

Appendix

Transformed shell principle coordinate can be expressed by:

$$\begin{aligned}
 \bar{Q}_{11}^n &= Q_{11}^n \cos^4 \theta + 2(Q_{12}^n + 2Q_{66}^n) \sin^2 \theta \cos^2 \theta + Q_{22}^n \sin^4 \theta \\
 \bar{Q}_{12}^n &= (Q_{11}^n + Q_{22}^n - 4Q_{66}^n) \sin^2 \theta \cos^2 \theta + Q_{12}^n (\sin^4 \theta + \cos^4 \theta) \\
 \bar{Q}_{22}^n &= Q_{11}^n \sin^4 \theta + 2(Q_{12}^n + 2Q_{66}^n) \sin^2 \theta \cos^2 \theta + Q_{22}^n \cos^4 \theta \\
 \bar{Q}_{66}^n &= (Q_{11}^n + Q_{22}^n - 2Q_{12}^n - 2Q_{66}^n) \sin^2 \theta \cos^2 \theta + Q_{66}^n (\sin^4 \theta + \cos^4 \theta) \\
 \bar{Q}_{44}^n &= Q_{44}^n \cos^2 \theta + Q_{55}^n \sin^2 \theta \\
 \bar{Q}_{55}^n &= Q_{55}^n \cos^2 \theta + Q_{44}^n \sin^2 \theta
 \end{aligned} \tag{57}$$

Where \bar{Q}_{ij} ($i, j = 1, 2, 3, 4, 5, 6$) presented the transformed reduce stiffness modulus.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{pmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{pmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} \end{Bmatrix} \tag{58a}$$

$$+ \begin{pmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{pmatrix} \begin{Bmatrix} \frac{w_0}{R_1} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2 \\ \frac{1}{2} \left(\frac{\partial w_0}{\partial y} \right)^2 + \frac{w_0}{R_2} \\ \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \end{Bmatrix} + \begin{Bmatrix} A_{31} \\ A_{32} \\ 0 \end{Bmatrix} \phi + \begin{Bmatrix} F_{31} \\ F_{32} \\ 0 \end{Bmatrix} \psi - \begin{Bmatrix} N_x^0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{pmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{pmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} \end{Bmatrix} \tag{58b}$$

$$+ \begin{pmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{pmatrix} \begin{Bmatrix} \frac{w_0}{R_1} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2 \\ \frac{1}{2} \left(\frac{\partial w_0}{\partial y} \right)^2 + \frac{w_0}{R_2} \\ \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \end{Bmatrix} + \begin{Bmatrix} E_{31} \\ E_{32} \\ 0 \end{Bmatrix} \phi + \begin{Bmatrix} Q_{31} \\ Q_{32} \\ 0 \end{Bmatrix} \psi - \begin{Bmatrix} M_x \\ M_y \\ 0 \end{Bmatrix}$$

Here, N_x, N_y, N_{xy} and M_x, M_y, M_{xy} expressed the total in-plane forced and moment resultants and P_x, P_y, P_{xy} and R_x, R_y are the third order stresses resultants can be written as:

$$P_x = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y) z^3 dz \tag{59a}$$

$$R_x, R_y = \int_{-h/2}^{h/2} (\tau_{xz}, \tau_{yz}) z^3 dz \quad (59b)$$

Also, normal forces and moments due to electrical field can be defined by:

$$M_x = \int_{-h/2}^{h/2} (e_{31} \frac{2V}{h} + q_{31} \frac{2\psi}{h}) z dz, M_y = \int_{-h/2}^{h/2} (e_{32} \frac{2V}{h} + q_{32} \frac{2\psi}{h}) z dz \quad (60)$$

$$\int_{-h_{n-1}}^{h_n} \begin{Bmatrix} D_x \\ D_y \end{Bmatrix} \cos(\xi z) dz = F_{11}^e \begin{Bmatrix} \frac{\partial \phi}{\partial x} \\ \frac{\partial \phi}{\partial y} \end{Bmatrix} \quad (61)$$

$$\int_{-h_{n-1}}^{h_n} \begin{Bmatrix} B_x \\ B_y \end{Bmatrix} \cos(\xi z) dz = R_{11}^e \begin{Bmatrix} \frac{\partial \psi}{\partial x} \\ \frac{\partial \psi}{\partial y} \end{Bmatrix} \quad (62)$$

$$\int_{-h_{n-1}}^{h_n} \xi \sin(\xi z) D_z dz = [A_{31} \left(\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} \right) - E_{31} \nabla^2 w] \quad (63)$$

$$\int_{-h_{n-1}}^{h_n} \xi \sin(\xi z) B_z dz = [G_{31} \left(\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} \right) - Q_{31} \nabla^2 w] \quad (64)$$

in which:

$$(A_{31}, E_{31}) = \int_{-h/2}^{h/2} e_{31} \xi \sin(\xi z) \{1, z\}_z dz, (A_{32}, E_{32}) = \int_{-h/2}^{h/2} e_{32} \xi \sin(\xi z) \{1, z\}_z dz \quad (65)$$

$$(F_{31}, F_{33}) = \int_{-h/2}^{h/2} \{s_{11} \cos^2(\xi z), s_{33} \xi^2 \sin^2(\xi z)\} dz \quad (66)$$

$$(G_{31}, Q_{31}) = \int_{-h/2}^{h/2} q_{31} \xi \sin(\xi z) \{1, z\}_z dz, (G_{32}, Q_{32}) = \int_{-h/2}^{h/2} q_{32} \xi \sin(\xi z) \{1, z\}_z dz \quad (67)$$

$$(R_{31}, R_{33}) = \int_{-h/2}^{h/2} \{d_{11} \cos^2(\xi z), d_{33} \xi^2 \sin^2(\xi z)\} dz \quad (68)$$

Motions equations of multiscale composite shell can be expressed in terms of $u, v, w, \varphi_x, \varphi_y$ displacements are obtained by Substituting Eq.(19) into (34) yields:

$$\begin{aligned}
& A_{11} \left(\frac{\partial^2 u_0}{\partial x^2} + \frac{1}{R_1} \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial y^2} \right) + A_{12} \left(\frac{\partial^2 v_0}{\partial x \partial y} + \frac{1}{R_2} \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x \partial y} \right) \\
& + A_{16} \left(\frac{\partial^2 v_0}{\partial y^2} + \frac{\partial^2 u_0}{\partial x \partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x \partial y} \right) + B_{11} \left(\frac{\partial^2 \varphi_x}{\partial x^2} \right) + B_{12} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} \right) \\
& + B_{16} \left(\frac{\partial^2 \varphi_x}{\partial y^2} + \frac{\partial^2 \varphi_y}{\partial x \partial y} \right) - s_1 E_{11} \left(\frac{\partial^2 \varphi_x}{\partial y^2} + \frac{\partial^3 w_0}{\partial x^3} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial x^2} \right) \\
& - s_1 E_{12} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} + \frac{\partial^3 w_0}{\partial x \partial y^2} - \frac{1}{R_2} \frac{\partial^2 v_0}{\partial x \partial y} \right) \\
& - s_1 E_{16} \left(\frac{\partial^2 \varphi_y}{\partial x^2} + \frac{\partial^2 \varphi_x}{\partial x \partial y} + 2 \frac{\partial^3 w_0}{\partial x^2 \partial y} - \frac{1}{R_2} \frac{\partial^2 v_0}{\partial y^2} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial x \partial y} \right) \\
& + A_{16} \left(\frac{\partial^2 u_0}{\partial x \partial y} + \frac{1}{R_1} \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x \partial y} \right) \\
& + A_{26} \left(\frac{\partial^2 v_0}{\partial y^2} + \frac{1}{R_2} \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial y^2} \right) \\
& + A_{66} \left(\frac{\partial^2 v_0}{\partial x \partial y} + \frac{\partial^2 u_0}{\partial y^2} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x^2} \right) + B_{16} \left(\frac{\partial^2 \varphi_x}{\partial x \partial y} \right) + B_{26} \left(\frac{\partial^2 \varphi_y}{\partial y^2} \right) \\
& + B_{66} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} + \frac{\partial^2 \varphi_x}{\partial y^2} \right) - s_1 E_{16} \left(\frac{\partial^2 \varphi_x}{\partial x \partial y} + \frac{\partial^3 w_0}{\partial y \partial x^2} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial x \partial y} \right) \\
& - s_1 E_{26} \left(\frac{\partial^2 \varphi_y}{\partial y^2} + \frac{\partial^3 w_0}{\partial x^3} - \frac{1}{R_2} \frac{\partial^2 v_0}{\partial y^2} \right) \\
& - s_1 E_{66} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} + \frac{\partial^2 \varphi_x}{\partial x^2} + 2 \frac{\partial^3 w_0}{\partial y^2 \partial x} - \frac{1}{R_2} \frac{\partial^2 v_0}{\partial x \partial y} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial y^2} \right) \\
& + \frac{1}{R_1} \left[\hat{A}_{45} \left(\varphi_y + \frac{\partial w_0}{\partial y} - \frac{v_0}{R_2} \right) + \hat{A}_{55} \left(\varphi_x + \frac{\partial w_0}{\partial x} - \frac{u_0}{R_1} \right) \right] + \frac{s_1}{R_1} \left[E_{11} \left(\frac{\partial^2 u_0}{\partial x^2} \right) \right. \\
& + \frac{1}{R_1} \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x^2} \left. \right) + E_{12} \left(\frac{\partial^2 v_0}{\partial x \partial y} + \frac{1}{R_1} \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial x \partial y} \right) \\
& + E_{11} \left(\frac{\partial^2 u_0}{\partial x^2} \right) \\
& + \frac{1}{R_1} \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x^2} \left. \right) + E_{16} \left(\frac{\partial^2 v_0}{\partial y^2} + \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial x^2} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x \partial y} \right) \\
& + F_{11} \left(\frac{\partial^2 \varphi_x}{\partial x^2} \right) + F_{12} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} \right) + F_{16} \left(\frac{\partial^2 \varphi_x}{\partial x^2} + \frac{\partial^2 \varphi_y}{\partial x \partial y} \right)
\end{aligned} \tag{69a}$$

$$\begin{aligned}
& -s_1 H_{11} \left(\frac{\partial^2 \varphi_y}{\partial x^2} + \frac{\partial^3 w_0}{\partial x^3} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial x^2} \right) - s_1 H_{12} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} + \frac{\partial^3 w_0}{\partial x \partial y^2} - \frac{1}{R_2} \frac{\partial^2 v_0}{\partial x \partial y} \right) \\
& - s_1 H_{16} \left(\frac{\partial^2 \varphi_y}{\partial y^2} + \frac{\partial^2 \varphi_x}{\partial x \partial y} + 2 \frac{\partial^3 w_0}{\partial y \partial x^2} - \frac{1}{R_2} \frac{\partial^2 v_0}{\partial x^2} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial x \partial y} \right) \\
& + E_{16} \left(\frac{\partial^2 u_0}{\partial x \partial y} + \frac{1}{R_1} \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x \partial y} \right) \\
& + E_{26} \left(\frac{\partial^2 v_0}{\partial y^2} + \frac{1}{R_2} \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial y^2} \right) \\
& + E_{66} \left(\frac{\partial^2 v_0}{\partial x \partial y} + \frac{\partial^2 u_0}{\partial y^2} + \frac{1}{R_2} \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial y^2} \right) \\
& + F_{16} \left(\frac{\partial^2 \varphi_x}{\partial x \partial y} \right) + F_{26} \left(\frac{\partial^2 \varphi_y}{\partial y^2} \right) + F_{66} \left(\frac{\partial^2 \varphi_x}{\partial y^2} + \frac{\partial^2 \varphi_y}{\partial x \partial y} \right) \\
& - s_1 H_{16} \left(\frac{\partial^2 \varphi_x}{\partial x \partial y} + \frac{\partial^3 w_0}{\partial x \partial y^2} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial x \partial y} \right) \\
& - s_1 H_{26} \left(\frac{\partial^2 \varphi_y}{\partial y^2} + \frac{\partial^3 w_0}{\partial x^3} - \frac{1}{R_2} \frac{\partial^2 v_0}{\partial y^2} \right) \\
& - s_1 H_{66} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} + \frac{\partial^2 \varphi_x}{\partial y^2} + 2 \frac{\partial^3 w_0}{\partial x \partial y^2} - \frac{1}{R_2} \frac{\partial^2 v_0}{\partial x \partial y} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial x \partial y} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial y^2} \right) \\
& + A_{31} \frac{\partial \phi}{\partial x} + G_{31} \frac{\partial \psi}{\partial x} = \bar{I}_0 \left(\frac{\partial^2 u_0}{\partial t^2} \right) - \bar{J}_1 \left(\frac{\partial^2 \varphi_x}{\partial t^2} \right) + s_1 \bar{I}_3 \frac{\partial^2}{\partial t^2} \left(\frac{\partial w_0}{\partial x} \right)
\end{aligned}$$

$$\begin{aligned}
& A_{11} \left(\frac{\partial^2 u_0}{\partial x^2} + \frac{1}{R_1} \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x^2} \right) + A_{26} \left(\frac{\partial^2 v_0}{\partial x \partial y} + \frac{1}{R_2} \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial x \partial y} \right) \\
& + A_{66} \left(\frac{\partial^2 v_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial x \partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x^2} \right) + B_{16} \left(\frac{\partial^2 \varphi_x}{\partial x^2} \right) \\
& + B_{26} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} \right) + B_{66} \left(\frac{\partial^2 \varphi_x}{\partial x^2} + \frac{\partial^2 \varphi_y}{\partial x \partial y} \right) \\
& - s_1 E_{16} \left(\frac{\partial^2 \varphi_x}{\partial x^2} + \frac{\partial^3 w_0}{\partial x^3} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial x^2} \right) \\
& - s_1 E_{26} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} + \frac{\partial^3 w_0}{\partial x \partial y^2} - \frac{1}{R_2} \frac{\partial^2 v_0}{\partial x \partial y} \right) \\
& - s_1 E_{66} \left(\frac{\partial^2 \varphi_y}{\partial x^2} + \frac{\partial^2 \varphi_x}{\partial x \partial y} + 2 \frac{\partial^3 w_0}{\partial x^2 \partial y} - \frac{1}{R_2} \frac{\partial^2 v_0}{\partial x^2} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial x \partial y} \right) \\
& + A_{12} \left(\frac{\partial^2 u_0}{\partial x \partial y} + \frac{1}{R_1} \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x \partial y} \right) \\
& + A_{22} \left(\frac{\partial^2 v_0}{\partial x^2} + \frac{1}{R_2} \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial y^2} \right)
\end{aligned}$$

(69b)

$$\begin{aligned}
& +A_{26} \left(\frac{\partial^2 v_0}{\partial x \partial y} + \frac{\partial^2 u_0}{\partial y^2} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x^2} \right) + B_{12} \left(\frac{\partial^2 \varphi_x}{\partial x \partial y} \right) + B_{22} \left(\frac{\partial^2 \varphi_y}{\partial y^2} \right) \\
& + B_{26} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} + \frac{\partial^2 \varphi_x}{\partial y^2} \right) - s_1 E_{12} \left(\frac{\partial^2 \varphi_x}{\partial x \partial y} + \frac{\partial^3 w_0}{\partial y \partial x^2} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial x \partial y} \right) \\
& - s_1 E_{22} \left(\frac{\partial^2 \varphi_y}{\partial y^2} + \frac{\partial^3 w_0}{\partial x^3} - \frac{1}{R_2} \frac{\partial^2 v_0}{\partial y^2} \right) \\
& - s_1 E_{26} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} + \frac{\partial^2 \varphi_x}{\partial x^2} + 2 \frac{\partial^3 w_0}{\partial y^2 \partial x} - \frac{1}{R_2} \frac{\partial^2 v_0}{\partial x \partial y} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial y^2} \right) \\
& + \frac{1}{R_2} \left[\hat{A}_{45} \left(\varphi_y + \frac{\partial w_0}{\partial y} - \frac{v_0}{R_2} \right) + \hat{A}_{55} \left(\varphi_x + \frac{\partial w_0}{\partial x} - \frac{u_0}{R_1} \right) \right] + \frac{s_1}{R_1} \left[E_{12} \left(\frac{\partial^2 u_0}{\partial x \partial y} \right) \right. \\
& \left. + \frac{1}{R_1} \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x^2} \right] + E_{12} \left(\frac{\partial^2 v_0}{\partial x \partial y} + \frac{1}{R_1} \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial x \partial y} \right) \\
& + E_{22} \left(\frac{\partial^2 u_0}{\partial x^2} \right) \\
& + \frac{1}{R_1} \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x \partial y} \left. \right) + E_{26} \left(\frac{\partial^2 v_0}{\partial x \partial y} + \frac{\partial^2 u_0}{\partial y^2} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x^2} \right) \\
& + F_{12} \left(\frac{\partial^2 \varphi_x}{\partial x \partial y} \right) + F_{22} \left(\frac{\partial^2 \varphi_y}{\partial y^2} \right) + F_{26} \left(\frac{\partial^2 \varphi_x}{\partial x \partial y} + \frac{\partial^2 \varphi_y}{\partial y^2} \right)
\end{aligned}$$

$$\begin{aligned}
& -s_1 H_{12} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} + \frac{\partial^3 w_0}{\partial x^2 \partial y} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial x \partial y} \right) - s_1 H_{22} \left(\frac{\partial^2 \varphi_y}{\partial x^2} + \frac{\partial^3 w_0}{\partial x^3} - \frac{1}{R_2} \frac{\partial^2 v_0}{\partial x^2} \right) \\
& - s_1 H_{26} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} + \frac{\partial^2 \varphi_x}{\partial y^2} + 2 \frac{\partial^3 w_0}{\partial x \partial y^2} - \frac{1}{R_2} \frac{\partial^2 v_0}{\partial x \partial y} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial y^2} \right) \\
& + E_{16} \left(\frac{\partial^2 u_0}{\partial y^2} + \frac{1}{R_1} \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x^2} \right) \\
& + E_{26} \left(\frac{\partial^2 v_0}{\partial y^2} + \frac{1}{R_2} \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial y^2} \right) \\
& + E_{66} \left(\frac{\partial^2 v_0}{\partial x \partial y} + \frac{\partial^2 u_0}{\partial y^2} + \frac{1}{R_2} \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x \partial y} \right) \\
& + F_{16} \left(\frac{\partial^2 \varphi_x}{\partial x^2} \right) + F_{26} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} \right) + F_{66} \left(\frac{\partial^2 \varphi_x}{\partial x^2} + \frac{\partial^2 \varphi_y}{\partial x \partial y} \right) \\
& - s_1 H_{16} \left(\frac{\partial^2 \varphi_x}{\partial x^2} + \frac{\partial^3 w_0}{\partial x \partial y^2} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial x \partial y} \right) \\
& - s_1 H_{16} \left(\frac{\partial^2 \varphi_x}{\partial x^2} + \frac{\partial^3 w_0}{\partial x^3} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial x^2} \right) \\
& - s_1 H_{66} \left(\frac{\partial^2 \varphi_y}{\partial x^2} + \frac{\partial^2 \varphi_x}{\partial x \partial y} + 2 \frac{\partial^3 w_0}{\partial y \partial x^2} - \frac{1}{R_2} \frac{\partial^2 v_0}{\partial x \partial y} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial x \partial y} \right) + A_{32} \frac{\partial \phi}{\partial x} \\
& + G_{32} \frac{\partial \psi}{\partial x} = \bar{I}_0 \left(\frac{\partial^2 v_0}{\partial t^2} \right) - \bar{J}_1 \left(\frac{\partial^2 \varphi_y}{\partial t^2} \right) + s_1 \bar{I}_3 \frac{\partial^2}{\partial t^2} \left(\frac{\partial w_0}{\partial y} \right)
\end{aligned}$$

$$\begin{aligned}
& \hat{A}_{45} \left(\frac{\partial^2 \varphi_y}{\partial x^2} + \frac{\partial^2 w_0}{\partial x \partial y} - \frac{1}{R_2} \frac{\partial v_0}{\partial x} \right) + \hat{A}_{55} \left(\frac{\partial \varphi_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2} - \frac{1}{R_1} \frac{\partial u_0}{\partial x} \right) \\
& + \hat{A}_{44} \left(\frac{\partial^2 \varphi_y}{\partial y^2} + \frac{\partial^2 w_0}{\partial y^2} - \frac{1}{R_2} \frac{\partial v_0}{\partial y} \right) + \hat{A}_{45} \left(\frac{\partial \varphi_x}{\partial y} + \frac{\partial^2 w_0}{\partial x \partial y} - \frac{1}{R_1} \frac{\partial u_0}{\partial y} \right) \\
& + s_1 [E_{11} \left(\frac{\partial^3 u_0}{\partial x^3} + \frac{1}{R_1} \frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial x^2} \frac{\partial^2 w_0}{\partial x^2} + \frac{\partial w_0}{\partial x} \frac{\partial^3 w_0}{\partial x^3} \right) \\
& + E_{12} \left(\frac{\partial^3 v_0}{\partial y \partial x^2} + \frac{1}{R_2} \frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial x \partial y} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial w_0}{\partial y} \frac{\partial^3 w_0}{\partial x^2 \partial y} \right) \\
& + E_{16} \left(\frac{\partial^3 v_0}{\partial x^3} + \frac{\partial^3 u_0}{\partial x^2 \partial y} + \frac{\partial w_0}{\partial y} \frac{\partial^3 w_0}{\partial x^3} + \frac{\partial^2 w_0}{\partial x^2} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^3 w_0}{\partial x^2 \partial y} \right) \\
& + F_{11} \frac{\partial^3 \varphi_x}{\partial x^3}
\end{aligned}$$

(69c)

$$\begin{aligned}
& +F_{12} \frac{\partial^3 \varphi_y}{\partial x^2 \partial y} + F_{16} \left(\frac{\partial^3 \varphi_y}{\partial x^3} + \frac{\partial^3 \varphi_x}{\partial x^2 \partial y} \right) - s_1 H_{11} \left(\frac{\partial^3 \varphi_x}{\partial x^3} + \frac{\partial^4 w_0}{\partial x^4} - \frac{1}{R_1} \frac{\partial^3 u_0}{\partial x^3} \right) \\
& - s_1 H_{12} \left(\frac{\partial^3 \varphi_y}{\partial x^2 \partial y} + \frac{\partial^4 w_0}{\partial x^2 \partial y^2} - \frac{1}{R_2} \frac{\partial^3 v_0}{\partial y \partial x^2} \right) \\
& - s_1 H_{16} \left(\frac{\partial^3 \varphi_y}{\partial x^3} + \frac{\partial^3 \varphi_x}{\partial x \partial y^2} + 2 \frac{\partial^4 w_0}{\partial x^3 \partial y} - \frac{1}{R_2} \frac{\partial^3 v_0}{\partial x^3} - \frac{1}{R_1} \frac{\partial^3 u_0}{\partial x^2 \partial y} \right) \\
& + E_{12} \left(\frac{\partial^3 u_0}{\partial x \partial y^2} + \frac{1}{R_1} \frac{\partial^2 w_0}{\partial y^2} + \frac{\partial^2 w_0}{\partial x \partial y} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^3 w_0}{\partial y^2 \partial x} \right) \\
& + E_{22} \left(\frac{\partial^3 v_0}{\partial y^3} + \frac{1}{R_2} \frac{\partial^2 w_0}{\partial y^2} + \frac{\partial^2 w_0}{\partial y^2} \frac{\partial^2 w_0}{\partial y^2} + \frac{\partial w_0}{\partial y} \frac{\partial^3 w_0}{\partial y^3} \right) \\
& + E_{26} \left(\frac{\partial^3 v_0}{\partial x \partial y^2} + \frac{\partial^3 u_0}{\partial y^3} + \frac{\partial^2 w_0}{\partial x \partial y} \frac{\partial^2 w_0}{\partial y^2} + \frac{\partial w_0}{\partial y} \frac{\partial^3 w_0}{\partial y^3} \right) + F_{12} \frac{\partial^3 \varphi_x}{\partial y^2 \partial x} \\
& + F_{22} \frac{\partial^3 \varphi_y}{\partial y^3} \\
& + F_{26} \left(\frac{\partial^3 \varphi_y}{\partial y^2 \partial x} + \frac{\partial^3 \varphi_x}{\partial y^3} \right) - s_1 H_{12} \left(\frac{\partial^3 \varphi_x}{\partial y^2 \partial x} + \frac{\partial^4 w_0}{\partial x^2 \partial y^2} - \frac{1}{R_1} \frac{\partial^3 v_0}{\partial x \partial y^2} \right) \\
& - s_1 H_{26} \left(\frac{\partial^3 \varphi_y}{\partial x \partial y^2} + \frac{\partial^3 \varphi_x}{\partial y^3} + 2 \frac{\partial^4 w_0}{\partial y^3 \partial x} - \frac{1}{R_2} \frac{\partial^3 v_0}{\partial x \partial y^2} - \frac{1}{R_1} \frac{\partial^3 u_0}{\partial y^3} \right) \\
& + 2E_{16} \left(\frac{\partial^3 u_0}{\partial y \partial x^2} + \frac{1}{R_1} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial^2 w_0}{\partial x \partial y} \frac{\partial^2 w_0}{\partial y^2} + \frac{\partial w_0}{\partial y} \frac{\partial^3 w_0}{\partial y^2 \partial x} \right) \\
& + 2E_{66} \left(\frac{\partial^3 v_0}{\partial y \partial x^2} + \frac{\partial^3 u_0}{\partial y^2 \partial x} + \frac{\partial w_0}{\partial y} \frac{\partial^3 w_0}{\partial y^2 \partial x} + \frac{\partial^2 w_0}{\partial y^2} \frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial x \partial y} \frac{\partial^2 w_0}{\partial x \partial y} \right. \\
& \left. + \frac{\partial w_0}{\partial x} \frac{\partial^3 w_0}{\partial y^2 \partial x} \right) + 2F_{16} \left(\frac{\partial^3 \varphi_x}{\partial x^2 \partial y} \right) + 2F_{66} \left(\frac{\partial^3 \varphi_y}{\partial x^2 \partial y} + \frac{\partial^3 \varphi_x}{\partial y^2 \partial x} \right) \\
& - 2s_1 H_{16} \left(\frac{\partial^3 \varphi_x}{\partial x^2 \partial y} + \frac{\partial^4 w_0}{\partial x^3 \partial y} - \frac{1}{R_1} \frac{\partial^3 u_0}{\partial y \partial x^2} \right) \\
& - 2s_1 H_{26} \left(\frac{\partial^3 \varphi_y}{\partial y^2 \partial x} + \frac{\partial^4 w_0}{\partial x \partial y^3} - \frac{1}{R_2} \frac{\partial^3 v_0}{\partial x \partial y^2} \right) \\
& - 2s_1 H_{66} \left(\frac{\partial^3 \varphi_y}{\partial y \partial x^2} + \frac{\partial^3 \varphi_x}{\partial x \partial y^2} + 2 \frac{\partial^4 w_0}{\partial y^2 \partial x^2} - \frac{1}{R_2} \frac{\partial^3 v_0}{\partial x \partial y^2} - \frac{1}{R_1} \frac{\partial^3 u_0}{\partial x \partial y^2} \right)]
\end{aligned}$$

$$\begin{aligned}
& +A_{26} \left(\frac{\partial v_0}{\partial y} + \frac{\partial u_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \right) + B_{12} \left(\frac{\partial \varphi_x}{\partial x} \right) + B_{22} \left(\frac{\partial \varphi_y}{\partial y} \right) + B_{26} \left(\frac{\partial \varphi_y}{\partial x} + \frac{\partial \varphi_x}{\partial y} \right) \\
& - s_1 E_{12} \left(\frac{\partial \varphi_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2} - \frac{1}{R_1} \frac{\partial u_0}{\partial x} \right) - s_1 E_{22} \left(\frac{\partial \varphi_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2} - \frac{1}{R_2} \frac{\partial v_0}{\partial y} \right) \\
& - s_1 E_{26} \left(\frac{\partial \varphi_y}{\partial x} + \frac{\partial \varphi_x}{\partial y} + 2 \frac{\partial^2 w_0}{\partial x \partial y} - \frac{1}{R_2} \frac{\partial u_0}{\partial y} - \frac{1}{R_2} \frac{\partial u_0}{\partial y} \right) \} + N_0^x \left(\frac{\partial^2 w_0}{\partial x^2} \right) \\
& + E_{31} \frac{\partial^2 \phi}{\partial x^2} + Q_{31} \frac{\partial^2 \psi}{\partial x^2} + E_{32} \frac{\partial^2 \phi}{\partial y^2} + Q_{32} \frac{\partial^2 \psi}{\partial x^2} \\
& = \bar{I}_0 \left(\frac{\partial^2 w_0}{\partial t^2} \right) + s_1^2 \bar{I}_6 \frac{\partial^2}{\partial t^2} \left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \right) + s_1 [\bar{I}_3 \frac{\partial^2}{\partial t^2} \left(\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} \right)] \\
& - \bar{J}_4 \frac{\partial^2}{\partial t^2} \left(\frac{\partial \varphi_y}{\partial x} + \frac{\partial \varphi_x}{\partial y} \right)
\end{aligned}$$

$$\begin{aligned}
& \hat{B}_{11} \left(\frac{\partial^2 u_0}{\partial x^2} + \frac{1}{R_1} \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x^2} \right) + \hat{B}_{12} \left(\frac{\partial^2 v_0}{\partial x \partial y} + \frac{1}{R_2} \frac{\partial w_0}{\partial x} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial x \partial y} \right) \\
& + \hat{B}_{16} \left(\frac{\partial^2 v_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial x \partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial x^2} \right) + \hat{D}_{11} \left(\frac{\partial^2 \varphi_x}{\partial x^2} \right) \\
& + \hat{D}_{12} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} \right) + \hat{B}_{16} \left(\frac{\partial^2 \varphi_x}{\partial x^2} + \frac{\partial^2 \varphi_y}{\partial x \partial y} \right) \\
& - s_1 \hat{F}_{26} \left(\frac{\partial^2 \varphi_y}{\partial y^2} + \frac{\partial^3 w_0}{\partial y^3} - \frac{1}{R_2} \frac{\partial^2 u_0}{\partial y^2} \right) \\
& - s_1 \hat{F}_{16} \left(\frac{\partial^2 \varphi_y}{\partial x \partial y} + \frac{\partial^3 w_0}{\partial y \partial x^2} - \frac{1}{R_1} \frac{\partial^2 v_0}{\partial x \partial y} \right) \\
& - s_1 \hat{F}_{66} \left(\frac{\partial^2 \varphi_x}{\partial y^2} + \frac{\partial^2 \varphi_y}{\partial x \partial y} + 2 \frac{\partial^3 w_0}{\partial y^2 \partial x} - \frac{1}{R_1} \frac{\partial^2 u_0}{\partial y^2} - \frac{1}{R_2} \frac{\partial^2 u_0}{\partial x \partial y} \right) \\
& + \hat{B}_{66} \left(\frac{\partial^2 v_0}{\partial x \partial y} + \frac{\partial^2 u_0}{\partial y^2} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial w_0}{\partial x} \frac{\partial^2 w_0}{\partial y^2} \right) \\
& + \hat{B}_{26} \left(\frac{\partial^2 v_0}{\partial y^2} + \frac{1}{R_2} \frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial y} \frac{\partial^2 w_0}{\partial y^2} \right)
\end{aligned} \tag{69d}$$

$$\begin{aligned}
& +\widehat{D}_{16}\frac{\partial^2\varphi_y}{\partial x\partial y}+\widehat{D}_{26}\frac{\partial^2\varphi_y}{\partial y^2}+\widehat{D}_{66}\left(\frac{\partial^2\varphi_y}{\partial x\partial y}+\frac{\partial^2\varphi_y}{\partial y^2}\right)-s_1\widehat{F}_{16}\left(\frac{\partial^2\varphi_x}{\partial x\partial y}+\frac{\partial^3w_0}{\partial y\partial x^2}-\frac{1}{R_1}\frac{\partial^2u_0}{\partial x\partial y}\right) \\
& -s_1\widehat{F}_{26}\left(\frac{\partial^2\varphi_y}{\partial y^2}+\frac{\partial^3w_0}{\partial x^3}-\frac{1}{R_2}\frac{\partial^2v_0}{\partial y^2}\right) \\
& -s_1\widehat{F}_{66}\left(\frac{\partial^2\varphi_y}{\partial x\partial y}+\frac{\partial^2\varphi_x}{\partial x^2}+2\frac{\partial^3w_0}{\partial y^2\partial x}-\frac{1}{R_2}\frac{\partial^2v_0}{\partial x\partial y}-\frac{1}{R_1}\frac{\partial^2u_0}{\partial y^2}\right) \\
& -\widehat{A}_{45}\left(\varphi_y+\frac{\partial w_0}{\partial y}-\frac{v_0}{R_2}\right)-\widehat{A}_{55}\left(\varphi_x+\frac{\partial w_0}{\partial x}-\frac{u_0}{R_1}\right)+E_{31}\frac{\partial^2\phi}{\partial x^2}+Q_{31}\frac{\partial^2\psi}{\partial x^2} \\
& =\bar{J}_1\frac{\partial^2u_0}{\partial t^2}+\bar{K}_2\frac{\partial^2\varphi_x}{\partial t^2}-s_1\bar{J}_4\frac{\partial^2}{\partial t^2}\left(\frac{\partial w_0}{\partial x}\right) \\
& \widehat{B}_{16}\left(\frac{\partial^2u_0}{\partial x^2}+\frac{1}{R_1}\frac{\partial w_0}{\partial x}+\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial x^2}\right)+\widehat{B}_{26}\left(\frac{\partial^2v_0}{\partial x\partial y}+\frac{1}{R_2}\frac{\partial w_0}{\partial x}+\frac{\partial w_0}{\partial y}\frac{\partial^2w_0}{\partial x\partial y}\right) \\
& +\widehat{B}_{66}\left(\frac{\partial^2v_0}{\partial x^2}+\frac{\partial^2u_0}{\partial x\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial x\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial x^2}\right)+\widehat{D}_{16}\left(\frac{\partial^2\varphi_x}{\partial x^2}\right) \\
& +\widehat{D}_{26}\left(\frac{\partial^2\varphi_y}{\partial x\partial y}\right)+\widehat{D}_{66}\left(\frac{\partial^2\varphi_x}{\partial x^2}+\frac{\partial^2\varphi_y}{\partial x\partial y}\right) \\
& -s_1\widehat{F}_{16}\left(\frac{\partial^2\varphi_y}{\partial x^2}+\frac{\partial^3w_0}{\partial x^3}-\frac{1}{R_2}\frac{\partial^2u_0}{\partial x^2}\right) \\
& -s_1\widehat{F}_{26}\left(\frac{\partial^2\varphi_y}{\partial x\partial y}+\frac{\partial^3w_0}{\partial x\partial y^2}-\frac{1}{R_1}\frac{\partial^2v_0}{\partial x\partial y}\right) \\
& -s_1\widehat{F}_{66}\left(\frac{\partial^2\varphi_x}{\partial x^2}+\frac{\partial^2\varphi_y}{\partial x\partial y}+2\frac{\partial^3w_0}{\partial y^2\partial x}-\frac{1}{R_2}\frac{\partial^2u_0}{\partial x^2}-\frac{1}{R_1}\frac{\partial^2u_0}{\partial x\partial y}\right) \\
& +\widehat{B}_{22}\left(\frac{\partial^2u_0}{\partial x\partial y}+\frac{1}{R_1}\frac{\partial w_0}{\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial x\partial y}\right) \\
& +\widehat{B}_{26}\left(\frac{\partial^2v_0}{\partial x\partial y}+\frac{\partial^2u_0}{\partial y^2}+\frac{\partial w_0}{\partial y}\frac{\partial^2w_0}{\partial x\partial y}+\frac{\partial w_0}{\partial x}\frac{\partial^2w_0}{\partial y^2}\right) \\
& +\widehat{D}_{12}\frac{\partial^2\varphi_y}{\partial x\partial y}+\widehat{D}_{22}\frac{\partial^2\varphi_y}{\partial y^2}+\widehat{D}_{26}\left(\frac{\partial^2\varphi_y}{\partial x\partial y}+\frac{\partial^2\varphi_y}{\partial y^2}\right)-s_1\widehat{F}_{12}\left(\frac{\partial^2\varphi_x}{\partial x\partial y}+\frac{\partial^3w_0}{\partial y\partial x^2}-\frac{1}{R_1}\frac{\partial^2u_0}{\partial x\partial y}\right) \\
& -s_1\widehat{F}_{22}\left(\frac{\partial^2\varphi_y}{\partial y^2}+\frac{\partial^3w_0}{\partial x^3}-\frac{1}{R_2}\frac{\partial^2v_0}{\partial y^2}\right) \\
& -s_1\widehat{F}_{26}\left(\frac{\partial^2\varphi_y}{\partial x\partial y}+\frac{\partial^2\varphi_x}{\partial y^2}+2\frac{\partial^3w_0}{\partial y^2\partial x}-\frac{1}{R_2}\frac{\partial^2v_0}{\partial x\partial y}-\frac{1}{R_1}\frac{\partial^2u_0}{\partial y^2}\right) \\
& -\widehat{A}_{44}\left(\varphi_y+\frac{\partial w_0}{\partial y}-\frac{v_0}{R_2}\right)-\widehat{A}_{45}\left(\varphi_x+\frac{\partial w_0}{\partial x}-\frac{u_0}{R_1}\right)+E_{32}\frac{\partial^2\phi}{\partial y^2}+Q_{32}\frac{\partial^2\psi}{\partial x^2} \\
& =\bar{J}_1\frac{\partial^2v_0}{\partial t^2}+\bar{K}_2\frac{\partial^2\varphi_y}{\partial t^2}-s_1\bar{J}_4\frac{\partial^2}{\partial t^2}\left(\frac{\partial w_0}{\partial y}\right)
\end{aligned} \tag{69e}$$

$$\begin{aligned}
A_{31} \left(\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} \right) - E_{31} \frac{\partial^2 w_0}{\partial x^2} + F_{33} \frac{\partial^2 w_0}{\partial x^2} + F_{31} \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) - G_{31} \phi - Q_{31} \psi &= 0 \\
A_{32} \left(\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} \right) - E_{32} \frac{\partial^2 w_0}{\partial y^2} + F_{33} \frac{\partial^2 w_0}{\partial x^2} + R_{11} \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) - G_{31} \phi - R_{31} \psi &= 0
\end{aligned} \tag{54}$$

Where the cross-sectional rigidities can be expressed as follows:

$$(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}) = \int_{h_k}^{h_{k+1}} \bar{Q}_{ij}^n (1, z, z^2, z^3, z^4, z^6) dz, \quad (i, j = 1, 2, 6) \tag{38}$$

And:

$$\bar{A}_{ij} = A_{ij} - 3s_1 D_{ij}, \bar{D}_{ij} = A_{ij} - 3s_1 F_{ij}, \hat{A}_{ij} = \bar{A}_{ij} - 3s_1 \bar{D}_{ij} (i, j = 4, 5) \tag{39a}$$

$$\hat{B}_{ij} = B_{ij} - s_1 E_{ij}, \hat{F}_{ij} = F_{ij} - s_1 H_{ij}, \hat{D}_{ij} = D_{ij} - s_1 F_{ij} \tag{39b}$$