Supplemental Appendix

This appendix offers proofs of pure strategy perfect Bayesian equilibria of both the unidimensional and multidimensional uncertainty games. I begin with the unidimensional uncertainty version.

1 Equilibria of the Unidimensional Uncertainty Game

Assume $P1$ and $P2$ share a common discount factor δ, which is common knowledge. $P1$ remains uncertain of whether $P2$ is aggressive or benign, but there is no uncertainty regarding time horizons. The manuscript describes four distinct equilibria, which I describe first here. I then present three additional non-cooperative pooling equilibria, some of which rely on nonintuitive off-path beliefs, and some of which are dominated by other equilibria. The omitted equilibria do not undermine the conclusions derived from Proposition 3.

Lemma 1: $P2^*_B = C_1, C_2$; $P2^*_A = D_1, D_2$; $P1^* = C_1, C_2|C, D_2|D$; $p_2|C_1 = 1$; $p_2|D_1 = 0$ constitutes a perfect Bayesian equilibrium when $p_1 \geq p^*$ and $\delta < 1 - \frac{H_{2A}}{E_{2A}}$.

Proof: This "cooperative separating equilibrium" obtains under two conditions: $U1_A$: $p_1 \geq p^*$ and $U1_B$: $\delta < 1 - \frac{H_{2A}}{E_{2A}}$.

In this equilibrium, $P2_B$'s expected utility equals $H_{2B}(1+\delta)$. Defection would yield a payoff of E_{2B}, which is strictly less than $H_{2B}(1+\delta)$ for a benign actor by assumption, as $H_{2B} > E_{2B}$. For the aggressive $P2$, its equilibrium strategy yields a payoff of E_{2A}. Conversely, cooperation would yield a payoff of $H_{2A} + \delta(E_{2A})$. Setting these payoffs equal, defection would be $P2_A$'s optimal strategy when $\delta < 1 - \frac{H_{2A}}{E_{2a}}$, which is given by assumption $U1_B$ above.

Given $P2$'s equilibrium strategies, $P1$'s expected utility for cooperation is:

$$U_1(C_1) = p_1(H_1(1 + \delta)) + (1 - p_1)(-S) \tag{1}$$

Its expected utility for defection is:

$$U_1(D_1) = p_1(E_1 + \delta H_1) \tag{2}$$

Setting these two equal and solving for p_1 yields the conditions under which $P1$ cooperates in round 1:

$$p_1 \geq \frac{S_1}{H_1 + S_1 - E_1} \tag{3}$$

The expression on the right hand side of this inequality is the p^* cutpoint, so this condition is satisfied by assumption $U1_B$.

Since $P2_A$ and $P2_B$ separate, $P1$ is able to fully update its beliefs, so $p_2|C_1 = 1$ and $p_2|D_1 = 0$. Given these posterior beliefs, $P1^* = C_2$ after observing cooperation and $P1^* = D_2$ after observing defection. $P2_A$ will defect in round 2 because defection is a dominant strategy for the aggressive type in a one-shot game. $P2_B$ will cooperate in round 2 as it knows $P1$ will reciprocate its own first round cooperation. ■

Lemma 2: $P2_B^* = C_1, C_2$; $P2_A^* = D_1, D_2$; $P1^* = D_1, C_2|C, D_2|D$; $p_2|C = 1$; $p_2|D = 0$ constitutes a perfect Bayesian equilibrium when $p_1 < p^*$ and $\frac{S_2}{H_{2B}} < \delta < \frac{S_2}{E_{2A}}$.

Proof: The "non-cooperative separating equilibrium" obtains under two conditions: $U2_A$: $p_1 < p^*$ and $U2_B$: $\frac{S_2}{H_{2B}} < \delta < \frac{S_2}{E_{2A}}$.

$P2_B$'s expected utility in equilibrium is $-S_2 + \delta H_{2B}$. Conversely, defection in round 1 would yield a payoff of zero. Setting these payoff equal shows that $P2_B$ will prefer to cooperate when $\delta > \frac{S_2}{H_{2B}}$. This condition is satisfied by assumption $U2_B$. For the aggressive type, its expected utility in equilibrium is zero. Cooperating in round one would yield a payoff of $-S_2 + \delta E_{2A}$. $P2_A$'s equilibrium strategy is supported when $\delta < \frac{S_2}{E_{2B}}$, which is again give by assumption $U2_B$.

Since $P2$'s equilibrium strategies mirror those described in Lemma 1, $P1$'s expected utilities are also the same. $P1$ will thus defect if $p_1 < p^*$, which is given by assumption $U2_A$.

Because benign and aggressive types separate, $p_2|C_1 = 1$ and $p_2|D_1 = 0$. As such, $P1^* = C_2$ after observing cooperation and $P1^* = D_2$ after observing defection. $P2_A$ will defect in round 2 because defection is a dominant strategy for the aggressive type in a one-shot game. $P2_B$ will cooperate in round 2 as it knows $P1$ will reciprocate its own first round cooperation. ■

Lemma 3: $P2_B^* = C_1, C_2$; $P2_A^* = C_1, D_2$; $P1^* = C_1, C_2|C, D_2|D$; $p_2|C = p_1$; $p_2|D < p^*$ constitutes a perfect Bayesian equilibrium when $p_1 \geq p^*$ and $\delta > 1 - \frac{H_{2A}}{E_{2A}}$.

Proof: The "cooperative pooling equilibrium" obtains under two conditions: $U3_A$: $p_1 \geq p^*$ and $U3_B$: $\delta > 1 - \frac{H_{2A}}{E_{2A}}$.

In this equilibrium, $P2_B$'s expected utility for cooperating is $(1+\delta)H_{2B}$. This is its highest possible payoff, so it is supportable in equilibrium. For $P2_A$, its equilibrium strategy yields a payoff of $H_{2B} + \delta E_{2B}$. Deviating to defection in round 1, given $P1$'s assumed off-path beliefs $(p_2|D < p^*)$, would yield a payoff of E_{2B}. Setting these payoffs equal and solving for δ reveals that $P2_A$'s equilibrium strategy is supported given the satisfaction of condition $U3_B$.

Given $P2$'s equilibrium strategies and assuming prior beliefs that support second round cooperation (i.e. $p_1 \geq p^*$), $P1$'s expected utility of cooperating in Round 1 is:

$$U_1(C_1) = p_1(H_1(1+\delta)) + (1-p_1)(H_1 - \delta S_1) \tag{4}$$

It's utility for defecting in round 1 would be:

$$U_1(D_1) = p_1(E_1 + \delta H_1) + (1-p_1)(E_1 - \delta S_1) \tag{5}$$

And since $H_1 > E_1$ for $P1$, cooperation is its optimal strategy. Given $P1$'s priors, which carry over in the pooling equilibria, $P1$ then again prefers to cooperate in round 2. $P2_A$ will of course defect in round 2. $P2_B$, knowing that $P1$'s optimistic priors will induce it to cooperate in round 2, will then cooperate itself. ■

Lemma 4: $P2^*_B = D_1, D_2$; $P2^*_A = D_1, D_2$; $P1^* = D_1, C_2|C, D_2|D$; $p_2|C \geq p_1$; $p_2|D = p_1$ constitutes a perfect Bayesian equilibrium when $p_1 < p^*$ and $\delta < \left\{\frac{S_2}{H_{2B}}, \frac{S_2}{E_{2A}}\right\}$.

Proof: This non-cooperative pooling equilibrium holds given the following conditions: $U4_A$: $p_1 < p^*$ and $U4_B$: $\delta < \left\{\frac{S_2}{H_{2B}}, \frac{S_2}{E_{2A}}\right\}$.

Given $P1$'s prior and off-path beliefs specified above, a benign $P2$'s equilibrium payoff here equals zero. Conversely, its payoff for cooperating in round 1 would be $\delta H_{2B} - S_2$. Defection is preferred when $\delta < \frac{S_2}{H_{2B}}$, which is given by condition $U4_B$. Similarly, the aggressive type's equilibrium payoff is also zero. Deviating to cooperation would yield it a payoff of $\delta E_{2B} - S_2$. Defection is preferred when $\delta < \frac{S_2}{E_{2A}}$, which is also given by condition $U4_B$.

$P1$ will defect in round 2 after observing $P2$ defect, as this action is uninformative and $p_1 < p^*$ by $U4_A$. In round 1, $P1$'s expected utility for cooperation is $-S_1$, while its expected utility for defection is zero. $P1$ thus defects in round 1. And as its pessimistic priors $p_1 < p^*$ remain unchanged following $P2$'s pooling behavior, $P1$ defects again in round 2. ■

1.1 Additional Equilibria

As mentioned in the text, there are three additional pure strategy pooling equilibria of the unidimensional uncertainty game that either rely on potentially nonintuitive off-path beliefs or are Pareto-dominated by other equilibria in the same parameter space. Importantly, each of these equilibria involve all players defecting in round 1. As such, they do *not* undermine the result described in Proposition 3 in the text. This section briefly describes each of these equilibria.

First, assuming $p_1 > p^*$, it is an equilibrium for all players to defect in round 1. $P1$ and $P2_B$ will then cooperate in round 2, while $P2_A$ defects. This holds across the entire parameter space when $p_1 > p^*$, but the players all receive lower utilities than they would in the cooperative pooling equilibrium described in the text. Second, assuming $p_1 < p^*$ it is an equilibrium for all players to defect in both rounds, so long as $P1$'s off-path beliefs are such that it would not reciprocate $P2$'s cooperation in round 2. If $p_2|C < p^*$, then complete defection is a perfect Bayesian equilibrium. Finally, it is an equilibrium for all players to defect in round 1, then for $P1$ to cooperate in round 2, while off-path it would defect in response to first round cooperation. This equilibrium holds when $p_1 > p^*$, but off-path would require $p_2|C_1 < p_2|D = p_1$, which would be ruled out by the intuitive criterion.

Again, these three pooling equilibria were omitted from the main text because they either relied on non-intuitive off-path beliefs, or they are Pareto dominated by other equilibria described in the text. And to reiterate, all of these equilibria involve first-round defection by all players, and as such do not undermine Proposition 3 as stated in the text. In the unidimensional uncertainty game, sustained cooperation across both rounds is *only* possible when $p_1 > p^*$.

2 Equilibria of the Multidimensional Uncertainty Game

This section proves the existence of ten distinct pure strategy equilibria in the multidimensional uncertainty game. Due to space constraints, the main text only described three of these: two "far-sighted misrepresentation equilibria" (FSME) and a single "short-sighted hedging equilibrium" (SSHE). I begin with these semi-separating equilibria, and move on the separating and pooling equilibria.[1]

Lemma 5: $P2^*_{SB} = C_1, C_2$; $P2^*_{LB} = C_1, C_2$; $P2^*_{SA} = D_1, D_2$; $P2^*_{LA} = C_1, D_2$; $P1^* = C_1, C_2|C, D_2|D$; $p_2|C = \frac{p_1}{1+q_1(p_1-1)}$; $p_2|D = 0$; $q_1 < \frac{H_1-E_1}{(1-p_1)(H_1+S_1-E_1)}$; $q_2|D = 1$; $q_2|C = \frac{q_1 p_1}{p_1+(1-q_1)p_1}$ constitutes a perfect Bayesian equilibrium when $p_1 \geq \left\{1-\frac{H_1-E_1}{q_1(H_1+S_1-E_1)}, \frac{S_1(1-q_1)}{H_1-E_1+S_1(1-q_1)}\right\}$ and $\underline{\delta}_2 < 1 - \frac{H_{2A}}{E_{2A}}$.

Proof: This equilibrium is referred to in the text as the cooperative far-sighted misrepresentation equilibrium. It holds under the following conditions:

[1] Recall that even the equilibria described here are not fully separating equilibria, as short-sighted and far-sighted types do not separate. Nevertheless, since the paper is concerned with $P1$'s ability to distinguish aggressive from benign types, I describe the strategy profiles with reference to their pooling/separating behavior according to aggressive/benign preferences.

$$M1_A \equiv p_1 \geq \left\{1 - \frac{H_1 - E_1}{q_1(H_1 + S_1 - E_1)}, \quad \frac{S_1(1-q_1)}{H_1 - E_1 + S_1(1-q_1)}\right\} \tag{6}$$

$$M1_B \equiv \underline{\delta}_2 < 1 - \frac{H_{2A}}{E_{2A}} < \overline{\delta}_2 \tag{7}$$

Assume $P1$ cooperates in round 1 and then reciprocates $P2$'s first round action. Knowing this, $P2_{SB}$'s expected utility for first round cooperation is $(1 + \underline{\delta}_2)H_{2B}$. This is its highest possible payoff, so this strategy is supported. $P2_{LB}$'s expected utility for first round cooperation is $(1 + \overline{\delta}_2)H_{2B}$, which is likewise its highest possible payoff. $P2_{LA}$'s expected utility for cooperation is $H_{2A} + \overline{\delta}_2 E_{2A}$. Conversely, its payoff for defection is E_{2A}. Cooperation is supported in equilibrium when $\overline{\delta}_2 > 1 - \frac{H_{2A}}{E_{2A}}$, which is given by assumption $M1_B$. $P2_{SA}$'s expected payoffs are identical, but swapping $\underline{\delta}_2$ for $\overline{\delta}_2$. Defection is supported in equilibrium when $\underline{\delta}_2 < 1 - \frac{H_{2A}}{E_{2A}}$, which is also given by assumption $M1_B$.

For $P1$, given $P2$'s equilibrium strategies, its expected utility for first round cooperation is:

$$U_1(C_1) = p_1(H_1(1 + \delta_1)) + q_1((1 - p_1) - S_1) + (1 - p_1)(1 - q_1)(H_1 - \delta_1 S_1) \tag{8}$$

Conversely, its payoff for defection is:

$$U_1(D_1) = p_1(E_1 + \delta_1 H_1) + (1 - p_1)(1 - q_1)(E_1 - \delta_1 S_1) \tag{9}$$

Setting these two equal and solving for p_1 shows that $P1$ will cooperate in round 1 in equilibrium any time:

$$p_1 > 1 - \frac{H_1 - E_1}{q_1(H_1 + S_1 - E_1)} \tag{10}$$

This is assumed by $M1_A$, so $P1$ cooperates in equilibrium. $P1$'s posterior beliefs are derived straightforwardly by applying Bayes' rule.

In round 2, both aggressive types of $P2$ will defect. $P1$ will cooperate if its posterior beliefs surpass p^*:

$$\frac{p_1}{1 + q_1(p_1 - 1)} \geq \frac{S_1}{H_1 + S_1 - E_1} \tag{11}$$

This holds when:

$$p_1 \geq \frac{S_1(1 - q_1)}{H_1 - E_1 + S_1(1 - q_1)} \tag{12}$$

This is given by assumption $M1_A$, so $P1$ cooperates in round 2. Knowing this, both benign types of $P2$ will also cooperate. ■

Lemma 6: $P2^*_{SB} = C_1, C_2$; $P2^*_{LB} = C_1, C_2$; $P2^*_{SA} = D_1, D_2$; $P2^*_{LA} = C_1, D_2$; $P1^* = D_1, C_2|C, D_2|D$; $p_2|C = \frac{p_1}{1+q_1(p_1-1)}$; $p_2|D = 0$; $q_1 < \frac{H_1-E_1}{(1-p_1)(H_1+S_1-E_1)}$; $q_2|D = 1$; $q_2|C = \frac{q_1 p_1}{p_1+(1-q_1)p_1}$ constitutes a perfect Bayesian equilibrium when $\frac{S_1(1-q_1)}{H_1+S_1-E_1-S_1 q_1} < p_1 < 1 - \frac{H_1-E_1}{q_1(H_1+S_1-E_1)}$ and $\frac{S_2}{H_{2B}} < \underline{\delta} < \frac{S_2}{E_{2A}} < \overline{\delta}$.

Proof: The non-cooperative far-sighted misrepresentation equilibrium in the text holds under the following conditions:

$$M2_A \equiv \frac{S_1(1-q_1)}{H_1 - E_1 + S_1(1-q_1)} < p_1 < 1 - \frac{H_1 - E_1}{q_1(H_1 + S_1 - E_1)} \tag{13}$$

$$M2_B \equiv \frac{S_2}{H_{2B}} < \underline{\delta} < \frac{S_2}{E_{2A}} < \overline{\delta} \tag{14}$$

Assume $P1$ adopts the equilibrium strategies described above. Given this, $P2_{SB}$'s expected utility for first round cooperation is $\underline{\delta}_2 H_{2B} - S_2$. Its expected utility for defection is zero. Cooperation is supported in equilibrium when $\underline{\delta}_2 > \frac{S_2}{H_{2B}}$, which is given by assumption $M2_B$. Similarly, $P2_{LB}$'s utility for cooperation is $\overline{\delta}_2 H_{2B} - S_2$, while its utility for defection is zero. Cooperation is again supported by $M2_B$.

For $P2_{LA}$, its expected utility for cooperation is $\overline{\delta}_2 E_{2A} - S_2$, while its payoff for defection is zero. Cooperation is supported in equilibrium when $\overline{\delta}_2 > \frac{S_2}{E_{2A}}$, which is given by assumption $M1_B$. $P2_{SA}$'s expected payoff for cooperation is $\underline{\delta}_2 E_{2A} - S_2$, while its payoff for defection is again zero. Defection holds in equilibrium when $\underline{\delta}_2 < \frac{S_2}{E_{2A}}$, which is again assumed by $M2_B$. Thus, $P2$'s first round strategies described above hold in equilibrium.

Assume $P2$ adopts the strategies specified above. For $P1$, its expected utilities are the same as in Lemma 5. It was shown above that defection would be supported in equilibrium any time:

$$p_1 < 1 - \frac{H_1 - E_1}{q_1(H_1 + S_1 - E_1)} \tag{15}$$

This is given by assumption $M2_A$ above. $P1$'s second round strategy is calculated the same way as Lemma 5. It will reciprocate defection, as $p_2|D_1 = 0$. Similarly, it will reciprocate cooperation so long as p_1 exceeds the threshold defined in equation 12 above. This is also assumed in $M2_A$. ■

Lemma 7: $P2^*_{SB} = D_1, D_2$; $P2^*_{LB} = C_1, C_2$; $P2^*_{SA} = D_1, D_2$; $P2^*_{LA} = D_1, D_2$; $P1^* = D_1, C_2|C, D_2|D$; $p_2|C = 1$; $p_2|D = \frac{p_1 q_1}{1-p_1+p_1 q_1}$; $q_1 < \frac{H_1-E_1}{(1-p_1)(H_1+S_1-E_1)}$; $q_1 < 1-\frac{S_2}{p_1(H_2-S_2)}$, $q_2|C = 0$; $q_2|D = \frac{q_1}{(1-p_1)+p_1 q_1}$ constitutes a perfect Bayesian equilibrium when $\underline{\delta}_2 < \frac{S_2}{H_{2B}} < \overline{\delta}_2 < \frac{S_2}{E_{2A}}$ and $p_1 < \left\{\frac{S_1}{(1-q_1)(H_1+S_1-E_1)}, \frac{S_1}{q_1(H_1+S_1)-E_1}\right\}$.

Proof: This is referred to as the short-sighted hedging equilibrium in the text. This equilibrium holds under the following conditions:

$$M3_A \equiv \underline{\delta}_2 < \frac{S_2}{H_{2B}} < \overline{\delta}_2 < \frac{S_2}{E_{2A}} \tag{16}$$

$$M3_B \equiv p_1 < \left\{\frac{S_1}{(1-q_1)(H_1+S_1-E_1)}, \quad \frac{S_1}{q_1(H_1+S_1)-E_1}\right\} \tag{17}$$

Assume $P1$ plays the strategies specified above. Given this, $P2_{SA}$'s expected utility for defection is zero. Its expected payoff for cooperation is $\underline{\delta}_2 E_{2A} - S_2$. Defection is supported when $\underline{\delta}_2 < \frac{S_2}{E_{2A}}$, which is given by assumption $M3_A$. For $P2_{LA}$, cooperation yields $\overline{\delta}_2 E_{2A} - S_2$, while defection would yield zero. Defection is preferred when $\overline{\delta}_2 < \frac{S_2}{E_{2A}}$, which is again given by $M3_A$.

For $P2_{SB}$, first round defection yields zero, while cooperation yields $\underline{\delta}_2 H_{2B} - S_2$. Defection is preferred when $\underline{\delta}_2 < \frac{S_2}{H_{2B}}$, which is assumed by $M3_A$. Finally, $P2_{LB}$ receives $\overline{\delta}_2 H_{2B} - S_2$ for cooperation, and zero for defection. Cooperation is preferred when $\overline{\delta}_2 \geq \frac{S_2}{H_{2B}}$, which is also assumed by $M3_A$.

Assuming all types of $P2$ adopt the strategies described above, $P1$'s expected utilities are as follows:

$$U_1(C_1) = (1-p_1)(-S_1) + p_1 q_1(-S_1) + p_1(1-q_1)(H_1(1+\delta_1)) \tag{18}$$

$$U_1(D_1) = p_1(1-q_1)(E_1 + \delta_1 H_1) \tag{19}$$

Setting these equal reveals that $P1$ will defect when:

$$p_1 < \frac{S_1}{(1-q_1)(H_1+S_1-E_1)} \tag{20}$$

This is assumed by $M3_B$, so $P1$ defects in round 1.

Given $P2$'s equilibrium strategies, $p_2|C = 1$, so $P1$ will cooperate in round 2 after observing $P2$ cooperate. Using Bayes' rule, $P1$'s posterior beliefs after observing defection are:

$$p_2|D_1 = \frac{p_1 q_1}{1 - p_1 + p_1 q_1} \tag{21}$$

Setting this equal to p^* reveals that $P1$ will defect in round 2 after observing defection in round 1 if:

$$p_1 < \frac{S_1}{q_1(H_1 + S_1) - E_1} \tag{22}$$

This is assumed by $M3_B$, so $P1^*|D_1 = D$. ■

Lemma 8: $P2^*_{SB} = C_1, C_2$; $P2^*_{LB} = C_1, C_2$; $P2^*_{SA} = D_1, D_2$; $P2^*_{LA} = D_1, D_2$; $P1^* = C_1, C_2|C, D_2|D$; $p_2|C = 1$; $p_2|D = 0$; $q_1 \in (0, 1)$; $q_2|C = q_1$; $q_2|D = q_1$ constitutes a perfect Bayesian equilibrium when $p_1 \geq p^*$ and $\overline{\delta}_2 < 1 - \frac{H_{2A}}{E_{2A}}$.

Proof: This separating (by preferences, not time horizons) equilibrium holds under two conditions:

$$M4_A \equiv p_1 \geq p^* \tag{23}$$

$$M4_B \equiv \overline{\delta}_2 < 1 - \frac{H_{2A}}{E_{2A}} \tag{24}$$

Assume $P1$ plays the equilibrium strategies described above. Given this, $P2_{SB}$ and $P2_{LB}$ both receive their maximum utility by cooperating in round 1. $P2_{LA}$ receives $H_{2A} + \overline{\delta}_2 E_{2A}$ for cooperating and E_{2A} if it defects. Defection is supported when $\overline{\delta}_2 < 1 - \frac{H_{2A}}{E_{2A}}$, which is assumed by $M4_B$. The same holds for $P2_{SA}$, swapping $\overline{\delta}_2$ for $\underline{\delta}_2$. When $\overline{\delta}_2$ falls below the threshold defined in $M4_B$, $\underline{\delta}_2$ necessarily does so as well.

Given $P2$'s equilibrium strategies, $P1$'s expected utilities in round 1 are as follows:

$$U_1(C_1) = p_1(H_1(1 + \delta_1)) + (1 - p_1)(-S_1) \tag{25}$$

$$U_1(D_1) = p_1(E_1 + \delta_1 H_1) \tag{26}$$

Setting these equal, $P1$ prefers to cooperate when $p_1 \geq \frac{S_1}{H_1 + S_1 - E_1}$, or $p_1 \geq p^*$.

Because both benign types of $P2$ cooperate while both aggressive types defect, $p_2|C = 1$ and $p_2|D = 0$. $P1$ thus cooperates in round 2 after observing cooperation, and defects after observing defection. Because far-sighted and short-sighted types behave alike, irrespective of their benign/aggressive strategies, $P1$ is unable to update its beliefs about q_1. Its equilibrium strategies, however, are completely independent of q_1. ∎

Lemma 9: $P2^*_{SB} = C_1, C_2$; $P2^*_{LB} = C_1, C_2$; $P2^*_{SA} = D_1, D_2$; $P2^*_{LA} = D_1, D_2$; $P1^* = D_1, C_2|C, D_2|D$; $p_2|C = 1$; $p_2|D = 0$; $q_1 \in (0, 1)$; $q_2|C = q_1$; $q_2|D = q_1$ constitutes a perfect Bayesian equilibrium when $p_1 < p^*$ and $\frac{S_2}{H_{2B}} < \{\underline{\delta}_2, \overline{\delta}_2\} < \frac{S_2}{E_{2A}}$.

Proof: This separating equilibrium holds under two conditions:

$$M5_A \equiv p_1 < p^* \tag{27}$$

$$M5_B \equiv \frac{S_2}{H_{2B}} < \{\underline{\delta}_2, \overline{\delta}_2\} < \frac{S_2}{E_{2A}} \tag{28}$$

Given the equilibrium strategies described above, $P2_{SB}$'s expected utility for first round cooperation is $\underline{\delta}_2 H_{2B} - S_2$, while defection yields an ultimate payoff of zero. Therefore, it will cooperate when $\underline{\delta}_2 > \frac{S_2}{H_{2B}}$. The far sighted benign type's calculus is the same, swapping δ_2 values. Cooperation is supported for both by $M5_B$. For aggressive types, both receive zero if they defect in round 1. Conversely, cooperation yields $\delta_2 E_{2A} - S_2$. Defection is preferred for both when $\{\underline{\delta}_2, \overline{\delta}_2\} < \frac{S_2}{E_{2A}}$, which is assumed by $M5_B$.

$P1$'s expected utilities are:

$$U_1(C_1) = p_1(H_1(1 + \delta_1)) + (1 - p_1)(-S_1) \tag{29}$$

$$U_1(D_1) = p_1(E_1 + \delta_1 H_1) \tag{30}$$

Setting these equal, $P1$ will defect when $p_1 < \frac{S_1}{H_1 + S_1 - E_1}$, or $p_1 < p^*$. This is assumed by $M5_A$. And because both benign types cooperate while both aggressive types defect, $p_2|C = 1$ and $p_2|D = 0$. $P1$ thus cooperates in round 2 after observing cooperation, and defects after observing defection. Because far-sighted and short-sighted types behave alike, irrespective of their benign/aggressive strategies, $P1$ is unable to update its beliefs about q_1. ∎

Lemma 10: $P2^*_{SB} = C_1, C_2$; $P2^*_{LB} = C_1, C_2$; $P2^*_{SA} = C_1, D_2$; $P2^*_{LA} = C_1, D_2$; $P1^* = C_1, C_2|C, D_2|D$; $p_2|C = p_1$; $p_2|D < p^*$; $q_1 \in (0, 1)$; $q_2|C = q_1$; $q_2|D \in (0, 1)$ constitutes a perfect Bayesian equilibrium when $p_1 \geq p^*$ and $\{\underline{\delta}_2, \overline{\delta}_2\} < 1 - \frac{H_{2A}}{E_{2A}}$.

Proof: This pooling equilibrium holds under the following conditions:

$$M6_A \equiv p_1 \geq p^* \tag{31}$$

$$M6_B \equiv \{\underline{\delta}_2, \overline{\delta}_2\} < 1 - \frac{H_{2A}}{E_{2A}} \tag{32}$$

Given these strategies, both benign types reap their highest possible payoff by cooperating in round 1. Aggressive types receive E_{2A} for defecting and $H_{2A} + \delta_2 E_{2A}$ for cooperating. Cooperation is thus supported in equilibrium for both aggressive types when $\{\underline{\delta}_2, \overline{\delta}_2\} < 1 - \frac{H_{2A}}{E_{2A}}$, which is given by $M6_B$.

For $P1$, given that $p_1 \geq p^*$ and $P2$'s pooling behavior prevents it from updating its priors, it will act based only on its first round utilities. And since it knows all four types of $P2$ will cooperate, it will cooperate as well. Since all four types pool, $p_2|C = p_1$, and $P1$ will cooperate in round 2. Off-path, this equilibrium assumes $p_2|D < p^*$, which would lead $P1$ to defect after observing first round defection. ■

Lemma 11: $P2^*_{SB} = D_1, D_2$; $P2^*_{LB} = D_1, D_2$; $P2^*_{SA} = D_1, D_2$; $P2^*_{LA} = D_1, D_2$; $P1^* = D_1, C_2|C, D_2|D$; $p_2|C > p^*$; $p_2|D = p_1$; $q_1 \in (0,1)$; $q_2|D = q_1$; $q_2|C \in (0,1)$ constitutes a perfect Bayesian equilibrium when $p_1 < p^*$ and $\{\underline{\delta}_2, \overline{\delta}_2\} < \left\{\frac{S_2}{H_{2B}}, \frac{S_2}{E_{2A}}\right\}$.

Proof: This non-cooperative pooling equilibrium occurs under the following conditions:

$$M7_A \equiv p_1 < p^* \tag{33}$$

$$M7_B \equiv \{\underline{\delta}_2, \overline{\delta}_2\} < \left\{\frac{S_2}{H_{2B}}, \frac{S_2}{E_{2A}}\right\} \tag{34}$$

Given $P1$'s first round defection, the assumption that $p_1 < p^*$ and $P2$'s pooling behavior, benign types' expected utility for defection is zero. Because it is assumed that $p_2|C > p^*$, cooperation would yield $\delta_2 H_{2B} - S_2$. Defection is supported when $\{\underline{\delta}_2, \overline{\delta}_2\} < \frac{S_2}{H_{2B}}$, which is assumed by $M7_B$. For aggressive types, defection also yields zero. Cooperation yields $\delta_2 E_{2A} - S_2$. Setting these equal, defection is supported when $\{\underline{\delta}_2, \overline{\delta}_2\} < \frac{S_2}{E_{2A}}$. This is also given by $M7_B$.

For $P1$, since both types of $P2$ defect, and its pessimistic prior beliefs carry over into round 2, it prefers to defect in round 1, as defection yields zero, while cooperation yields $-S_1$. $P1$'s priors thus carry over in equilibrium: $p_2|D = p_1 < p^*$, so $P1$ defects in round 2. It is assumed that $p_2|C > p^*$, so off the equilibrium path $P1$ would reciprocate cooperation. In equilibrium, $q_2|D = q_1$. And since $P1$'s second round decision is unaffected by $P2$'s time horizons, its off-path beliefs are irrelevant: $q_2|C \in (0,1)$. ■

Lemma 12: $P2^*_{SB} = D_1, D_2$; $P2^*_{LB} = D_1, D_2$; $P2^*_{SA} = D_1, D_2$; $P2^*_{LA} = D_1, D_2$; $P1^* = D_1, C_2|C, C_2|D$; $p_2|C > p^*$; $p_2|D > p^*$; $q_1 \in (0,1)$; $q_2|D = q_1$; $q_2|C \in (0,1)$ constitutes a perfect Bayesian equilibrium when $p_1 \geq p^*$.

Proof: This non-cooperative pooling equilibrium holds under a single condition: $p_1 \geq p^*$. Given $P1$'s equilibrium strategy, benign $P2$'s expected utility for defection is $\delta_2 H_{2B}$, while their expected utility for cooperation is $\delta_2 H_{2B} - S_2$. Both thus prefer to defect. Aggressive types' expected utilities for defection is $\delta_2 E_{2A}$, while cooperation yields $\delta_2 E_{2A} - S_2$. Again, defection is preferred.

For $P1$, given that all types of $P2$ will defect in round 1, its preferred first round strategy is defection. In equilibrium, its prior beliefs carry over due to $P2$'s pooling behavior, so $p_2|D = p_1$. The second round interaction then becomes, in effect, a one-round game in which it prefers to cooperate *iff* $p_1 \geq p^*$. So, in equilibrium, $P1$ prefers to cooperate in round 2 after observing defection in round 1 when $p_1 \geq p^*$. Off-path, $P1$ also prefers to cooperate after observing cooperation in round 1, so it is assumed that $p_2|C \geq p^*$. q_1 is immaterial in this equilibrium. ■

Lemma 13: $P2^*_{SB} = D_1, D_2$; $P2^*_{LB} = D_1, D_2$; $P2^*_{SA} = D_1, D_2$; $P2^*_{LA} = D_1, D_2$; $P1^* = D_1, D_2|C, D_2|D$; $p_2|C < p^*$; $p_2|D < p^*$; $q_1 \in (0,1)$; $q_2|D = q_1$; $q_2|C \in (0,1)$ constitutes a perfect Bayesian equilibrium when $p_1 < p^*$.

Proof: This non-cooperative pooling equilibrium holds any time $p_1 < p^*$. Given $P1$'s equilibrium strategy, all four types of $P2$'s expected utility for defection is zero, while their expected utility for cooperation is $-S_2$. Both thus prefer to defect in round 1. And since $P1$ defects in round 2 regardless of $P2$'s strategy in round 1, all four types will again defect in round 2.

$P1$ faces a similar calculus. Knowing that all four types of $P2$ will defect in round 1, it will also defect. Its posterior beliefs then carry over, so $p_2|D = p_1$. $P1$ will thus defect in round 2 whenever $p_1 < p^*$, which is assumed. Off-path, it is assumed that $p_2|C < p^*$, so $P1$ will defect in round 2 even if $P2$ cooperates in round 1. ■

Lemma 14: $P2^*_{SB} = D_1, C_2$; $P2^*_{LB} = D_1, C_2$; $P2^*_{SA} = D_1, D_2$; $P2^*_{LA} = D_1, D_2$; $P1^* = D_1, D_2|C, C_2|D$; $p_2|C < p^*$; $p_2|D = p_1$; $q_1 \in (0,1)$; $q_2|D = q_1$; $q_2|C \in (0,1)$ constitutes a perfect Bayesian equilibrium when $p_1 > p^*$.

Proof: This non-cooperative pooling equilibrium holds any time $p_1 > p^*$. It depends on implausible off-path beliefs such that $p_2|C < p_2|D = p_1$. Given $P1$'s equilibrium strategy, benign types' expected utility for defection equals $\delta_2 H_{2B}$, while their expected utility for cooperation is $-S_2$. Defection is preferred. For aggressive types, the expected utility for defection is $\delta_2 E_{2A}$, while cooperation yields $-S_2$. Again, defection is preferred.

For $P1$, first round defection is preferred because it knows that all four types of $P2$ will defect. In round 2, $P1$ will then cooperate after observing defection, as $p_1 > p^*$ and its prior beliefs carry over in equilibrium. Off path, $P1$ defects in response to cooperation given its non-intuitive beliefs: $p_2|C < \{p_1, p^*\}$. In equilibrium, because $P1$ cooperates in round 2, both benign types of $P2$ will cooperate. Both aggressive types will therefore defect. ■

3 Propositions

This section offers formal proofs of the informal propositions offered in the text. I restate these proposition more formally here, before offering the proof.

Proposition 1a: In the far-sighted misrepresentation equilibrium, $p_2|C - p_1$ is increasing in q_1.

Proof: In the FSME, only the short-sighted aggressive type defects in round 1. All others cooperate. Given this, using Bayes' rule, $P1$'s posterior beliefs after observing C are:

$$p_2|C = \frac{p_1}{1 + q_1(p_1 - 1)} \tag{35}$$

Taking the partial derivative with respect to q_1 yields:

$$\frac{\partial u}{\partial q_1}\left(\frac{p_1}{1 + q_1(p_1 - 1)}\right) = \frac{p_1(1 - p_1)}{1 + q_1(p_1 - 1)^2} \tag{36}$$

This expression is positive for all $\{q_1, p_1\} > 0$, so $p_2|C$ is increasing in q_1. As such, $p_2|C - p_1$ is also increasing in q_1. ■

Corollary 1a: The range of p_1 values supporting the far-sighted misrepresentation equilibrium is increasing in q_1.

Proof: Follows from Proposition 1a.

Proposition 1b: The range of p_1 values supporting the cooperative FSME is shrinking in q_1, while the range of p_1 values supporting the non-cooperative FSME is increasing in q_1.

Proof: As described in the text, the cutpoint separating the cooperative and non-cooperative FSME is:

$$p_1 = 1 - \frac{H_1 - E_1}{q_1(H_1 + S_1 - E_1)} \tag{37}$$

Taking the partial of this expression w.r.t. q_1 yields:

$$\frac{\partial u}{\partial q_1}\left(1 - \frac{H_1 - E_1}{q_1(H_1 + S_1 - E_1)}\right) = \frac{H_1 - E_1}{q_1^2(H_1 + S_1 - E_1)} \tag{38}$$

The latter expression is strictly positive given the assumption that $H_1 > E_1$, so the cutpoint above which $P1$ cooperates in round 1 is increasing in q_1. This shrinks the parameter space supporting first round cooperation, and expands the parameter space supporting first round defection. ■

Proposition 2: In the short-sighted hedging equilibrium, $|p_1 - p_2|D$ is decreasing in q_1.

Proof: In the SSHE, $P1$'s posterior beliefs after observing defection are defined using Bayes' rule as:

$$p_2|D = \frac{p_1 q_1}{1 - p_1 + p_1 q_1} \tag{39}$$

This expression is strictly less than p_1, so lower values represent more informative signals. Taking the partial w.r.t. q_1 yields:

$$\frac{\partial u}{\partial q_1}\left(\frac{p_1 q_1}{1 - p_1 + p_1 q_1}\right) = \frac{p_1(1 - p_1)}{(1 + (q_1 - 1)p_1)^2} \tag{40}$$

This expression is strictly positive for all $\{q_1, p_1\} > 0$, so $P1$ revises its prior beliefs downward to a lesser degree as q_1 increases. ■

Proposition 3: Mutual cooperation across both rounds of the game is possible across a wider range of p_1 values under the FSME than is possible in the unidimensional uncertainty game.

Proof: Lemma 1 and Lemma 3 above demonstrated that, in the unidimensional uncertainty game, mutual cooperation across both rounds of the game is only possible when $p_1 > p^*$. Conversely, Lemma 5 demonstrates that mutual cooperation across both rounds is possible in the multidimensional uncertainty game when:

$$p_1 \geq \left\{1 - \frac{H_1 - E_1}{q_1(H_1 + S_1 - E_1)}, \quad \frac{S_1(1 - q_1)}{H_1 - E_1 + S_1(1 - q_1)}\right\} \tag{41}$$

Both of these expressions are strictly less than p^*. Since sustained cooperation is possible in the FSME when p_1 is greater than these thresholds, this implies that cooperation is possible across both rounds under a wider range of p_1 values as compared to the unidimensional uncertainty model. ■