^2};$$

$$xPH = \frac{-2a + ad + d}{(3+a)(1-a)};$$

$$xPI = \frac{2d + 3ad + a^2d - 6a}{(6-d-ad)(1-a)};$$

$$xNH = \frac{-2a + ad + d}{3+a};$$

$$xSH = \frac{(1+a)(a+d)}{3+a};$$

$$xSIL = \frac{2d(1+a)}{6-d-ad};$$

Verifying (59).

Simplify[D[p, a]]

$$-\frac{-6 + a^2(-2+d) + 5d + 2ad}{2(a(-2+d) + d)^2}$$

Verifying (61).

Simplify[D[xPH, a]]

$$\frac{-6 + a^2(-2+d) + 5d + 2ad}{(-1+a)^2(3+a)^2}$$

Verifying (63).

Simplify[D[xPI, a]]

$$-\frac{3\left(12 - 4(3+a)d + (1+a)^2 d^2\right)}{(-1+a)^2(-6+d+ad)^2}$$

Verifying (65).

Simplify[D[xNH, a]]

$$\frac{2(-3+d)}{(3+a)^2}$$

Verifying (67).

Simplify[D[xSH, a]]

$$\frac{3 + 6a + a^2 + 2d}{(3+a)^2}$$

Verifying (69).

Simplify[D[xSIL, a]]

$$\frac{12\,d}{\left(-6 + d + a\,d\right)^2}$$

Verifying (70).

Simplify[D[p, d]]

$$\frac{a\left(a^2\,d^2 + d\left(-12 + 5\,d\right) + 6\,a\left(2 - 2\,d + d^2\right)\right)}{2\,d^2\left(a\left(-2 + d\right) + d\right)^2}$$

The sufficiency of (71).

Reduce$\left[\left\{\dfrac{a\left(a^2\,d^2 + d\left(-12 + 5\,d\right) + 6\,a\left(2 - 2\,d + d^2\right)\right)}{2\,d^2\left(a\left(-2 + d\right) + d\right)^2} < 0,\ 0 \le a < 1,\ 0 \le d \le 1\right\},\ d\right]$

$$0 < a < 1\ \&\&\ \left(\frac{6}{5 + a} - 2\sqrt{3}\ \sqrt{-\frac{-3 + 2\,a + a^2}{\left(1 + a\right)\left(5 + a\right)^2}} < d < \frac{2\,a}{1 + a}\ ||\ \frac{2\,a}{1 + a} < d \le 1\right)$$

Verifying (72).

Simplify[D[xPH, d]]

$$\frac{1 + a}{3 - 2\,a - a^2}$$

Verifying (73).

Simplify[D[xPI, d]]

$$-\frac{12\left(1 + a\right)}{\left(-1 + a\right)\left(-6 + d + a\,d\right)^2}$$

Verifying (74).

Simplify[D[xNH, d]]

$$\frac{1 + a}{3 + a}$$

Verifying (75).

Simplify[D[xSH, d]]

$$\frac{1 + a}{3 + a}$$

Verifying (76).

Simplify[D[xSIL, d]]

$$\frac{12\left(1 + a\right)}{\left(-6 + d + a\,d\right)^2}$$