The Bright Side of Having an Enemy

WEB APPENDIX A

DERIVATION OF NECESSARY AND SUFFICIENT CONDITIONS FOR THE EQUILIBRIUM.

We will first show that the retailers' profit functions are quasi-concave, which implies that a local maximizer of a retailer's profit function is also a global maximizer. Then, we will obtain the retailers' equilibrium prices as a function of the manufacturers' wholesale prices. Finally, we will obtain the necessary and sufficient conditions under which the price pair (w_1^*, w_2^*) from the main text is an equilibrium, and we will demonstrate that given w_i^*, manufacturer j's profit function is quasi-concave, and hence, manufacturer j will not have any profitable non-local deviations from the price w_j^*.

Retailer 1's and retailer 2's profit functions are given by $\pi_{R1} = D_1(p_1 - w_1)$ and $\pi_{R2} = D_2(p_2 - w_2)$, respectively, where D_1 and D_2 are given by

$$D_1 = \begin{cases} 1 - \frac{p_1}{q_1} & \text{if } p_1 < \frac{q_1}{q_2} p_2 \\ 1 - \frac{p_1 - p_2}{q_1 - q_2} & \text{if } \frac{q_1}{q_2} p_2 \le p_1 \le q_1 - q_2 + p_2 \\ 0 & \text{if } p_1 > q_1 - q_2 + p_2 \end{cases}$$

$$D_2 = \begin{cases} 1 - \frac{p_2}{q_2} & \text{if } p_2 < q_2 - q_1 + p_1 \\ \frac{p_1 - p_2}{q_1 - q_2} - \frac{p_2}{q_2} & \text{if } q_2 - q_1 + p_1 \le p_2 \le \frac{q_2}{q_1} p_1 \\ 0 & \text{if } p_2 > \frac{q_2}{q_1} p_1 \end{cases}$$

Note that π_{Ri} is continuous and piecewise concave, i.e., π_{Ri} is concave on each interval on which π_{Ri} takes a specific functional form. Moreover, π_{Ri} is differentiable everywhere except a finite number of kink points. Hence, to show that π_{Ri} is quasi-concave, it is sufficient to show that at a given kink point p_i', if $\frac{\partial_- \pi_{Ri}}{\partial p_i}|_{p_i=p_i'} < 0$, then $\frac{\partial_+ \pi_{Ri}}{\partial p_i}|_{p_i=p_i'} \leq 0$, where $\frac{\partial_- \pi_{Ri}}{\partial p_i}|_{p_i=p_i'}$ and $\frac{\partial_+ \pi_{Ri}}{\partial p_i}|_{p_i=p_i'}$ are the left and right derivatives of π_{Ri} at the kink point p_i'.

The function π_{R1} has two kink points, $p_1' \equiv \frac{q_1}{q_2} p_2$ and $p_1'' \equiv q_1 - q_2 + p_2$. One can show that $\frac{\partial_- \pi_{R1}}{\partial p_1}|_{p_1'} = 1 - \frac{2p_2}{q_2} + \frac{w_1}{q_1}$ and $\frac{\partial_+ \pi_{R1}}{\partial p_1}|_{p_1'} = \frac{p_2(-2q_1+q_2)+q_2(q_1-q_2+w_1)}{(q_1-q_2)q_2}$. We need to show that if $\frac{\partial_- \pi_{R1}}{\partial p_1}|_{p_1'} < 0$, then $\frac{\partial_+ \pi_{Ri}}{\partial p_i}|_{p_1'} \leq 0$. The inequality $\frac{\partial_- \pi_{R1}}{\partial p_1}|_{p_1'} < 0$ is equivalent to $p_2 > \frac{q_1 q_2 + q_2 w_1}{2q_1}$, and $\frac{\partial_+ \pi_{Ri}}{\partial p_i}|_{p_1'} \leq 0$ is equivalent to $p_2 \geq \frac{q_1 q_2 - q_2^2 + q_2 w_1}{2q_1 - q_2}$. Since $q_1 > w_1$ and $q_1 > q_2$, one can show that $\frac{q_1 q_2 + q_2 w_1}{2q_1} > \frac{q_1 q_2 - q_2^2 + q_2 w_1}{2q_1 - q_2}$, which implies that if $\frac{\partial_- \pi_{R1}}{\partial p_1}|_{p_1'} < 0$, then $\frac{\partial_+ \pi_{Ri}}{\partial p_i}|_{p_1'} \leq 0$. Also, note that $\frac{\partial_+ \pi_{R1}}{\partial p_1}|_{p_1''} = 0$. Hence, we found that at either kink point if the left-derivative of π_{R1} is negative, then the right derivative at that point is not positive, which implies that π_{R1} is quasi-concave. A similar argument shows that π_{R2} is quasi-concave.

Quasi-concavity of π_{Ri} implies that a local maximizer of π_{Ri} satisfying the first-order condition ($\frac{\partial \pi_{Ri}}{\partial p_i} = 0$) is also a global maximizer. Hence, using the first-order conditions, one can show that the retailers' equilibrium prices are as follows:

$$p_1^* = \begin{cases} \frac{q_1+w_1}{2} & \text{if } 0 \leq w_1 < \max\{0, \frac{2q_1 w_2 - q_1 q_2}{q_2}\} \\ \frac{q_1}{q_2} w_2 & \text{if } \max\{0, \frac{2q_1 w_2 - q_1 q_2}{q_2}\} \leq w_1 < \max\{0, \frac{q_2^2 - q_1 q_2 + 2q_1 w_2 - q_2 w_2}{q_2}\} \\ \frac{q_1(2q_1 - 2q_2 + 2w_1 + w_2)}{4q_1 - q_2} & \text{if } \max\{0, \frac{q_2^2 - q_1 q_2 + 2q_1 w_2 - q_2 w_2}{q_2}\} \leq \mathrm{w}_1 \leq \frac{2q_1^2 - 2q_1 q_2 + q_1 w_2}{2q_1 - q_2} \\ w_1 & \text{if } \frac{2q_1^2 - 2q_1 q_2 + q_1 w_2}{2q_1 - q_2} < w_1 \end{cases}$$

$$p_2^* = \begin{cases} \frac{q_2+w_2}{2} & \text{if } 0 \le w_2 < \max\{0, q_2+2w_1-2q_1\} \\ w_1+q_2-q_1 & \text{if } \max\{0, q_2+2w_1-2q_1\} \le w_2 < \max\{0, \frac{2q_1(q_2+w_1-q_1)-q_2w_1}{q_1}\} \\ \frac{q_1(q_2+2w_2)-q_2(q_2-w_1)}{4q_1-q_2} & \text{if } \max\{0, \frac{2q_1q_2+2q_1w_1-2q_1^2-q_2w_1}{q_1}\} \le w_2 \le \frac{q_1q_2-q_2^2+q_2w_1}{2q_1-q_2} \\ w_2 & \text{if } \frac{q_1q_2-q_2^2+q_2w_1}{2q_1-q_2} < w_2 \end{cases}$$

Using the equilibrium retail prices (p_1^*, p_2^*), we can obtain the manufacturers' subgame equilibrium profit functions:

$$\pi_{M1} = \begin{cases} \frac{(w_1-c_1)(q_1-2q_2-w_1+2w_2)}{2(q_1-q_2)} & \text{if } 0 \le w_1 < \max\{0, \frac{2q_1w_2-q_1q_2}{q_2}\} \\ \frac{(w_1-c_1)(q_2-w_2)}{q_2} & \text{if } \max\{0, \frac{2q_1w_2-q_1q_2}{q_2}\} \le w_1 < \max\{0, \frac{q_2^2+2q_1w_2-q_1q_2-q_2w_2}{q_2}\} \\ \frac{(w_1-c_1)(2q_1^2+q_2w_1-q_1(2q_2+2w_1-w_2))}{4q_1^2-5q_1q_2+q_2^2} & \text{if } \max\{0, \frac{q_2^2+2q_1w_2-q_1q_2-q_2w_2}{q_2}\} \le w_1 \le \frac{2q_1^2-2q_1q_2+q_1w_2}{2q_1-q_2} \\ 0 & \text{if } \frac{2q_1^2-2q_1q_2+q_1w_2}{2q_1-q_2} < w_1 \end{cases}$$

$$\pi_{M2} =$$

$$\begin{cases} \frac{(w_2-c_2)(q_2-w_2)}{2q_2} & \text{if } 0 \le w_2 < \max\{0, q_2+2w_1-2q_1\} \\ \frac{(q_1-w_1)(w_2-c_2)}{q_2} & \text{if } \max\{0, q_2+2w_1-2q_1\} \le w_2 < \max\{0, \frac{2q_1(q_2+w_1-q_1)-q_2w_1}{q_1}\} \\ \frac{q_1(w_2-c_2)(q_1(q_2-2w_2)+q_2(w_1+w_2-q_2))}{(q_1-q_2)(4q_1-q_2)q_2} & \text{if } \max\{0, \frac{2q_1(q_2+w_1-q_1)-q_2w_1}{q_1}\} \le w_2 \le \frac{q_1q_2-q_2^2+q_2w_1}{2q_1-q_2} \\ 0 & \text{if } \frac{q_1q_2-q_2^2+q_2w_1}{2q_1-q_2} < w_2 \end{cases}$$

We are looking for an equilibrium where each manufacturer has a positive market share. Hence, the equilibrium wholesale prices (w_1^*, w_2^*) must satisfy $\max\{0, \frac{q_2^2+2q_1w_2^*-q_1q_2-q_2w_2^*}{q_2}\} < w_1^* < \frac{2q_1^2-2q_1q_2+q_1w_2^*}{2q_1-q_2}$, or equivalently, $\max\{0, \frac{2q_1q_2+2q_1w_1^*-2q_1^2-q_2w_1^*}{q_1}\} < w_2^* < \frac{q_1q_2-q_2^2+q_2w_1^*}{2q_1-q_2}$. Let us assume that such an equilibrium exists. Then, the equilibrium prices must satisfy the first order conditions, $\frac{\partial \pi_{M1}}{\partial w_1} = 0$ and $\frac{\partial \pi_{M2}}{\partial w_2} = 0$, where $\pi_{M1} = \frac{(w_1-c_1)(2q_1^2+q_2w_1-q_1(2q_2+2w_1-w_2))}{4q_1^2-5q_1q_2+q_2^2}$ and $\pi_{M2} = \frac{q_1(w_2-c_2)(q_1(q_2-2w_2)+q_2(w_1+w_2-q_2))}{(q_1-q_2)(4q_1-q_2)q_2}$. Solving the first-order conditions, we find that

$$w_1^* = \frac{2c_1(2q_1-q_2)^2+q_1(2q_1c_2+8q_1^2-q_2c_2-11q_1q_2+3q_2^2)}{16q_1^2-17q_1q_2+4q_2^2},$$

$$w_2^* = \frac{2c_2(2q_1-q_2)^2+q_2(2q_1c_1+6q_1^2-q_2c_1-8q_1q_2+2q_2^2)}{16q_1^2-17q_1q_2+4q_2^2}.$$

Note that the wholesale prices (w_1^*, w_2^*) are the same as in the main text. We need to verify two things. First, (w_1^*, w_2^*) must satisfy $\max\{0, \frac{q_2^2+2q_1w_2^*-q_1q_2-q_2w_2^*}{q_2}\} < w_1^* < \frac{2q_1^2-2q_1q_2+q_1w_2^*}{2q_1-q_2}$, so that under (w_1^*, w_2^*) each manufacturer has a positive market share. Second, we need to show that given manufacturer i's wholesale price w_i^*, manufacturer j's profit function π_{Mj} is quasi-concave in w_j, which will imply that manufacturer j does not have any profitable non-local deviations from w_j^*.

Straightforward algebra shows that $\max\{0, \frac{q_2^2+2q_1w_2^*-q_1q_2-q_2w_2^*}{q_2}\} < w_1^* < \frac{2q_1^2-2q_1q_2+q_1w_2^*}{2q_1-q_2}$ is equivalent to $c_2 \in (\underline{c}, \bar{c})$, where $\underline{c} \equiv \max\{0, \frac{8c_1q_1^2-8q_1^3-9c_1q_1q_2+11q_1^2q_2+2c_1q_2^2-3q_1q_2^2}{2q_1^2-q_1q_2}\}$ and $\bar{c} \equiv \frac{2c_1q_1q_2+6q_1^2q_2-c_1q_2^2-8q_1q_2^2+2q_2^3}{8q_1^2-9q_1q_2+2q_2^2}$. To see that the interval $(\underline{c}, \bar{c})$ is non-empty, note that when $q_1 = 1$, $q_2 = 0.5$, $c_1 = 0.4$, we have $\underline{c} = 0$ and $\bar{c} = 0.3875$.

Next, let us show that if $c_2 \in (\underline{c}, \bar{c})$ and $w_i = w_i^*$, then π_{Mj} is quasi-concave, where $i, j \in \{1,2\}$ and $i \neq j$. Consider the case with $j = 1$ and $i = 2$. From the expression for π_{M1}, we can see that π_{M1} is continuous and piecewise concave, i.e., π_{M1} is concave on each interval on which π_{M1} takes a specific functional form. Moreover, π_{M1} is differentiable everywhere except a finite number of kink points. Hence, to show that π_{M1} is quasi-concave, it is sufficient to show that at a given kink point $\widetilde{w}_1$, if $\frac{\partial_- \pi_{M1}}{\partial w_1}|_{w_1=\widetilde{w}_1} < 0$, then $\frac{\partial_+ \pi_{M1}}{\partial w_1}|_{w_1=\widetilde{w}_1} \leq 0$, where $\frac{\partial_- \pi_{M1}}{\partial w_1}|_{w_1=\widetilde{w}_1}$ and $\frac{\partial_+ \pi_{M1}}{\partial w_1}|_{w_1=\widetilde{w}_1}$ are the left and right derivatives of π_{M1} at the kink point $\widetilde{w}_1$. The function π_{M1} has up to three kink points, $w_1' \equiv \frac{2q_1w_2^*-q_1q_2}{q_2}$, $w_1'' \equiv \frac{q_2^2+2q_1w_2^*-q_1q_2-q_2w_2^*}{q_2}$, and $w_1''' \equiv \frac{2q_1^2-2q_1q_2+q_1w_2^*}{2q_1-q_2}$. Consider the first kink point $w_1' \equiv \frac{2q_1w_2^*-q_1q_2}{q_2}$, and let us assume that it exists, i.e., $\frac{2q_1w_2^*-q_1q_2}{q_2} > 0$. One can show that after substituting in the expression for w_2^*, $\frac{\partial_-\pi_{M1}}{\partial w_1}|_{w_1=w_1'}$ is as follows:

$$\frac{\partial_-\pi_{M1}}{\partial w_1}|_{w_1=w_1'} = \frac{q_2(q_1(24q_1^2-19q_1q_2+4q_2^2)+c_1(8q_1^2-13q_1q_2+4q_2^2))-8c_2q_1(2q_1-q_2)^2}{2q_1q_2(16q_1^2-17q_1q_2+4q_2^2)}$$

Further, $\frac{\partial\pi_{M1}}{\partial w_1}|_{w_1=w_1'} > 0$ if and only if $c_2 < \tilde{c}$, where $\tilde{c} \equiv \frac{8c_1q_1^2q_2+24q_1^3q_2-13c_1q_1q_2^2-19q_1^2q_2^2+4c_1q_2^3+4q_1q_2^3}{32q_1^3-32q_1^2q_2+8q_1q_2^2}$. Finally, one can show that $\bar{c} < \tilde{c}$. Hence, if $c_2 \in (\underline{c}, \bar{c})$, then $\frac{\partial_-\pi_{M1}}{\partial w_1}|_{w_1=w_1'} > 0$.

Now, consider the second kink point $w_1'' \equiv \frac{q_2^2+2q_1w_2^*-q_1q_2-q_2w_2^*}{q_2}$, and let us assume that it exists, i.e., $\frac{q_2^2+2q_1w_2^*-q_1q_2-q_2w_2^*}{q_2} > 0$. Since for $w_1 \in [\max\{0, \frac{2q_1w_2-q_1q_2}{q_2}\}, w_1'')$, π_{M1} is a strictly increasing function in w_1, it must be that $\frac{\partial_-\pi_{M1}}{\partial w_1}|_{w_1=w_1''} > 0$.

Finally, for the third kink point $w_1''' \equiv \frac{2q_1^2-2q_1q_2+q_1w_2^*}{2q_1-q_2}$, one can easily show that $\frac{\partial_-\pi_{M1}}{\partial w_1}|_{w_1=w_1'''} < 0$. However, since for $w_1 > w_1'''$, we have $\pi_{M1} = 0$, it follows that $\frac{\partial_+\pi_{M1}}{\partial w_1}|_{w_1=w_1''} = 0$.

Hence, the conditions for quasi-concavity are satisfied at all kink points of π_{M1}, i.e., π_{M1} is a quasi-concave function. A similar proof shows that π_{M2} is also quasi-concave.

Since w_i^* is a local maximizer of manufacturer i's profit function π_{Mi} (since w_i^* was obtained using the first-order conditions) and since π_{Mi} is a quasi-concave function, it follows that w_i^* is a global maximizer of π_{Mi}, i.e., manufacturer i does not have any profitable non-local deviations from w_i^*. ■

PROOF OF PROPOSITION 5.

We start with Lemma W1, which characterizes the equilibrium outcome when manufacturer 1 sells two products through retailer 1.

LEMMA W1. *Suppose that the monopolist manufacturer 1 sells products 1 and 2. In equilibrium, the wholesale prices are* $\widehat{w}_1 = \frac{q_1+c_1}{2}$ *and* $\widehat{w}_2 = \frac{q_2+c_2}{2}$*, and the retail prices are* $\hat{p}_1 = \frac{3q_1+c_1}{4}$ *and* $\hat{p}_2 = \frac{3q_2+c_2}{4}$*. Manufacturer 1's equilibrium profit is*

$$\hat{\pi}_{M1} = \frac{q_1c_2^2-2c_1c_2q_2+q_2(c_1^2-2q_1c_1+q_1^2+2q_2c_1-q_1q_2)}{8(q_1-q_2)q_2}. \tag{W1}$$

Proof of Lemma W1. We find the subgame perfect equilibrium by backward induction. Given the wholesale prices w_1 and w_2, retailer 1's profit can be readily derived as $\pi_{R1} = (1 - \frac{p_1-p_2}{q_1-q_2})(p_1 - w_1) + (\frac{p_1-p_2}{q_1-q_2} - \frac{p_2}{q_2})(p_2 - w_2)$. Retailer 1 chooses p_1 and p_2 to maximize π_{R1}. Solving the first-order conditions ($\frac{\partial \pi_{R1}}{\partial p_1} = 0$ and $\frac{\partial \pi_{R1}}{\partial p_2} = 0$), we find retailer 1's optimal retail prices: $\hat{p}_1(w_1, w_2) = \frac{w_1+q_1}{2}$ and $\hat{p}_2(w_1, w_2) = \frac{w_2+q_2}{2}$. Manufacturer 1 anticipates retailer 1's best-response to its wholesale prices, and chooses (w_1, w_2) to maximize $\pi_{M1} = (1 - \frac{\hat{p}_1(w_1,w_2)-\hat{p}_2(w_1,w_2)}{q_1-q_2})(w_1 - c_1) + (\frac{\hat{p}_1(w_1,w_2)-\hat{p}_2(w_1,w_2)}{q_1-q_2} - \frac{\hat{p}_2(w_1,w_2)}{q_2})(w_2 - c_2)$. Solving the first order conditions, we find that manufacturer 1's optimal wholesale prices are given by $\widehat{w}_1 = \frac{q_1+c_1}{2}$ and $\widehat{w}_2 = \frac{q_2+c_2}{2}$. Using $(\widehat{w}_1, \widehat{w}_2)$, one can readily find retailer 1's equilibrium prices: $\hat{p}_1 \equiv \hat{p}_1(\widehat{w}_1, \widehat{w}_2) = \frac{3q_1+c_1}{4}$ and $\hat{p}_2 \equiv \hat{p}_2(\widehat{w}_1, \widehat{w}_2) = \frac{3q_2+c_2}{4}$. Plugging the equilibrium retail and wholesale prices into manufacturer 1's profit function, one can show that $\hat{\pi}_{M1} = \frac{q_1c_2^2-2c_1c_2q_2+q_2(c_1^2-2q_1c_1+q_1^2+2q_2c_1-q_1q_2)}{8(q_1-q_2)q_2}$. □

One can verify that $\max\{0, c_1 - q_1 + q_2\} < c_2 < \frac{c_1q_2}{q_1}$ is the necessary and sufficient condition for the existence of the equilibrium in Lemma W1 (i.e., both products have a positive market share). Note that $(\max\{0, c_1 - q_1 + q_2\}, \frac{c_1 q_2}{q_1}) \subset (\underline{c}, \bar{c})$, where $\underline{c}$ and $\bar{c}$ are defined in the main text. In the rest of this proof, we assume that $\max\{0, c_1 - q_1 + q_2\} < c_2 < \frac{c_1 q_2}{q_1}$.

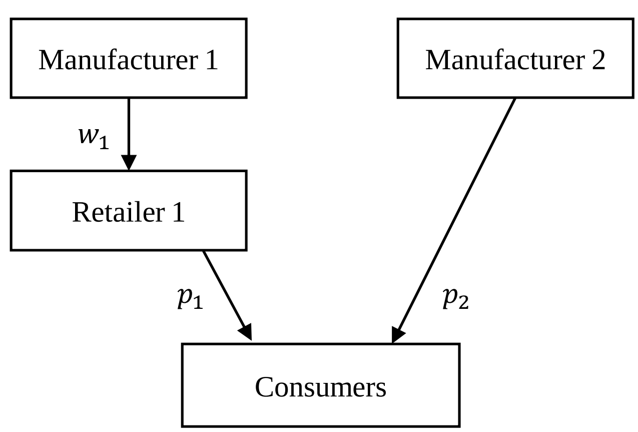
If manufacturer 1 does not spin off product 2, then its equilibrium profit is $\pi_{M1}^{*\text{NoSpinoff}} = \hat{\pi}_{M1}$ as specified in the equation (W1). If manufacturer 2 spins off product 2, its equilibrium profit is $\pi_{M1}^{*\text{Spinoff}} \equiv \pi_{M1}^{*}$, where π_{M1}^{*} is as in equation (3) in the Appendix of the main paper.

Define $\Delta\pi \equiv \pi_{M1}^{*\text{Spinoff}} - \pi_{M1}^{*\text{NoSpinoff}}$. Straightforward algebra shows that $\Delta\pi|_{c_2 = \frac{c_1 q_2}{q_1}} = \frac{(q_1 - c_1)^2 q_2 (128 q_1^4 - 404 q_1^3 q_2 + 361 q_1^2 q_2^2 - 128 q_1 q_2^3 + 16 q_2^4)}{8 q_1 (4 q_1 - q_2)(16 q_1^2 - 17 q_1 q_2 + 4 q_2^2)^2}$. One can show that if $q_2 < \hat{q}$, then $\Delta\pi|_{c_2 = \frac{c_1 q_2}{q_1}} > 0$, where $\hat{q} \approx 0.509 q_1$. By continuity of $\Delta\pi$ in c_2, it follows that if $q_2 < \hat{q}$, then there exists $\hat{c} \in [\max\{0, c_1 - q_1 + q_2\}, \frac{c_1 q_2}{q_1})$ such that if $c_2 \in (\hat{c}, \frac{c_1 q_2}{q_1})$, then $\Delta\pi > 0$, or equivalently, $\pi_{M1}^{*\text{Spinoff}} > \pi_{M1}^{*\text{NoSpinoff}}$. In words, if $q_2 < \hat{q}$ and $c_2 \in (\hat{c}, \frac{c_1 q_2}{q_1})$, manufacturer 2's exit will make manufacturer 1 worse off even if manufacturer 1 can produce both the high-quality and low-quality products. ■

EQUILIBRIUM WHEN MANUFACTURER 2 IS SELLING DIRECTLY TO CONSUMERS.

Figure W1 illustrates the manufacturers' channel structures.

Figure W1 Manufacturers' Distribution Channels



Retailer 1's and manufacturer 2's profit functions are given by $\pi_{R1}^D(p_1,p_2)=(1-\frac{p_1-p_2}{q_1-q_2})(p_1-w_1)$ and $\pi_{M2}^D(p_1,p_2)=(\frac{p_1-p_2}{q_1-q_2}-\frac{p_2}{q_2})(p_2-c_2)$, respectively. Note that the superscript D indicates that manufacturer 2 sells directly to consumers. Retailer 1 chooses p_1 to maximize π_{R1}^D and manufacturer 2 chooses p_2 to maximize π_{M2}^D. Solving the first-order conditions ($\frac{\partial\pi_{R1}^D}{\partial p_1}=0$ and $\frac{\partial\pi_{M2}^D}{\partial p_2}=0$), we obtain the subgame equilibrium prices, $p_1^D(w_1)=\frac{q_1(c_2+2(w_1+q_1-q_2))}{4q_1-q_2}$ and $p_2^D(w_1)=\frac{2c_2q_1+(w_1+q_1-q_2)q_2}{4q_1-q_2}$. Manufacturer 1 anticipates retail prices $p_1^D(w_1)$ and $p_2^D(w_1)$, and chooses its wholesale price w_1 to maximize $\pi_{M1}^D(w_1)=(1-\frac{p_1^*(w_1)-p_2^*(w_1)}{q_1-q_2})(w_1-c_1)$. One can show that manufacturer 1's optimal wholesale price is $w_1^D=\frac{q_1(c_2+2q_1-2q_2)+c_1(2q_1-q_2)}{4q_1-2q_2}$, which allows us to also determine the equilibrium retail price for each product:

$$p_1^D=\frac{q_1(c_1(2q_1-q_2)+(c_2+2q_1-2q_2)(3q_1-q_2))}{8q_1^2-6q_1q_2+q_2^2},$$
$$p_2^D=\frac{2c_2q_1(4q_1-2q_2)+q_2(2c_1q_1+c_2q_1+6q_1^2-c_1q_2-8q_1q_2+2q_2^2)}{(4q_1-q_2)(4q_1-2q_2)}. \tag{W2}$$

Finally, using the equilibrium wholesale and retail prices, we easily obtain the manufacturers' and retailer 1's equilibrium profits:

$$\pi_{M1}^D=\frac{(q_1(c_2+2q_1-2q_2)-c_1(2q_1-q_2))^2}{4(2q_1-q_2)(4q_1^2-5q_1q_2+q_2^2)},$$
$$\pi_{M2}^D=\frac{q_1(q_2\left(2c_1q_1+c_2q_1+6q_1^2-c_1q_2-8q_1q_2+2q_2^2\right)-c_2(2q_1-q_2)(4q_1-2q_2))^2}{2(q_1-q_2)q_2(4q_1-q_2)^2(2q_1-q_2)}, \tag{W3}$$
$$\pi_{R1}^D=\frac{(q_1(c_2+2q_1-2q_2)-c_1(2q_1-q_2))^2}{4(q_1-q_2)(4q_1-q_2)^2}.$$

■

PROOF OF PROPOSITION 6.

First, we will prove that when manufacturer 2 sells directly to consumers, its exit can make both manufacturer 1 and retailer 1 worse off. If manufacturer 2 is in the market, we know that manufacturer 1's and retailer 1's equilibrium profits are π_{M1}^D and π_{R1}^D, respectively (see equation (W3)). Recall from the main text that if manufacturer 2 exits the market, manufacturer 1's and retailer 1's profits are given by $\pi_{M1}^{**} = \frac{(q_1-c_1)^2}{8q_1}$ and $\pi_{R1}^{**} = \frac{(q_1-c_1)^2}{16q_1}$, respectively. Define $\Delta\pi_{M1} \equiv \pi_{M1}^D - \pi_{M1}^{**}$ and $\Delta\pi_{R1} \equiv \pi_{R1}^D - \pi_{R1}^{**}$. We will find conditions under which $\Delta\pi_{M1} > 0$ and $\Delta\pi_{R1} > 0$. Note that we focus on parameter values such that in equilibrium each manufacturer has a positive market share. One can show that the necessary and sufficient condition for such an equilibrium is given by $\max\{0, \kappa\} < c_2 < \bar{c}$, where $\kappa \equiv \frac{2c_1q_1-2q_1^2-c_1q_2+2q_1q_2}{q_1}$ and $\bar{c} = \frac{2c_1q_1q_2+6q_1^2q_2-c_1q_2^2-8q_1q_2^2+2q_2^3}{8q_1^2-9q_1q_2+2q_2^2}$. One can also show that there exist q' and q'' such that *i)* if $q_2 < q'$, then $\Delta\pi_{M1}|_{c_2=\bar{c}} = \frac{(q_1-c_1)^2q_2(32q_1^3-57q_1^2q_2+28q_1q_2^2-4q_2^3)}{8q_1(8q_1^2-9q_1q_2+2q_2^2)^2} > 0$, and *ii)* if $q_2 < q''$, then $\Delta\pi_{R1}|_{c_2=\bar{c}} = \frac{(q_1-c_1)^2q_2(16q_1^3-33q_1^2q_2+20q_1q_2^2-4q_2^3)}{16q_1(8q_1^2-9q_1q_2+2q_2^2)^2} > 0$. By continuity of $\Delta\pi_{M1}$ and $\Delta\pi_{R1}$, it follows that if $q_2 < \min\{q', q''\}$, then there exists $\hat{c} < \bar{c}$ such that if $c_2 \in (\hat{c}, \bar{c})$, then $\Delta\pi_{M1} > 0$ and $\Delta\pi_{R1} > 0$.

It remains to show that manufacturer 1 can become better off when product 2's quality increases. Recall that when manufacturer 2 is in the market, manufacturer 1's profit is π_{M1}^D as in the equation (W3). We will show that there exists $\tilde{c}$ such that if $c_2 \in (\tilde{c}, \bar{c})$, then $\frac{\partial\pi_{M1}^D}{\partial q_2} > 0$. Straightforward algebra shows that

$$\frac{\partial\pi_{M1}^D}{\partial q_2} = \frac{(q_1(2q_2-c_2-2q_1)+c_1(2q_1-q_2))(c_1(12q_1^3-14q_1^2q_2+6q_1q_2^2-q_2^3)+q_1(4q_1^3+14c_2q_1q_2-14c_2q_1^2-3c_2q_2^2-6q_1q_2^2+2q_2^3))}{4(2q_1-q_2)^2(4q_1^2-5q_1q_2+q_2^2)^2}$$

One can show that if $c_2 \in (\tilde{c}, \bar{c})$, then $\frac{\partial\pi_{M1}^D}{\partial q_2} > 0$, where

$$\tilde{c} \equiv \frac{12c_1q_1^3+4q_1^4-14c_1q_1^2q_2+6c_1q_1q_2^2-6q_1^2q_2^2-c_1q_2^3+2q_1q_2^3}{14q_1^3-14q_1^2q_2+3q_1q_2^2}.$$

The following numerical example verifies non-emptiness of $(\tilde{c},\bar{c})$. Suppose $q_1 = 1$, $q_2 = 0.85$, and $c_1 = 0.05$. One can verify that $0.3 \in (\tilde{c},\bar{c})$, and if $c_2 = 0.3$, then $\frac{\partial \pi_{M1}^{*D}}{\partial q_2} = 0.09 > 0$. ■

WEB APPENDIX B

EXTENSION WITH CONSUMER HETEROGENEITY ONLY IN HORIZONTAL PREFERENCES.

We use a standard horizontal-differentiation model where manufacturer 1's product is "located" at 0 and manufacturer 2's product is at 1, and consumers are uniformly distributed on the line segment between 0 and 1. The total number of consumers is normalized to one. Each manufacturer sells through its exclusive retailer. If a consumer purchases manufacturer 1's product, she will obtain a utility $q_1 - p_1 - tx$, and if she purchases manufacturer 2's product, her utility will be $q_2 - p_2 - t(1-x)$, where q_i is manufacturer i's product quality, p_i is the retail price, $x \in [0,1]$ represents the consumer's horizontal preference and t measures the strength of the consumers' horizontal preferences. One can readily derive product 1's demand D_1:

If $p_2 \leq q_2 - t$, then

$$D_1 = \begin{cases} 1 & \text{if } p_1 \leq q_1 - q_2 + p_2 - t \\ \frac{q_1 - q_2 - p_1 + p_2 + t}{2t} & \text{if } q_1 - q_2 + p_2 - t < p_1 < q_1 - q_2 + p_2 + t \\ 0 & \text{if } p_1 \geq q_1 - q_2 + p_2 + t \end{cases}$$

If $q_2 - t < p_2 < q_2$, then

$$D_1 = \begin{cases} 1 & \text{if } p_1 \leq q_1 - q_2 + p_2 - t \\ \frac{q_1 - q_2 - p_1 + p_2 + t}{2t} & \text{if } q_1 - q_2 + p_2 - t < p_1 < q_1 + q_2 - p_2 - t \\ \frac{q_1 - p_1}{t} & \text{if } q_1 + q_2 - p_2 - t \leq p_1 < q_1 \\ 0 & \text{if } p_1 \geq q_1 \end{cases}$$

The demand function for product 2 can be obtained in a similar way. To ensure quasi-concavity of retailer i's profit function, we assume that $t < \frac{q_i - c_i}{2}$ for $i = 1,2$. We will focus on the parameter region in which interior solutions exist, and in equilibrium the market is fully covered, and each manufacturer has a positive market share. We use backward induction to find the equilibrium.

Retailer i chooses its price p_i to maximize π_{Ri}, where $\pi_{R1} = \frac{q_1 - q_2 - p_1 + p_2 + t}{2t}(p_1 - w_1)$ and $\pi_{R2} = \frac{q_2 - q_1 + p_1 - p_2 + t}{2t}(p_2 - w_2)$. Solving the first-order conditions ($\frac{\partial \pi_{R1}}{\partial p_1} = 0$ and $\frac{\partial \pi_{R2}}{\partial p_2} = 0$), we obtain the subgame equilibrium retail prices:

$$p_1^* = \frac{3t + q_1 - q_2 + 2w_1 + w_2}{3}$$

$$p_2^* = \frac{3t - q_1 + q_2 + w_1 + 2w_2}{3}$$

Using the subgame equilibrium retail prices (p_1^*, p_2^*), we can obtain the manufacturers' profit functions: $\pi_{M1} = \frac{(w_1 - c_1)(3t + q_1 - q_2 - w_1 + w_2)}{6t}$ and $\pi_{M2} = \frac{(w_2 - c_2)(3t - q_1 + q_2 + w_1 - w_2)}{6t}$. Solving the first-order conditions ($\frac{\partial \pi_{M1}}{\partial w_1} = 0$ and $\frac{\partial \pi_{M2}}{\partial w_2} = 0$), we find the equilibrium wholesale prices:

$$w_1^* = \frac{q_1 - q_2 + 2c_1 + c_2 + 9t}{3}$$

$$w_2^* = \frac{q_2 - q_1 + c_1 + 2c_2 + 9t}{3}$$

Finally, substituting the equilibrium wholesale prices into the manufacturers' profit functions, we find the manufacturers' equilibrium profits:

$$\pi_{M1}^* = \frac{(q_1 - q_2 - c_1 + c_2 + 9t)^2}{54t}$$

$$\pi_{M2}^* = \frac{(q_2 - q_1 + c_1 - c_2 + 9t)^2}{54t}$$

A necessary condition for the existence of the equilibrium where the market is fully covered and both manufacturers have positive market shares is that $\underline{t} < t < \bar{t}$, where $\underline{t} \equiv \max\{\frac{q_1 - c_1 - q_2 + c_2}{9}, \frac{q_2 - c_2 - q_1 + c_1}{9}\}$ and $\bar{t} \equiv \frac{q_1 - c_1 + q_2 - c_2}{9}$. We will assume that t satisfies these inequalities.

Now, let us consider the situation in which manufacturer 2 and retailer 2 exit the market. Demand for manufacturer 1's product becomes

$$\widetilde{D}_1 = \begin{cases} 1 & \text{if } p_1 < q_1 - t \\ \frac{q_1 - p_1}{t} & \text{if } q_1 - t \le p_1 \le q_1 \\ 0 & \text{if } p_1 > q_1 \end{cases}$$

Retailer 1 chooses its price p_1 to maximize its profit $\pi_{R1} = \widetilde{D}_1(p_1 - w_1)$. One can show that retailer 1's subgame equilibrium price is

$$p_1^{**} = \begin{cases} q_1 - t & \text{if } w_1 \le q_1 - 2t \\ \frac{q_1 + w_1}{2} & \text{if } q_1 - 2t < w_1 < q_1 \end{cases}$$

Anticipating retailer 1's optimal pricing of p_1^{**}, manufacturer 1 chooses w_1 to maximize its profit, $\pi_{M1} = \widetilde{D}_1(p_1^{**})\,(w_1 - c_1)$. Manufacturer 1's optimal price is

$$w_1^{**} = \begin{cases} q_1 - 2t & \text{if } t \le \frac{q_1 - c_1}{4} \\ \frac{q_1 + c_1}{2} & \text{if } \frac{q_1 - c_1}{4} < t \end{cases}$$

Plugging manufacturer 1's and retailer 1's equilibrium prices into manufacturer 1's profit function, we obtain

$$\pi_{M1}^{**} = \begin{cases} q_1 - c_1 - 2t & \text{if } t \le \frac{q_1 - c_1}{4} \\ \frac{(q_1 - c_1)^2}{8t} & \text{if } \frac{q_1 - c_1}{4} < t \end{cases}$$

Recall that we are assuming that $\underline{t} < t < \bar{t}$. One can readily show that when $\underline{t} < t < \bar{t}$, we have $\pi_{M1}^{**} > \pi_{M1}^{*}$, i.e., the competitor's exit makes manufacturer 1 better off.[1] Note that the effect of one firm's exit on the other firm's demand function in the current horizontal differentiation model directly contrasts the effect in our core model. In the standard horizontal-differentiation model, the competitor's presence in the market makes the firm's demand curve *less* steep, because

[1] Define $\Delta\pi \equiv \pi_{M1}^{*} - \pi_{M1}^{**}$. If $\frac{q_1 - c_1}{4} < t < \bar{t}$, then $\Delta\pi = \frac{4(q_1 - c_1 - q_2 + c_2 + 9t)^2 - 27(q_1 - c_1)^2}{216t}$, and $\Delta\pi > 0$ if and only if $4(q_1 - c_1 - q_2 + c_2 + 9t)^2 > 27(q_1 - c_1)^2$. Note that $t > \underline{t}$ implies that $q_1 - c_1 - q_2 + c_2 + 9t > 0$. Hence, $\Delta\pi > 0$ if and only if $2(q_1 - c_1 - q_2 + c_2 + 9t) > 3\sqrt{3}(q_1 - c_1)$, which is equivalent to $t > \frac{(3\sqrt{3} - 2)(q_1 - c_1) + 2(q_2 - c_2)}{18}$. Finally, since $q_i > c_i > 0$, one can show that $\frac{(3\sqrt{3} - 2)(q_1 - c_1) + 2(q_2 - c_2)}{18} > \bar{t}$. Hence, if $\frac{q_1 - c_1}{4} < t < \bar{t}$, then $\Delta\pi < 0$. Similarly, one can show that $\Delta\pi < 0$ for $\underline{t} < t < \min\{\frac{q_1 - c_1}{4}, \bar{t}\}$.

$\frac{dD_1}{dp_1} > \frac{d\tilde{D}_1}{dp_1}$, where $\frac{dD_1}{dp_1} = -\frac{1}{2t}$ and $\frac{d\tilde{D}_1}{dp_1} = -\frac{1}{t}$. So, one of the key conditions under which, in our core model, manufacturer 1 can benefit from the competitor's presence is not satisfied in the horizontal-differentiation model where consumers are heterogeneous *only* in terms of horizontal preferences. Intuitively, in the standard horizontal differentiation setting, when manufacturer 2 is present in the market, consumers' preferences for the two products are not "aligned" in the sense that consumers with a higher valuation for product 2 have a lower valuation for product 1, and vice versa. Hence, for the marginal consumer to switch to product 1, product 1's price will need to be decreased enough to compensate for both the consumer's lower valuation for product 1 and higher valuation for product 2. However, after manufacturer 2's exit, the non-buying consumers' alternative is to opt for the outside option. Hence, to attract the marginal consumer, product 1's price will need to be reduced just enough to compensate for the consumer's lower valuation for product 1, i.e., the price cut needed to attract an additional customer becomes less. More mathematically, this means that after manufacturer 2's exit, product 1's demand curve becomes steeper.

The above analysis of a standard horizontally differentiated market provides a boundary condition for our results, and helps to contrast our quality-differentiation model with the standard horizontal-differentiation model. ■

WEB APPENDIX C

ROBUSTNESS CHECKS

EXTENSION WITH $q_2 > q_1$.

Our main model has assumed that manufacturer 2's quality is lower than that of manufacturer 1. We change that assumption here, and we will demonstrate that manufacturer 2's exit can make both manufacturer 1 and retailer 1 worse off even when manufacturer 2's quality is higher than that of manufacturer 1. Further, we will show that an increase in manufacturer 2's quality level can make manufacturer 1 better off.

The analysis is very similar to our analysis in the main model, so we will be very brief here. When both manufacturer 1 and manufacturer 2 are present in the market, one can show that manufacturer 1's and retailer 1's equilibrium profits are given by

$$\pi_{M1}^{*} = \frac{q_2(2q_2-q_1)(c_1(9q_1q_2-2q_1^2-8q_2^2)+q_1(2c_2q_2+6q_2^2-c_2q_1-8q_2q_1+2q_1^2))^2}{(q_2-q_1)(4q_2-q_1)q_1(16q_2^2-17q_1q_2+4q_1^2)^2}$$

$$\pi_{R1}^{*} = \frac{q_2(2q_2-q_1)^2(c_1(9q_1q_2-2q_1^2-8q_2^2)+q_1(2c_2q_2+6q_2^2-c_2q_1-8q_1q_2+2q_1^2))^2}{(q_2-q_1)q_1(4q_2-q_1)^2(16q_2^2-17q_1q_2+4q_1^2)^2}$$

To ensure that in equilibrium both manufacturers have positive market share, we are assuming that $c_2 \in (\underline{c}, \bar{c})$, where $\underline{c} \equiv \max\{0, \frac{2q_1^3-2c_1q_1^2+9c_1q_1q_2-8q_1^2q_2-8c_1q_2^2+6q_1q_2^2}{q_1^2-2q_1q_2}\}$ and $\bar{c} \equiv \frac{3q_1^2q_2+2c_1q_2^2-c_1q_1q_2-11q_1q_2^2+8q_2^3}{2q_1^2-9q_1q_2+8q_2^2}$.

Recall from the main text that if manufacturer 2 exits the market, then manufacturer 1's and retailer 1's profits are given by $\pi_{M1}^{**} = \frac{(q_1-c_1)^2}{8q_1}$, and $\pi_{R1}^{**} = \frac{(q_1-c_1)^2}{16q_1}$, respectively. We will show that there exists $\hat{q}$, such that if $q_2 > \hat{q}$, then there is $\hat{c} \in (\underline{c}, \bar{c})$, such that if $c_2 > \hat{c}$, then $\pi_{M1}^{**} < \pi_{M1}^{*}$ and $\pi_{R1}^{**} < \pi_{R1}^{*}$, i.e., manufacturer 2's exit will make manufacturer 1 and retailer 1 worse off. Define $\Delta\pi_{M1} \equiv \pi_{M1}^{*} - \pi_{M1}^{**}$ and $\Delta\pi_{R1} \equiv \pi_{R1}^{*} - \pi_{R1}^{**}$. One can show that $\Delta\pi_{M1}|_{c_2=\bar{c}} =$

$\frac{(q_1-c_1)^2(32q_2^3-4q_1^3+28q_1^2q_2-57q_1q_2^2)}{8(2q_1^2-9q_1q_2+8q_2^2)^2}$ and $\Delta\pi_{R1}|_{c_2=\bar{c}} = \frac{(q_1-c_1)^2(16q_2^3-4q_1^3+20q_1^2q_2-33q_1q_2^2)}{16(2q_1^2-9q_1q_2+8q_2^2)^2}$. Straightforward algebra shows that there exists $\hat{q} > q_1$ such that if $q_2 > \hat{q}$, then $\Delta\pi_{M1}|_{c_2=\bar{c}} > 0$ and $\Delta\pi_{R1}|_{c_2=\bar{c}} > 0$. By continuity of $\Delta\pi_{M1}|_{c_2=\bar{c}}$ and $\Delta\pi_{R1}|_{c_2=\bar{c}}$ in c_2, it follows that there exists $\hat{c} \in [\underline{c}, \bar{c})$, such that if $c_2 \in (\hat{c}, \bar{c})$, then $\Delta\pi_{M1} > 0$ and $\Delta\pi_{R1} > 0$, i.e., manufacturer 2's exit will make both manufacturer 1 and retailer 1 worse off.

Note that the intuition for why manufacturer 2's exit can make manufacturer 1 and retailer 1 worse off is consistent with that in our main model. Namely, after manufacturer 2's exit, the demand curve for manufacturer 1's product becomes less steep, because its slope changes from $-\frac{1}{q_1-q_2}-\frac{1}{q_2}$ to $-\frac{1}{q_2}$. Less steep demand worsens the double-marginalization problem within manufacturer 1's channel, reducing manufacturer 1's profit. Further, since after manufacturer 2's exit, manufacturer 1 will set a relatively high wholesale price, retailer 1 can also become worse off than when manufacturer 2 is present in the market.

To show that an increase in manufacturer 2's product quality q_2 can make manufacturer 1 better off, consider a numerical example with $q_1 = 0.6$, $c_1 = 0.2$, and $c_2 = 0.25$. Then, if $q_2 = 0.8$, in equilibrium both manufacturers have positive market shares, and $\frac{\partial\pi_{M1}^*}{\partial q_2}|_{q_2=0.8} = 0.003 > 0$, i.e., a marginal increase in manufacturer 2's quality will lead to a higher profit for manufacturer 1.

Figure W2 graphically illustrates the parameter region in which manufacturer 2's exit will make manufacturer 1 and retailer 1 worse off.

Figure W2 Effect of Competitor's Exit on Manufacturer 1 and Retailer 1 when $q_2 > q_1$ ($q_1 = 0.5$, $c_1 = 0.2$)

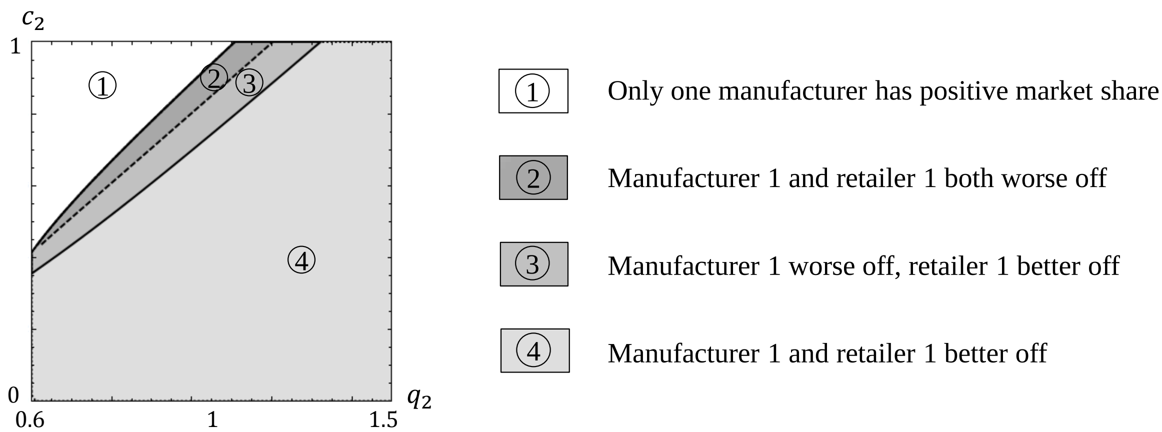
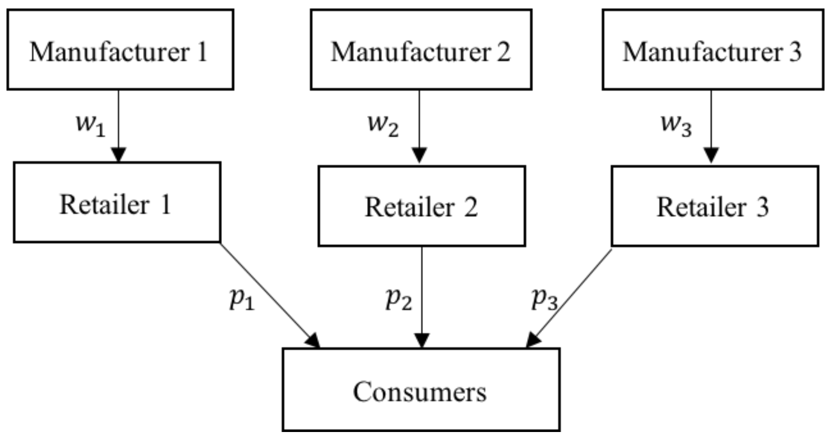


Figure W2 shows that when manufacturer 2 is not too competitive (q_2 is small enough and c_2 is sufficiently high, i.e., region 2 in Figure W2), its exit will make manufacturer 1 and retailer 1 worse off. Note that manufacturer 1's exit will also make the consumers worse off since the retail price will increase. Thus, when manufacturer 2 is not very competitive (region 2 in Figure W2), its exit can lead to an *all-lose* outcome for the manufacturers, retailers and the consumers. ■

EXTENSION WITH 3 MANUFACTURERS.

Using a numerical example, we will show that the exit of a competitor (or competitors) can make both manufacturer 1 and retailer 1 worse off even when initially there are three manufacturers in the market. Figure W3 illustrates the market structure.

Figure W3 Alternative Market Structure with Three Manufacturers



More specifically, let us assume that $q_1 = 1$, $q_2 = 0.5$, $q_3 = 0.3$, $c_1 = 0.4$, $c_2 = 0.35$, and $c_3 = 0.2$. To find the equilibrium of the game, we use backward induction. Given the manufacturers' wholesale prices w_1, w_2 and w_3, retailers i will choose its retail price p_i to maximize its profit π_{Ri}, where $\pi_{R1} = (1 - \frac{p_1 - p_2}{1-0.5})(p_1 - w_1)$, $\pi_{R2} = (\frac{p_1 - p_2}{1-0.5} - \frac{p_2 - p_3}{0.5-0.3})(p_2 - w_2)$ and $\pi_{R3} = (\frac{p_2 - p_3}{0.5-0.3} - \frac{p_3}{0.3})(p_3 - w_3)$. Solving the first-order conditions ($\frac{\partial \pi_{Ri}}{\partial p_i} = 0$ for $i = 1,2,3$), we can find the retailers' subgame equilibrium prices, $p_1^* = \frac{25+50w_1+28w_2+10w_3}{92}$, $p_2^* = \frac{1+2w_1+14w_2+5w_3}{23}$ and $p_3^* = \frac{3+6w_1+42w_2+130w_3}{230}$. Using the retail prices, we can readily obtain the manufacturers' profit functions, $\pi_{M1} = \frac{(28w_2+5(5+2w_3)-42w_1)(w_1-0.4)}{46}$, $\pi_{M2} = \frac{7(1+2w_1-9w_2+5w_3)(w_2-0.35)}{23}$ and $\pi_{M3} = \frac{5(3+6w_1+42w_2-100w_3)(w_3-0.2)}{138}$. Solving the first-order conditions ($\frac{\partial \pi_{Mi}}{\partial w_i} = 0$ for $i = 1,2,3$), we find the manufacturers' equilibrium wholesale prices: $w_1^* \approx 0.643$, $w_2^* \approx 0.36$ and $w_3^* \approx 0.21$. One can readily verify that in equilibrium each manufacturer has a positive market share. Using the equilibrium retail and wholesale prices, we find that $\pi_{M1}^* \approx 0.053$ and $\pi_{R1}^* \approx 0.024$.

First, let us assume that manufacturer 2 exits the market due to some exogenous shock, so that only manufacturers 1 and 3 remain in the market. We solve the game by using backward induction. Given the manufacturers' wholesale prices w_1 and w_3, retailer $i \in \{1,3\}$ chooses its retail price p_i to maximize its profit, where $(1 - \frac{p_1 - p_2}{1-0.3})(p_1 - w_1)$ and $\pi_{R3} = (\frac{p_2 - p_3}{1-0.3} - \frac{p_3}{0.3})(p_3 - w_3)$. Solving the first-order conditions ($\frac{\partial \pi_{Ri}}{\partial p_i} = 0$ for $i = 1,3$), we can find the retailers' subgame equilibrium prices, $p_1^{**} = \frac{2(7+10w_1+5w_3)}{37}$ and $p_3^{**} = \frac{21+30w_1+200w_3}{370}$. Manufacturer $i \in \{1,3\}$ correctly anticipates the retailers' subgame equilibrium prices (p_1^{**}, p_3^{**}), and chooses its wholesale price to maximize its profit π_{Mi}, where $\pi_{M1} = \frac{10(2(7+5w_3)-17w_1)(w_1-0.4)}{259}$ and $\pi_{M3} = \frac{10(21+30w_1-170w_3)(w_3-0.2)}{777}$. Solving the first-order conditions ($\frac{\partial \pi_{Mi}}{\partial w_i} = 0$ for $i = 1,3$), we find the manufacturers' equilibrium wholesale prices: $w_1^{**} \approx 0.677$ and $w_3^{**} \approx 0.221$. Using the equilibrium wholesale and retail prices, we can readily find manufacturer 1's and retailer 1's equilibrium profits: $\pi_{M1}^{**} \approx 0.05$ and $\pi_{R1}^{**} \approx 0.023$. Notice that $\pi_{M1}^* > \pi_{M1}^{**}$ and $\pi_{R1}^* > \pi_{R1}^{**}$, i.e., manufacturer 2's exit will make manufacturer 1 and retailer 1 worse off.

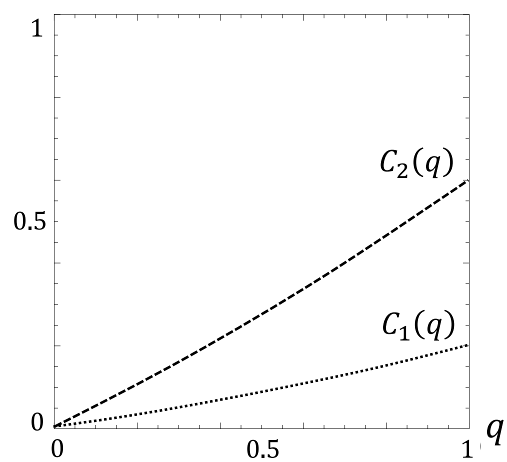
Second, let us assume that some exogenous shock forces both manufacturer 2 and manufacturer 3 to exit the market. In that situation, manufacturer 1's and retailer 1's profit is given by $\pi_{M1}^{***} = \frac{(q_1 - c_1)^2}{8q_1}$ and $\pi_{R1}^{***} = \frac{(q_1 - c_1)^2}{16q_1}$. Substituting in the numerical values of q_1 and c_1, we find that $\pi_{M1}^{***} = 0.045$ and $\pi_{R1}^{***} = 0.022$. Notice that $\pi_{M1}^* > \pi_{M1}^{***}$ and $\pi_{R1}^* > \pi_{R1}^{***}$, i.e., manufacturer 2's and manufacturer 3's exit from the market will make manufacturer 1 and retailer 1 worse off.

■

EXTENSION WITH ENDOGENOUS PRODUCT QUALITY.

We analyze a numerical example with manufacturer 1's and manufacturer 2's marginal costs being $C_1(q_1) = 0.1q_1 + 0.1q_1^2$ and $C_2(q_2) = \alpha_2 q_2 + 0.1q_2^2$. We assume that $\alpha_2 > 0.1$ (i.e., manufacturer 2 is less cost-efficient than manufacturer 1). Note that the function C_i is increasing in q_i (i.e., $C_i' > 0$) and is convex (i.e., $C_i'' > 0$), which are the standard assumptions in the literature (e.g., Mussa and Rosen 1978). To ensure that in equilibrium manufacturer 2 has a positive market share, we focus on $\alpha_2 < \bar{\alpha}$, where $\bar{\alpha} \approx 0.82$. Figure W4 illustrates the firms' cost functions.

Figure W4 Manufacturers' Marginal Cost as a Function of Quality ($\boldsymbol{\alpha_2 = 0.5}$)



Let π_{M1}^* and π_{R1}^* denote manufacturer 1's and retailer 1's equilibrium profits when manufacturer 2 is in the market, and let π_{M1}^{**} and π_{R1}^{**} denote manufacturer 1's and retailer 1's equilibrium profits after manufacturer 2 exits the market. We determine the parameter region for α_2 in which $\pi_{M1}^* > \pi_{M1}^{**}$ and $\pi_{R1}^* > \pi_{R1}^{**}$, i.e., manufacturer 2's exit will make both manufacturer 1 and retailer 1 worse off.

First, consider the situation where both manufacturer 1 and manufacturer 2 are present in the market. Given q_1 and q_2, one can show that the manufacturers' and retailers' subgame equilibrium profits are

$$\pi_{M1}^{NE}(q_1,q_2)=\frac{q_1^2(2q_1-q_2)(8q_1^3-9q_1^2(8+q_2)+q_1q_2(101-20\alpha_2)+q_2^2(q_2+10\alpha_2-28))^2}{100(4q_1^2-5q_1q_2+q_2^2)(16q_1^2-17q_1q_2+4q_2^2)^2},$$

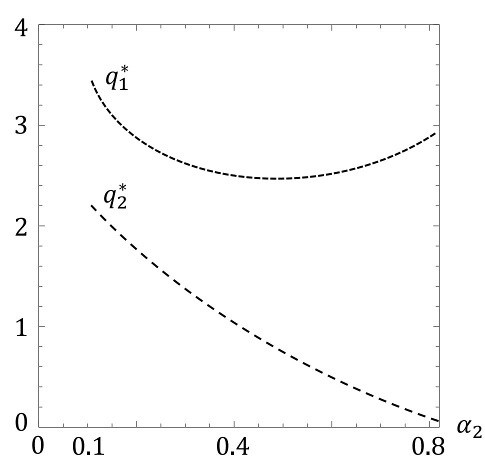
$$\pi_{M2}^{NE}(q_1,q_2)=\frac{q_1(2q_1-q_2)q_2(2q_1^3-2q_2^2(q_2-10(1-\alpha_2))+q_1^2(62-9q_2-80\alpha_2)+9q_1q_2(q_2+10\alpha_2-9))^2}{100(4q_1^2-5q_1q_2+q_2^2)(16q_1^2-17q_1q_2+4q_2^2)^2},$$

$$\pi_{R1}^{NE}(q_1,q_2)=\frac{q_1^2(2q_1-q_2)^2(8q_1^3-9q_1^2(8+q_2)+q_1q_2(101-20\alpha_2)+q_2^2(q_2+10\alpha_2-28))^2}{100(q_1-q_2)(4q_1-q_2)^2(16q_1^2-17q_1q_2+4q_2^2)^2},$$

$$\pi_{R2}^{NE}(q_1,q_2)=\frac{q_1q_2(2q_1-q_2)^2(2q_1^3-2q_2^2(q_2-10(1-\alpha_2))+q_1^2(62-9q_2-80\alpha_2)+9q_1q_2(-9+q_2+10\alpha_2))^2}{100(q_1-q_2)(4q_1-q_2)^2(16q_1^2-17q_1q_2+4q_2^2)^2}.$$

The superscript *NE* stands for "No Exit" and indicates that manufacturer 2 is in the market. The equilibrium quality levels (q_1^*,q_2^*) satisfy the first-order conditions $\frac{\partial\pi_{M1}^{NE}(q_1,q_2)}{\partial q_1}=0$ and $\frac{\partial\pi_{M2}^{NE}(q_1,q_2)}{\partial q_2}=0$. Unfortunately, solving the firms' equilibrium quality levels in parametric form turns out to be analytically intractable. But we are able to determine the equilibrium outcome numerically; Figure W5 illustrates manufacturers' equilibrium quality levels (q_1^*,q_2^*) as a function of α_2.

Figure W5 Manufacturers' Equilibrium Quality Levels



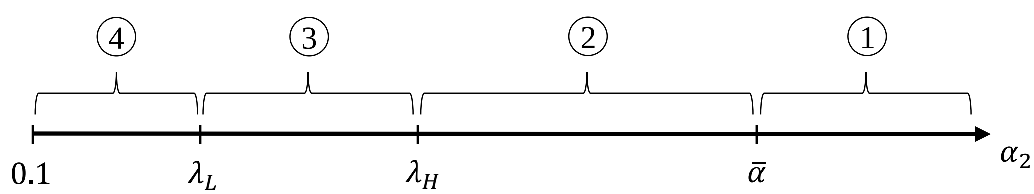
Using equilibrium quality levels (q_1^*,q_2^*), one can easily determine manufacturer 1's and retailer 1's equilibrium profits, $\pi_{M1}^*=\pi_{M1}^{NE}(q_1^*,q_2^*)$ and $\pi_{R1}^*=\pi_{R1}^{NE}(q_1^*,q_2^*)$.

Now, suppose that manufacturer 2 exits the market. Given q_1, one can show that manufacturer 1's and retailer 1's subgame equilibrium profits are $\pi_{M1}^{E}(q_1) = \frac{q_1(0.9-0.1q_1)^2}{8}$ and $\pi_{R1}^{E}(q_1) = \frac{q_1(0.9-0.1q_1)^2}{16}$.

The superscript *E* stands for "Exit" and indicates that manufacturer 1 has exited the market.

We examine two situations. First, let us assume that at the time of manufacturer 2's exit from the market, manufacturer 1's product has already been designed or produced at the previously anticipated optimal level (q_1^*), and manufacturer 1 can no longer adjust its product quality, in which case manufacturer 1's and retailer 1's equilibrium profits are $\pi_{M1}^{**} = \pi_{M1}^{E}(q_1^*)$ and $\pi_{R1}^{**} = \pi_{R1}^{E}(q_1^*)$. Then, one can show that there exist $\lambda_L \approx 0.241$ and $\lambda_H \approx 0.48$ such that $\pi_{M1}^{*} > \pi_{M1}^{**}$ if and only if $\alpha_2 \in (\lambda_L, \bar{\alpha})$, and that $\pi_{R1}^{*} > \pi_{R1}^{**}$ if and only if $\alpha_2 \in (\lambda_H, \bar{\alpha})$. Figure W6 illustrates these results.

Figure W6 Effect of Manufacturer 2's Exit (no adjustment in quality)



1 Only manufacturer 1 has positive market share

2 Manufacturer 1 and retailer 1 worse off

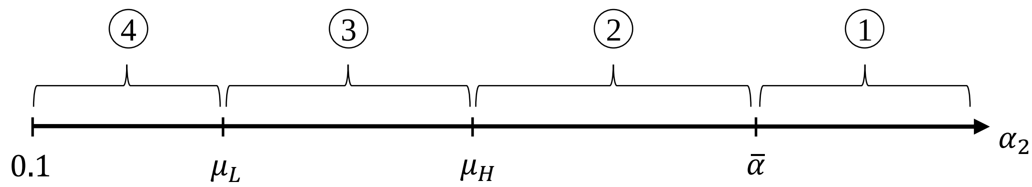
3 Manufacturer 1 worse off, retailer 1 better off

4 Manufacturer 1 and retailer 1 better off

Second, let us assume that when manufacturer 2 exits the market, manufacturer 1 can freely change its product quality. Manufacturer 1's optimal quality, q_1^{**}, will maximize manufacturer 1's profit, $\pi_{M1}^{E}(q_1)$. Solving the first-order condition ($\frac{\partial \pi_{M1}^{E}}{\partial q_1} = 0$), we find that $q_1^{**} = 3$, $\pi_{M1}^{**} = 0.135$ and $\pi_{R1}^{**} = 0.0675$. Comparing π_{M1}^{*} with π_{M1}^{**} and π_{R1}^{*} with π_{R1}^{**}, one can show that there exist

$\mu_L \approx 0.243$ and $\mu_H \approx 0.51$ such that $\pi^*_{M1} > \pi^{**}_{M1}$ if and only if $\alpha_2 \in (\mu_L, \bar{\alpha})$, and that $\pi^*_{R1} > \pi^{**}_{R1}$ if and only if $\alpha_2 \in (\mu_H, \bar{\alpha})$. Figure W7 illustrates these results.

Figure W7 Effect of Manufacturer 2's Exit (with quality adjustment)



1 Only manufacturer 1 has positive market share

2 Manufacturer 1 and retailer 1 worse off

3 Manufacturer 1 worse off, retailer 1 better off

4 Manufacturer 1 and retailer 1 better off

Next, let us show that when manufacturer 1 can adjust its product quality, manufacturer 2's exit may lead to an *increase* in manufacturer 1's quality, a *decrease* in manufacturer 1's unit sales, and also that an increase in manufacturer 2's cost efficiency (due to a reduction in α_2 or k_2) can *increase* manufacturer 1's profit. To do this, we analyze an example where $C_1(q_1) = 0.1q_1 + 0.1q_1^2$ and $C_2(q_2) = 0.6q_2 + 0.1q_2^2$.

When manufacturer 2 is in the market, using the first-order conditions $\frac{\partial \pi^{NE}_{M1}(q_1,q_2)}{\partial q_1} = 0$ and $\frac{\partial \pi^{NE}_{M2}(q_1,q_2)}{\partial q_2} = 0$, one can show that the manufacturers' equilibrium quality levels are $q_1^* \approx 2.75$ and $q_2^* \approx 0.61$. Also, the manufacturers' wholesale prices are $w_1^* \approx 1.846$ and $w_2^* \approx 0.45$, the retailers' prices are $p_1^* \approx 2.229$ and $p_2^* \approx 0.472$, and manufacturer 1 and retailer 1's equilibrium profits are $\pi^*_{M1} \approx 0.145$ and $\pi^*_{R1} \approx 0.0686$, respectively. Using the equilibrium quality levels and retail prices, one can derive manufacturer 1's equilibrium unit sales to be 0.18.

When manufacturer 2 exits the market, manufacturer 1's optimal quality level *increases* to $q_1^{**} = 3$ (where $q_1^{**} = \arg\max_{q_1} \pi_{M1}^E(q_1)$), and in equilibrium product 1's wholesale and retail prices are $w_1^{**} = 2.1$ and $p_1^{**} = 2.55$, and manufacturer 1's and retailer 1's profits are $\pi_{M1}^{**} = 0.135$ and $\pi_{R1}^{**} = 0.0675$, respectively. Product 1's unit sales will *decrease* to 0.15. Notice that both manufacturer 1 and retailer 1 are *worse off* if manufacturer 2 exits the market, i.e., $\pi_{M1}^{**} < \pi_{M1}^{*}$ and $\pi_{R1}^{**} < \pi_{R1}^{*}$.

It remains to show that manufacturer 1 can benefit if manufacturer 2 becomes more cost-efficient (due to a decrease in α_2 or k_2) and increases its equilibrium quality level. In particular, if α_2 decreases from 0.6 to 0.59, we find that manufacturer 2's equilibrium quality increases to $q_2^* \approx 0.633743$ and π_{M1}^* *increases* to 0.146132. Similarly, if k_2 decreases from 0.1 to 0.099, then manufacturer 2's equilibrium quality increases to $q_2^* \approx 0.613951$ and π_{M1}^* *increases* to 0.145957. Thus, an increase in manufacturer 2's cost efficiency (and a subsequent increase in product 2's quality) can make manufacturer 1 *better off*. ■

1. Extension with retailer 2 joining manufacturer 1's channel after manufacturer 2's exit.

Let us assume that after manufacturer 2's exit, manufacturer 1 will offer retailer 2 the same wholesale contract that is offered to retailer 1, and retailer 2 will decide whether to join the channel to sell manufacturer 1's product. We assume that, to join manufacturer 1's channel, retailer 2 has to incur some fixed cost $f > 0$, e.g., contracting or setting up the logistics, etc. For example, starting a Toyota dealership in the U.S. requires an investment between 0.5 and 15 million dollars, depending on the location (Mellott 2017). Moreover, we introduce consumers' horizontal preferences into our core vertical differentiation model. Namely, we assume that consumers are unfirmly distributed on the line segment $[0,1]$, with the two retailers located at the center. If a consumer located at $l \in [0,1]$ buys from retailer i, she will obtain a utility $\theta q - p_i - t\, d(i, l)$, where θ is the consumer's marginal willingness to pay for quality, $q \in \{q_1, q_2\}$ is the quality of the product that retailer i sells, p_i is retailer i's price, $d(i, l)$ is the distance between the consumer and retailer i, and t measures the strength of the consumer's horizontal preferences. To be consistent with our core model, we assume that θ is uniformly distributed on $[0,1]$. After manufacturer 2's exit, if retailer 2 decides to sell manufacturer 1's product, it will choose its retail price simultaneously with retailer 1.

LEMMA W2. *Given that retailers are at the same location, for any $f > 0$, after manufacturer 2 exits the market, retailer 2 will strictly prefer not to join manufacturer 1's channel.*

Proof of Lemma W2. If retailer 2 joins manufacturer 1's channel, then both retailer 1 and retailer 2 will be selling manufacturer 1's product. Consumers will buy from the retailer that charges the lowest price. If the retailers' prices are equal, then each retailer will serve half of the demand. We will show that in equilibrium $p_1^* = p_2^* = w_1$. By contraposition, first suppose that

$p_1^* = p_2^* > w_1$. Then, retailer 1 will benefit by deviating to a price $p_1^* - \epsilon$ and stealing retailer 2's share of the market, where $\epsilon > 0$ is arbitrarily small. Second, suppose that $p_1^* > p_2^*$ and $p_2^* = w_1$. Then, retailer 2 has a profitable deviation to $p_2^* + \epsilon$, which will give retailer 2 strictly higher (and positive) profit because its profit margin will become positive rather than zero. Third, if $p_1^* > p_2^*$ and $p_2^* > w_1$, then retailer 1 has a profitable deviation to $p_2^* - \epsilon$, which will allow retailer 1 to earn positive profit rather than zero. A similar argument works for the case $p_1^* < p_2^*$. Hence, the only situation in which neither retailer has a profitable deviation is when $p_1^* = p_2^* = w_1$.

Thus, each retailer's equilibrium markup is zero. Since retailer 2 has to incur a fixed cost $f > 0$ to join manufacturer 1's channel, it follows that retailer 2's profit is $-f < 0$. Hence, retailer 2 will strictly prefer not to join manufacturer 1's channel. □

Clearly, Lemma W2 depends on the lack of differentiation between the two retailers (aside from the differentiation of the products they sell). However, one may intuit that after manufacturer 2's exit, if retailer 2 joins manufacturer 1's channel, both retailers would have incentives to differentiate from each other since now they will have no differentiation based on the products they sell. To capture this idea, we assume that after manufacturer 2's exit, the two retailers will maximally differentiate from each other by moving to the opposite ends of the line segment. Note that maximal differentiation by the retailers is an exogenous assumption to ensure that the model is analytically tractable. In the Robustness Checks section of the main paper, we discuss what will happen if the retailers do not maximally differentiate. Let us also assume that no fixed cost is needed, i.e., $f = 0$; note that assuming a positive f will make our result more likely to happen. To make the analysis analytically tractable, we analyze a numerical example with $q_1 = 1$, $q_2 = 0.5$, $c_1 = 0.4$, $c_2 = 0.3$, and $t = 0.5$.

Let us first analyze the situation in which manufacturer 2 is present in the market. We solve the game by backward induction. Demand for each manufacturer's product is:

$$D_1 = \begin{cases} \frac{7}{8} - p_1 & \text{if } p_1 \le \min\{0.75, 2p_2\} \\ \frac{7}{8} - 2p_1^2 - 8p_2^2 + p_1(8p_2 - 1) & \text{if } 2p_2 < p_1 < 0.75 \\ 2(1-p_1)^2 & \text{if } 0.75 < p_1 < 2p_2 \\ 2 - 8p_2^2 - p_1(4 - 8p_2) & \text{if } \max\{0.75, 2p_2\} \le p_1 < 0.5 + p_2 \\ 0 & \text{if } p_1 \ge 0.5 + p_2 \end{cases} \quad \text{(W4)}$$

$$D_2 = \begin{cases} (1-2p_2)^2 & \text{if } p_2 < p_1 - 0.5 \\ 4(p_1 - 2p_2)^2 & \text{if } p_1 - 0.5 \le p_2 \le \frac{1}{2}p_1 \\ 0 & \text{if } p_2 > \frac{1}{2}p_1 \end{cases} \quad \text{(W5)}$$

Let us assume that the retailers' equilibrium prices satisfy $2p_2^* < p_1^* < 0.75$. Later we will verify that the equilibrium prices actually satisfy those inequalities. Given the manufacturers' wholesale prices, retailer 1 will choose its price p_1 to maximize its profit $\pi_{R1} = (\frac{7}{8} - 2p_1^2 - 8p_2^2 + p_1(8p_2 - 1))(p_1 - w_1)$, and retailer 2 will choose p_2 to maximize its profit $\pi_{R2} = 4(p_1 - 2p_2)^2(p_2 - w_2)$. Using the first-order conditions, one can show that

$$\begin{aligned} p_1^* &= \frac{12w_1 + 40w_2 - 9 + 3\sqrt{37 + 16w_1^2 + w_1(8 - 64w_2) - 80w_2 + 64w_2^2}}{32} \\ p_2^* &= \frac{4w_1 + 56w_2 - 3 + 3\sqrt{37 + 16w_1^2 + w_1(8 - 64w_2) - 80w_2 + 64w_2^2}}{64} \end{aligned} \quad \text{(W6)}$$

Each manufacturer correctly anticipates the retailers' subgame equilibrium prices (p_1^*, p_2^*) and chooses its wholesale price to maximize its profit π_{Mi}, where $\pi_{M1} = (\frac{7}{8} - 2(p_1^*)^2 - 8(p_2^*)^2 + p_1^*(8p_2^* - 1))(w_1 - 0.4)$ and $\pi_{M2} = 4(p_1^* - 2p_2^*)^2(w_2 - 0.3)$. Solving the first-order conditions, one can show that the manufacturers' equilibrium wholesale prices are $w_1^* \approx 0.6377$ and $w_2^* \approx 0.3263$. Plugging (w_1^*, w_2^*) into the expressions for p_1^* and p_2^* in (W6), we find that

$$p_1^* \approx 0.74 \quad \text{(W7)}$$

$p_2^* \approx 0.34$

Notice that p_1^* and p_2^* in (W7) indeed satisfy $2p_2^* < p_1^* < 0.75$, as we initially assumed. One can also show that the retailers and manufacturers do not have any profitable non-local deviations. Using the equilibrium wholesale and retail prices, we can obtain manufacturer 1's and retailer 1's equilibrium profits: $\pi_{M1}^* \approx 0.0303$ and $\pi_{R1}^* = 0.0131$.

Now, let us analyze the situation where manufacturer 2 exits the market and retailer 2 joins manufacturer 1's channel. Since the retailers are symmetric, we will write out the demand function only for retailer 1.

$$\widetilde{D}_1 = \begin{cases} 1 - \frac{2p_1+0.5}{2} & \text{if } p_1 \leq p_2 - t \\ \frac{7+12p_1^2+12p_2-4p_2^2-4p_1(5+2p_2)}{16} & \text{if } p_2 - t < p_1 < p_2 + t \\ 0 & \text{if } p_1 \geq p_2 + t \end{cases} \tag{W8}$$

Given manufacturer 1's wholesale price, retailers 1 and 2 choose their prices to maximize $\pi_{R1} = \frac{7+12p_1^2+12p_2-4p_2^2-4p_1(5+2p_2)}{16}(p_1 - w_1)$ and $\pi_{R2} = \frac{7+12p_2^2+12p_1-4p_1^2-4p_2(5+2p_1)}{16}(p_2 - w_1)$, respectively. Using the first-order conditions, we find that the retailers' equilibrium prices are

$$p_1^{**} = \frac{7+4w_1-\sqrt{21-24\,w_1+16w_1^2}}{8}$$
$$p_2^{**} = \frac{7+4w_1-\sqrt{21-24\,w_1+16w_1^2}}{8} \tag{W9}$$

Manufacturer 1 correctly anticipates the retailers' subgame equilibrium prices (p_1^{**}, p_2^{**}) and chooses its wholesale price w_1 to maximize its profit $\pi_{M1} = \frac{(w_1-0.4)(\sqrt{21-24\,w_1+16w_1^2}-4w_1)}{8}$. Solving the first-order condition ($\frac{d\pi_{M1}}{dw_1} = 0$), one can show that manufacturer 1's optimal price is $w_1^{**} = 0.6215$. Plugging w_1^{**} into (W9), we find the equilibrium retail prices

$$p_1^{**} = p_2^{**} = 0.748 \tag{W10}$$

Using the equilibrium wholesale and retail prices, we can readily find manufacturer 1's and its retailers' equilibrium profits: $\pi_{M1}^{**} \approx 0.0281$ and $\pi_{R1}^{**} = \pi_{R2}^{**} \approx 0.008$.

Since $\pi_{M1}^{*} > \pi_{M1}^{**}$, it follows that manufacturer 2's exit makes manufacturer 1 worse off. Further, since $\pi_{R1}^{*} > \pi_{R1}^{**}$, retailer 1 also becomes worse off when manufacturer 2 exits the market. Note that after manufacturer 2's exit from the market, the retail demand curve for manufacturer 1's product becomes less steep, which is also qualitatively the same as what happens in our core model. To show this, let us evaluate product 1's retail demand function at the equilibrium retail prices before and after manufacturer 2's exit. Using the demand functions in (W4) and (W8) and the equilibrium retail prices in (W7) and (W10), we find that $\frac{\partial D_1}{\partial p_1}|_{(p_1^*, p_2^*)} \approx -1.235$ and $\frac{\partial \tilde{D}_1}{\partial p_1}|_{(p_1^{**}, p_2^{**})} \approx -0.502$, which confirms that after manufacturer 2's exit, the steepness of product 1's demand curve decreases. ■

2. Extension with retailer 2 joining manufacturer 1's channel after manufacturer 2's exit.

Suppose that after manufacturer 2's exit, retailer 2 will join manufacturer 1's channel, but the ex-ante and ex-post differentiation between the retailers will be the same, i.e., we do not require that the retailers establish more differentiation after manufacturer 2's exit.

For this extension, we adopt the same assumptions as in our core model for the quality dimension, but allow each retailer $i \in \{1,2\}$ to have a segment of loyal customers of size δ_i (who will not consider buying from the other retailer). Such an assumption is commonly used in the literature to represent differentiation between firms (e.g., Narasimhan 1988, Iyer and Pazgal 2003). There is also a segment of switchers of size one, who can buy from either retailer. This model is very complex to analyze, since it involves mixed strategy pricing equilibria. However, one can solve the model using numerical examples. Those examples seem to suggest that as the size of retailer 1's loyal segment increases and the size of retailer 2's loyal segment decreases, manufacturer 2's exit becomes more likely to make manufacturer 1 worse off. Intuitively, after manufacturer 2's exit, retail competition for the switcher segment will be very intense because each retailer will be selling the same product. However, if retailer 1's loyal segment (δ_1) is large, retailer 1 will focus on serving its loyal segment rather than compete for switchers, which will worsen the double-marginalization problem for manufacturer 1. Further, if retailer 2's loyal segment (δ_2) is small, then manufacturer 1's market-expansion benefit after manufacturer 2's exit will also be small. Hence, one can intuit that if δ_1 is large enough and δ_2 is small enough, then manufacturer 2's exit can potentially make manufacturer 1 worse off. To illustrate this, we provide a numerical example with $q_1 = 1$, $q_2 = 0.85$, $c_1 = 0.75$, $c_2 = 0.74$, $\delta_1 = 1$ and $\delta_2 = 0$.

First, let us analyze the situation where manufacturer 2 is in the market. The retailers' demand functions are as follows:

$$D_1 = \begin{cases} 2(1-p_1) & \text{if } p_1 \le \frac{1}{0.85}p_2 \\ (1-p_1) + (1-\frac{p_1-p_2}{1-0.85}) & \text{if } \frac{1}{0.85}p_2 < p_1 < 0.15 + p_2 \\ (1-p_1) & \text{if } 0.15 + p_2 \le p_1 \le 1 \\ 0 & \text{if } p_1 > 1 \end{cases} \quad \text{(W11)}$$

$$D_2 = \begin{cases} 1-\frac{p_2}{0.85} & \text{if } p_2 < p_1 - 0.15 \\ \frac{p_1-p_2}{1-0.85} - \frac{p_2}{0.85} & \text{if } p_1 - 0.15 < p_2 < 0.85p_1 \\ 0 & \text{if } p_2 \ge 0.85p_1 \end{cases} \quad \text{(W12)}$$

We solve the game by backward induction, and we will focus on the equilibrium where both manufacturers have positive market shares in the consumer segment with switchers (i.e., equilibrium retail prices will satisfy $\frac{1}{0.85}p_2^* < p_1^* < 0.15 + p_2^*$, or equivalently, $p_1^* - 0.15 < p_2^* < 0.85p_1^*$). Given the wholesale prices, retailer $i \in \{1,2\}$ will choose a price p_i to maximize its profit π_{Ri}, where $\pi_{R1} = ((1-p_1) + (1-\frac{p_1-p_2}{1-0.85}))(p_1 - w_1)$ and $\pi_{R2} = (\frac{p_1-p_2}{1-0.85} - \frac{p_2}{0.85})(p_2 - w_2)$. Solving the first-order conditions ($\frac{\partial \pi_{R1}}{\partial p_1} = 0, \frac{\partial \pi_{R2}}{\partial p_2} = 0$), we find the equilibrium retail prices: $p_1^* = \frac{2(6+23w_1+10w_2)}{75}$ and $p_2^* = \frac{102+391w_1+920w_2}{1500}$. Manufacturer $i \in \{1,2\}$ correctly anticipates the retailers' subgame equilibrium prices, and chooses its wholesale price to maximize π_{Mi}, where $\pi_{M1} = ((1-p_1^*) + (1-\frac{p_1^*-p_2^*}{1-0.85}))(w_1 - 0.75)$ and $\pi_{M2} = (\frac{p_1^*-p_2^*}{1-0.85} - \frac{p_2^*}{0.85})(w_2 - 0.74)$. Solving the first-order conditions ($\frac{\partial \pi_{M1}}{\partial w_1} = 0, \frac{\partial \pi_{M2}}{\partial w_2} = 0$), we find the manufacturers' equilibrium prices: $w_1^* = \frac{24887}{29730}$ and $w_2^* = \frac{880117}{1189200}$. Using (w_1^*, w_2^*), we can readily obtain the equilibrium retail prices: $p_1^* = \frac{3883241}{4459500}$, $p_2^* = \frac{4125746}{5574375}$. One can verify that $\frac{1}{0.85}p_2^* < p_1^* < 0.15 + p_2^*$, which implies that each manufacturer indeed has a positive market share in the consumer segment with switchers.

Using the equilibrium wholesale and retail prices, we find that

$$\pi_{M1}^* = \frac{17890301347}{795485610000} \approx 0.0224898$$
$$\pi_{R1}^* = \frac{518818739063}{59661420750000} \approx 0.00869 \tag{W13}$$

Let us verify that retailers and manufacturers do not have any profitable non-local deviations from their equilibrium prices. First, let us consider retailer 1's possible deviations. Note that we have already shown that p_1^* is optimal in the range $[\frac{1}{0.85}p_2^*, 0.15 + p_2^*]$. So, we need to consider two other ranges of deviations. Retailer 1 can deviate to a lower price p_1' to push the competitor out of the market (i.e., $p_1' < \frac{1}{0.85}p_2^*$) or to a higher price p_1' to serve only its loyal customers rather than compete for switchers (i.e., $p_1' > 0.15 + p_2^*$). For $p_1' < \frac{1}{0.85}p_2^*$, retailer 1's profit is $\pi_{R1} = 2(1 - p_1')(p_1' - w_1^*)$. One can show that π_{R1} is increasing in p_1' for all $p_1' < \frac{1}{0.85}p_2^*$. Since π_{R1} is a continuous function, it must be that $\pi_{R1}|_{p_1=\frac{1}{0.85}p_2^*} > \pi_{R1}|_{p_1=p_1'}$ for all $p_1' < \frac{1}{0.85}p_2^*$. Since p_1^* maximizes π_{R1} on $[\frac{1}{0.85}p_2^*, 0.15 + p_2^*]$ (because p_1^* was obtained using the first-order condition), it follows that $\pi_{R1}|_{p_1=p_1^*} > \pi_{R1}|_{p_1=\frac{1}{0.85}p_2^*}$. Combining $\pi_{R1}|_{p_1=p_1^*} > \pi_{R1}|_{p_1=\frac{1}{0.85}p_2^*}$ and $\pi_{R1}|_{p_1=\frac{1}{0.85}p_2^*} > \pi_{R1}|_{p_1=p_1'}$, it follows that $\pi_{R1}|_{p_1=p_1^*} > \pi_{R1}|_{p_1=p_1'}$ for all $p_1' < \frac{1}{0.85}p_2^*$, i.e., retailer 1 will not be able to improve its profit if it deviates from p_1^* to $p_1' < \frac{1}{0.85}p_2^*$. Next, for $0.15 + p_2^* < p_1' \leq 1$, retailer 1's profit is $\pi_{R1} = (1 - p_1')(p_1' - w_1^*)$. Using the first order condition, one can readily show that retailer 1's optimal price on the interval $(0.15 + p_2^*, 1]$ is $p_1' = \frac{1+w_1^*}{2}$, which gives retailer 1 a profit of $\pi_{R1}' \equiv \frac{(1-w_1^*)^2}{4} \approx 0.007$. Recall that $\pi_{R1}^* \approx 0.008$. Since $\pi_{R1}^* > \pi_{R1}'$, retailer 1 does not have a profitable deviation from p_1^* to any $p_1' \in (0.15 + p_2^*, 1]$.

Now, let us verify that given manufacturer 2's wholesale price $w_2^* = \frac{880117}{1189200}$, manufacturer 1 does not have any profitable non-local deviations from $w_1^* = \frac{24887}{29730}$. Manufacturer 1 can potentially deviate to a lower price to induce retailer 1 to push the competitor out of the market, or manufacturer 1 can deviate to a higher price to induce retailer 1 to serve only its loyal customers rather than compete for the switcher segment. First, suppose that manufacturer 1 deviates to some w_1' so that in equilibrium, retailer 2 has zero market share. Let (p_1', p_2') be the equilibrium retail prices after manufacturer 1's deviation to w_1'. To find manufacturer 1's profit under this deviation, we will need to find (p_1', p_2'). From equation (W12), we can see that retailer 2 will have zero market share only if $p_1' \leq \frac{1}{0.85} p_2'$. Also, if in a subgame equilibrium, retailer 2 has zero market share, then $p_2' = w_2^*$, i.e., retailer 2's price will equal its marginal cost, which is w_2^*. To find p_1', we maximize retailer 1's profit $\pi_{R1} = 2(1 - p_1)(p_1 - w_1')$ subject to $p_1 \leq \frac{1}{0.85} w_2^*$. We find that for any $w_1' \in [0.75, 1]$, the solution to the maximization problem is $p_1' = \frac{1}{0.85} w_2^*$. A necessary condition for $p_1' = \frac{1}{0.85} w_2^*$ to be retailer 1's *global* maximizer is that $\frac{\partial_+ \pi_{R1}}{\partial p_1}\big|_{p_1 = \frac{1}{0.85} w_2^*} < 0$, i.e., the right-derivative of retailer 1's profit function at the point $\frac{1}{0.85} w_2^*$ needs to be negative since otherwise retailer 1 will benefit by charging a price above $\frac{1}{0.85} w_2^*$. Since $\frac{\partial_+ \pi_{R1}}{\partial p_1}\big|_{p_1 = \frac{1}{0.85} w_2^*} = \frac{23 w_1'}{3} - \frac{19458473}{3032460}$, we have $\frac{\partial_+ \pi_{R1}}{\partial p_1}\big|_{p_1 = \frac{1}{0.85} w_2^*} < 0$ if and only if $w_1' < \frac{19458473}{23248860}$.[2] In words, to push the competitor out of the market, manufacturer 1's deviation price w_1' will need to be below $\frac{19458473}{23248860}$. So, suppose that

[2] To see that $\frac{\partial_+ \pi_{R1}}{\partial p_1}\big|_{p_1 = \frac{1}{0.85} w_2^*} = \frac{23 w_1'}{3} - \frac{19458473}{3032460}$, using equation (W11) we find that if $p_1 \in (\frac{1}{0.85} w_2^*, 0.15 + w_2^*)$, then $\pi_{R1} = ((1 - p_1) + (1 - \frac{p_1 - w_2^*}{1 - 0.85}))(p_1 - w_1')$. After substituting in $w_2^* = \frac{880117}{1189200}$, we differentiate π_{R1} with respect to p_1 and take the limit as $p_1 \to \frac{1}{0.85} w_2^*$. The resulting expression is $\frac{\partial_+ \pi_{R1}}{\partial p_1}\big|_{p_1 = \frac{1}{0.85} w_2^*} = \frac{23 w_1'}{3} - \frac{19458473}{3032460}$.

manufacturer 1's deviation price satisfies $w_1' < \frac{19458473}{23248860}$, which leads to an equilibrium where retailer 2 has zero market share, and hence, the equilibrium retail prices are given by $(p_1', p_2') = (\frac{1}{0.85} w_2^*, w_2^*)$. Plugging (p_1', p_2') into manufacturer 1's profit function, we obtain manufacturer 1's deviation profit $\pi_{M1}' = 2(1 - \frac{1}{0.85} w_2^*)(w_1' - 0.75)$. Since π_{M1}' is increasing in w_1', it follows that $\pi_{M1}' < 2(1 - \frac{1}{0.85} w_2^*)(\frac{19458473}{23248860} - 0.75)$ for all $w_1' \in [0.75, \frac{19458473}{23248860})$. Plugging in $w_2^* = \frac{880117}{1189200}$ into the last inequality, we find that $\pi_{M1}' < \frac{66064746271}{2937551583150} \approx 0.0224897$. Note that $\frac{66064746271}{2937551583150} < \pi_{M1}^*$, where $\pi_{M1}^* = \frac{17890301347}{795485610000} \approx 0.0224898$. Hence, it follows that $\pi_{M1}' < \pi_{M1}^*$ for all $w_1' < \frac{19458473}{23248860}$, i.e., manufacturer 1 cannot improve its profit by deviating to a price that will push the competitor out of the market. Next, suppose that manufacturer 1 deviates to a price w_1'' so that retailer 1 serves only its loyal customer segment without serving any consumers in the switcher segment. Hence, manufacturer 1's deviation profit is $\pi_{M1}'' = (1 - p_1'')(w_1'' - 0.75)$, where $p_1'' \geq w_1''$ is the price that retailer 1 will charge after manufacturer 2's deviation to w_1''. Note that $\pi_{M1}'' \leq (1 - w_1'')(w_1'' - 0.75)$. Moreover, one can easily show that for any $w_1'' \in [0.75, 1]$, we have $(1 - w_1'')(w_1'' - 0.75) < \pi_{M1}^*$. Note that the maximum of the function $(1 - w_1'')(w_1'' - 0.75)$ is $\frac{1}{64}$, which is achieved at $w_1'' = \frac{7}{8}$. Also note that $\frac{1}{64} < \pi_{M1}^*$, where $\pi_{M1}^* \approx 0.0224898$. From the inequalities $\pi_{M1}'' \leq (1 - w_1'')(w_1'' - 0.75)$, $(1 - w_1'')(w_1'' - 0.75) \leq \frac{1}{64}$ and $\frac{1}{64} < \pi_{M1}^*$, it follows that $\pi_{M1}'' < \pi_{M1}^*$, i.e., if manufacturer 1 deviates to a price w_1'' such that retailer 1 will serve only its loyal customers, then manufacturer 1 will become worse off than when manufacturer 1 does not deviate. A similar proof also shows that retailer 2 and manufacturer 2 do not have any profitable non-local deviations.

Next, suppose that manufacturer 2 exits the market due to an exogenous shock, and after manufacturer 2's exit, retailer 2 will join manufacturer 1's distribution channel. Given manufacturer 1's wholesale price w_1, retailers 1 and 2 simultaneously choose their retail prices to maximize their profits.

If the retailers are charging the same price, then we assume that the segment with switchers is divided equally between the retailers. Retailer 1's demand function is

$$\widetilde{D}_1 = \begin{cases} 2(1-p_1) & \text{if } p_1 < p_2 \\ (1-p_1) + \frac{(1-p_1)}{2} & \text{if } p_1 = p_2 \\ (1-p_1) & \text{if } p_2 < p_1 \leq 1 \\ 0 & \text{if } p_1 > 1 \end{cases} \tag{W14}$$

Similarly, retailer 2's demand function is

$$\widetilde{D}_2 = \begin{cases} (1-p_1) & \text{if } p_2 < p_1 \\ \frac{(1-p_1)}{2} & \text{if } p_2 = p_1 \\ 0 & \text{if } p_1 < p_2 < 1 \end{cases} \tag{W15}$$

Retailer i chooses its price to maximize its profit π_{Ri}, where $\pi_{R1} = \widetilde{D}_1(p_1 - w_1)$ and $\pi_{R2} = \widetilde{D}_2(p_2 - w_1)$. One can readily show that retailer i's best-response to the competitor's price is as follows:

$$BR_1 = \begin{cases} \frac{1+w_1}{2} & \text{if } p_2 \leq \frac{1+w_1}{2} - \frac{\sqrt{2}(1-w_1)}{4} \text{ or } p_2 \geq \frac{1+w_1}{2} \\ p_2 - \epsilon & \text{if } \frac{1+w_1}{2} - \frac{\sqrt{2}(1-w_1)}{4} < p_2 < \frac{1+w_1}{2} \end{cases}$$

$$BR_2 = \begin{cases} w_1 & \text{if } p_1 \leq w_1 \\ p_1 - \epsilon & \text{if } w_1 < p_1 \leq \frac{1+w_1}{2} \\ \frac{1+w_1}{2} & \text{if } p_1 > \frac{1+w_1}{2} \end{cases}$$

where $\epsilon > 0$ is arbitrarily close to zero. It turns out that BR_1 and BR_2 do not intersect, and hence, there is no pure-strategy equilibrium in retail prices. So, we will find the mixed strategy equilibrium in retail prices.

One can show that the support of retailer i's price distribution is an interval with endpoints $\underline{\lambda_i} < \bar{\lambda}_i$, such that $\underline{\lambda_1} = \underline{\lambda_2}$ and $\bar{\lambda}_1 = \bar{\lambda}_2 = \frac{1+w_1}{2}$.[3] To find the actual equilibrium probability distribution functions for the retailers' prices, we will use the property that each price within the support of retailer i's price distribution must give retailer i the same expected profit (since otherwise retailer i will have a profitable deviation). Let F_i be the cumulative distribution function (cdf) for retailer i's equilibrium price distribution. As we will demonstrate below, retailer 1's price distribution will actually have an atom (i.e., a mass point) at $\frac{1+w_1}{2}$. Let z_1 be the probability with which retailer 1 sets its price equal to $\frac{1+w_1}{2}$.

If retailer 1 charges $p_1 = \frac{1+w_1}{2}$, then its expected profit is $\mathbb{E}(\pi_{R1}|_{p_1=\frac{1+w_1}{2}}) = \frac{(1-w_1)^2}{4}$. If retailer 1 charges $p_1 \in [\underline{\lambda}, \frac{1+w_1}{2})$, then its expected profit is $\mathbb{E}(\pi_{R1}|_{p_1}) = (1-p_1)(p_1-w_1) + (1-F_2(p_1))(1-p_1)(p_1-w_1)$. Setting $\mathbb{E}(\pi_{R1}|_{p_1})$ equal to $\mathbb{E}(\pi_{R1}|_{p_1=\frac{1+w_1}{2}})$ yields $F_2(p_1) = \frac{8p_1(1+w_1)-1-8p_1^2-6w_1-w_1^2}{4(1-p_1)(p_1-w_1)}$. Note that $\underline{\lambda}$ must satisfy $F_2(\underline{\lambda}) = 0$, solving which we obtain $\underline{\lambda} = \frac{2-\sqrt{2}(1-w_1)+2w_1}{4}$. To summarize, retailer 2's price distribution is given by

$$F_2(p) = \begin{cases} \frac{8p(1+w_1)-1-8p^2-6w_1-w_1^2}{4(1-p)(p-w_1)} & \text{if } \frac{2-\sqrt{2}(1-w_1)+2w_1}{4} \leq p \leq \frac{1+w_1}{2} \\ 1 & \text{if } p \geq \frac{1+w_1}{2} \end{cases} \tag{W16}$$

[3] $\underline{\lambda_1} = \underline{\lambda_2}$ can be shown using a proof by contradiction. Namely, let us first assume that $\underline{\lambda_1} < \underline{\lambda_2}$. If $p_1 = \underline{\lambda_1}$, then retailer 1 receives a profit of $\pi_{R1}|_{p_1=\underline{\lambda_1}} = 2(1-\underline{\lambda_1})(\underline{\lambda_1}-w_1)$. If retailer 1 charges $p_1' \equiv (\underline{\lambda_1}+\underline{\lambda_2})/2$, then its profit is $\pi_{R1}|_{p_1=p_1'} = 2(1-p_1')(p_1'-w_1)$. Since the function $2(1-p_1)(p_1-w_1)$ is increasing for all $p_1 < \frac{1+w_1}{2}$ and since $\underline{\lambda_1} < p_1' < \frac{1+w_1}{2}$, it follows that $\pi_{R1}|_{p_1=p_1'} > \pi_{R1}|_{p_1=\underline{\lambda_1}}$, i.e., retailer 1 has a profitable deviation, which is a contradiction to $\underline{\lambda_1}$ being the lower endpoint of the support of retailer 1's equilibrium price distribution. Similarly, one can show that $\underline{\lambda_1} > \underline{\lambda_2}$ also leads to a contradiction. Hence, it must be that $\underline{\lambda_1} = \underline{\lambda_2}$. Proofs that retailer i's price distribution is an interval and that $\bar{\lambda}_1 = \bar{\lambda}_2$ are rather standard in the literature, so we will not present them here (for example, see Narasimhan 1988).

Next, if retailer 2 charges a $p_2 = \frac{1+w_1}{2}$, then its expected profit is $\mathbb{E}(\pi_{R2}|_{p_2=\frac{1+w_1}{2}}) = z_1 \frac{(1-w_1)^2}{4}$, where z_1 is the probability with which retailer 1 charges $p_1 = \frac{1+w_1}{2}$. If retailer 2 charges $p_2 \in [\underline{\lambda}, \frac{1+w_1}{2})$, then its expected profit is $\mathbb{E}(\pi_{R2}|_{p_2}) = (1 - F_1(p_2))(1 - p_2)(p_2 - w_1)$. Setting $\mathbb{E}(\pi_{R2}|_{p_2})$ equal to $\mathbb{E}(\pi_{R2}|_{p_2=\frac{1+w_1}{2}})$ yields $F_1(p_2) = \frac{4p_2(1+w_1)-4p_2^2-2w_1(2-z_1)-z_1-w_1^2 z_1}{4(1-p_2)(p_2-w_1)}$. Note that we must have $F_1(\underline{\lambda}) = 0$. Using $\underline{\lambda} = \frac{2-\sqrt{2}(1-w_1)+2w_1}{4}$, one can easily show that $F_1(\underline{\lambda}) = 1 - 2z_1$. Solving $F_1(\underline{\lambda}) = 0$ gives us $z_1 = \frac{1}{2}$. Plugging $z_1 = \frac{1}{2}$ into the expression for $F_1(p_2)$, we find that $F_1(p_2) = \frac{8p_1(1+w_1)-1-8p_1^2-6w_1-w_1^2}{8(1-p_1)(p_1-w_1)}$. To summarize, retailer 1's price distribution is given by

$$F_1(p) = \begin{cases} \frac{8p(1+w_1)-1-8p^2-6w_1-w_1^2}{8(1-p)(p-w_1)} & \text{if } \frac{2-\sqrt{2}(1-w_1)+2w_1}{4} \le p < \frac{1+w_1}{2} \\ 1 & \text{if } p \ge \frac{1+w_1}{2} \end{cases} \tag{W17}$$

Manufacturer 1's expected profit is given by

$$\mathbb{E}(\pi_{M1}) = \mathbb{E}\big((1-p_1) + (1-\min\{p_1,p_2\})\big)(w_1 - 0.75)$$

$$= [\tfrac{1}{2}(1 - \tfrac{1+w_1}{2}) + \int_{\underline{\lambda}}^{\frac{1+w_1}{2}} (1-p)\, dF_1(p) + \int_{\underline{\lambda}}^{\frac{1+w_1}{2}} (1-p)\, dG(p)](w_1 - 0.75)$$

where $G(p) = 1 - (1 - F_1(p))(1 - F_2(p))$ is the cumulative distribution function for the random variable $\min\{p_1, p_2\}$. Differentiating $F_1(p)$ and $G(p)$, we can find that $dF_1(p) = \frac{(1-2p+w_1)(1-w_1)^2}{41(1-p)^2(p-w_1)^2}$ and $dG(p) = \frac{(1-w_1)^2(1-4p^3+w_1+w_1^2+w_1^3+6p^2(1+w_1)-4p(1+w_1+w_1^2))}{16(1-p)^3(p-w_1)^3}$. Plugging the expressions for $dF_1(p)$ and $dG(p)$ into $\mathbb{E}(\pi_{M1})$ and simplifying the expression, we obtain

$$\mathbb{E}(\pi_{M1}) = [\tfrac{1}{2}(1 - \tfrac{1+w_1}{2}) + \int_{\underline{\lambda}}^{\frac{1+w_1}{2}} (1-p) \frac{(1-2p+w_1)(1-w_1)^4}{16(1-p)^3(p-w_1)^3} dp](w_1 - 0.75) \tag{W18}$$

The antiderivative of $(1-p)\frac{(1-2p+w_1)(1-w_1)^4}{16(1-p)^3(p-w_1)^3}$ is

$$A(p) \equiv \frac{1-w_1}{32}(2\,ln(1-p) - 2\,ln(p-w_1) - \frac{2(1-w_1)}{1-p} - \frac{(1-w_1)^2}{(p-w_1)^2})$$

Hence, $\int_{\underline{\lambda}}^{\frac{1+w_1}{2}}(1-p)\frac{(1-2p+w_1)(1-w_1)^4}{16(1-p)^3(p-w_1)^3}dp = A(\frac{1+w_1}{2}) - A(\underline{\lambda})$. One can show that

$$A(\frac{1+w_1}{2}) - A(\underline{\lambda}) = \frac{(1-w_1)(12+(2-\sqrt{2})(ln(2-\sqrt{2})-ln(2+\sqrt{2})))}{16(2-\sqrt{2})}$$

Substituting $\int_{\underline{\lambda}}^{\frac{1+w_1}{2}}(1-p)\frac{(1-2p+w_1)(1-w_1)^4}{16(1-p)^3(p-w_1)^3}dp$ with $A(\frac{1+w_1}{2}) - A(\underline{\lambda})$ in the equation (W18), we obtain

$$\mathbb{E}(\pi_{M1}) = [\frac{1}{2}(1-\frac{1+w_1}{2}) + \frac{(1-w_1)(12+(2-\sqrt{2})(ln(2-\sqrt{2})-ln(2+\sqrt{2})))}{16(2-\sqrt{2})}](w_1 - 0.75)$$

Note that $\mathbb{E}(\pi_{M1})$ is concave (since the second derivative is $-2.84 < 0$). Solving the first-order condition ($\frac{\partial\mathbb{E}(\pi_{M1})}{\partial w_1} = 0$), we find that $w_1^{**} = \frac{7}{8}$. Using w_1^{**}, we find manufacturer 1's equilibrium expected profit:

$$\mathbb{E}(\pi_{M1}^{**}) = \frac{4(5-\sqrt{2})+(2-\sqrt{2})(ln(2-\sqrt{2})-ln(2+\sqrt{2}))}{1024(2-\sqrt{2})} \approx 0.02219 \qquad \text{(W19)}$$

Since retailer 1 receives the *same* expected profit for any given price in the support of retailer 1's price distribution, we can find retailer 1's expected profit by simply computing its profit when $p_1 = \frac{1+w_1^{**}}{2}$, which is given by

$$\mathbb{E}(\pi_{R1}^{**}) = \frac{(1-w_1^{**})^2}{4} \approx 0.0039 \qquad \text{(W20)}$$

Finally, let us compare manufacturer 1's and retailer 1's profits before manufacturer 2's exit (i.e., π_{M1}^* and π_{R1}^*) with their expected profits after manufacturer 2's exit (i.e., $\mathbb{E}(\pi_{M1}^{**})$ and $\mathbb{E}(\pi_{R1}^{**})$). Recall from equation (W13) that $\pi_{M1}^* \approx 0.0224898$, $\pi_{R1}^* \approx 0.00869$. Hence, $\pi_{M1}^* > \mathbb{E}(\pi_{M1}^{**})$ and $\pi_{R1}^* > \mathbb{E}(\pi_{R1}^{**})$, i.e., manufacturer 2's exit makes both manufacturer 1 and retailer 1 worse off.

The above result is counterintuitive: after manufacturer 2 exits the market and retailer 2 starts selling manufacturer 1's product, why doesn't price competition between retailers 1 and 2 become even more intense, leading to higher sales and profit for manufacturer 1? Intuitively, after manufacturer 2's exit, switchers will buy from the retailer that offers the lowest price. Hence, if both retailers compete for switchers, they will have to offer deep discounts to increase the likelihood of being the lowest-price retailer. However, since retailer 1's loyal segment is large enough, retailer 1 prefers charging a high price (probabilistically, on average) to focus on its loyal segment, rather than competing head-on for the switcher segment. Retailer 1 charging a high price induces retailer 2 to also increase its price. Thus, retailer 1's focus on its loyal customers leads to higher average retail prices and lower unit sales for manufacturer 1. By contrast, before manufacturer 2's exit, the retailers are differentiated by the qualities of the products that they sell. Hence, both retailers compete for the switchers without having to reduce their prices too much, because retailer 1 serves switchers with high willingness to pay, while retailer 2 serves switchers with lower willingness to pay. Furthermore, note that after manufacturer 2's exit, the demand curve for manufacturer 1's product will become less steep, which also reduces the retailers' incentives to decrease their prices, worsening the double-marginalization problem in manufacturer 1's channel. The reason is that a marginal decrease in retail price will lead to fewer additional sales for the retailer than when the demand curve is steeper. Intuitively, when manufacturer 2 is in the market, the marginal consumers are relatively price sensitive because if the price of one product increases, then the consumers will tend to buy the other product. By contrast, after manufacturer 2's exit, the marginal consumers become less price sensitive because if they do not buy manufacturer 1's product, they can opt only for the outside option.

Note that retailer 2 joining manufacturer 1's channel after manufacturer 2's exit makes manufacturer 1 more likely to benefit from manufacturer 2's exit. However, qualitatively, the result in the above numerical example is consistent with our core model: manufacturer 2's exit from the market increases manufacturer 1's demand and allows manufacturer 1 to charge a higher wholesale price, but it also makes the demand curve for manufacturer 1's product less steep, which can worsen the double-marginalization problem within manufacturer 1's channel. If manufacturer 2 is not very strong (e.g., due to its small size and low economies of scale), then its exit will make manufacturer 1 worse off, because worsened double-marginalization problem will dominate the positive effect of manufacturer 1 becoming monopolist. ■

EXTENSION WITH ASYMMETRIC RETAILING COST.

We extend our base model by assuming that retailer i has a selling cost s_i, where $s_1 > s_2$. To simplify the analysis, we assume that $s_2 = 0$. We will show that if s_1 is not too large, then our core model's main results will hold, i.e., manufacturer 2's exit can make manufacturer 1 and retailer 1 worse off, and an increase in manufacturer 2's quality can raise manufacturer 1's profit.

To ensure that in equilibrium both manufacturers have a positive market share, we will assume that $c_2 \in (\underline{c}, \bar{c})$, where $\underline{c} \equiv \max\{0, \frac{8c_1q_1^2-8q_1^3-9c_1q_1q_2+11q_1^2q_2+2c_1q_2^2-3q_1q_2^2+8q_1^2s_1-9q_1q_2s_1+2q_2^2s_1}{2q_1^2-q_1q_2}\}$ and $\bar{c} \equiv \frac{2c_1q_1q_2+6q_1^2q_2-c_1q_2^2-8q_1q_2^2+2q_2^3+2q_1q_2s_1-q_2^2s_1}{8q_1^2-9q_1q_2+2q_2^2}$.

First, let us find the manufacturer 1's and retailer 1's equilibrium profits when manufacturer 2 is in the market. We solve the game by backward induction. Given the wholesale prices, retailer 1's and retailer 2's profits are $\pi_{R1}(p_1, p_2) = (1 - \frac{p_1-p_2}{q_1-q_2})(p_1 - s_1 - w_1)$ and $\pi_{R2}(p_1, p_2) = (\frac{p_1-p_2}{q_1-q_2} - \frac{p_2}{q_2})(p_2 - w_2)$, respectively. Solving the first-order conditions ($\frac{\partial \pi_{R1}}{\partial p_1} = 0$ and $\frac{\partial \pi_{R2}}{\partial p_2} = 0$), we obtain the equilibrium retail prices $p_1^*(w_1, w_2)$ and $p_2^*(w_1, w_2)$ for the subgame conditional on the wholesale prices w_1 and w_2. Anticipating the retailers' optimal pricing decisions, manufacturer 1 chooses w_1 to maximize its profit $\pi_{M1}(w_1, w_2) = (1 - \frac{p_1^*(w_1,w_2)-p_2^*(w_1,w_2)}{q_1-q_2})(w_1 - c_1)$, and manufacturer 2 chooses w_2 to maximize its profit $\pi_{M2}(w_1, w_2) = (\frac{p_1^*(w_1,w_2)-p_2^*(w_1,w_2)}{q_1-q_2} - \frac{p_2^*(w_1,w_2)}{q_2})(w_2 - c_2)$. Solving the first order conditions ($\frac{\partial \pi_{M1}}{\partial w_1} = 0$ and $\frac{\partial \pi_{M2}}{\partial w_2} = 0$), one can readily find the manufacturers' equilibrium wholesale prices (w_1^* and w_2^*), from which one can also obtain the equilibrium retail prices (p_1^* and p_2^*). Using these equilibrium prices, we derive manufacturer 1's and retailer 1's equilibrium profits:

$$\pi_{M1}^*(s_1) = \frac{(2q_1-q_2)(11q_1^2q_2-8q_1^3-3q_1q_2^2-c_2q_1(2q_1-q_2)+c_1(8q_1^2-9q_1q_2+2q_2^2)+8q_1^2s_1-9q_1q_2s_1+2q_2^2s_1)^2}{(4q_1^2-5q_1q_2+q_2^2)(16q_1^2-17q_1q_2+4q_2^2)^2}$$

$$\pi_{R1}^{*}(s_1) = \frac{(2q_1-q_2)^2(11q_1^2q_2-8q_1^3-3q_1q_2^2-c_2q_1(2q_1-q_2)+c_1(8q_1^2-9q_1q_2+2q_2^2)+8q_1^2s_1-9q_1q_2s_1+2q_2^2s_1)^2}{(q_1-q_2)(4q_1-q_2)^2(16q_1^2-17q_1q_2+4q_2^2)^2}$$

Second, one can show that after manufacturer 2 exits the market, manufacturer 1's and retailer 1's equilibrium profits are $\pi_{M1}^{**}(s_1) = \frac{(q_1-c_1)(q_1-c_1-2s_1)}{8q_1}$ and $\pi_{R1}^{**}(s_1) = \frac{(q_1-c_1-s_1)^2}{16q_1}$, respectively.

Define $\Delta\pi_{M1}(s_1) \equiv \pi_{M1}^{*}(s_1) - \pi_{M1}^{**}(s_1)$ and $\Delta\pi_{R1}(s_1) \equiv \pi_{R1}^{*}(s_1) - \pi_{R1}^{**}(s_1)$. Recall that we are assuming that $c_2 \in (\underline{c}, \bar{c})$ to ensure that in equilibrium both manufacturers have a positive market share. One can show that there exists $\hat{q}$ such that if $q_2 < \hat{q}$, then $\Delta\pi_{M1}(0)|_{c_2=\bar{c}} > 0$. By continuity of $\Delta\pi_{M1}(0)$ in c_2, it follows that there exists $\hat{c} \in (\underline{c}, \bar{c})$ such that if $c_2 \in (\hat{c}, \bar{c})$, then $\Delta\pi_{M1}(0) > 0$. Since $\Delta\pi_{M1}(s_1)$ is continuous in s_1, there must exist $\hat{s} > 0$ such that if $s_1 < \hat{s}$ then $\Delta\pi_{M1}(s_1) > 0$. In words, if manufacturer 2's product is not very competitive ($q_2 < \hat{q}$ and $c_2 > \hat{c}$) and retailer 1's selling cost is not too high ($s_1 < \hat{s}$), then manufacturer 2's exit makes manufacturer 1 worse off ($\Delta\pi_{M1}(s_1) > 0$). Using a similar proof, one can also show that if manufacturer 2's product is not very competitive and retailer 1's selling cost is not too high, then manufacturer 2's exit will make retailer 1 worse off ($\Delta\pi_{R1}(s_1) > 0$).

It remains to show that an increase in q_2 can lead to an increase in $\pi_{M1}^{*}(s_1)$. One can show that there exists $\tilde{c}$ such that $\frac{\partial\pi_{M1}^{*}(0)}{\partial q_2} > 0$ if $c_2 \in (\tilde{c}, \bar{c})$, where

$$\tilde{c} \equiv \max\{0, \frac{c_1(640q_1^6-1968q_1{}^5q_2+2470q_1^4q_2^2-1608q_1^3q_2^3+573q_1^2q_2^4-106q_1q_2^5+8q_2^6)-128q_1^7+752q_1^6q_2-1410q_1^5q_2^2+1226q_1^4q_2^3-557q_1^3q_2^4+129q_1^2q_2^5-12q_1q_2^6}{480q_1^6-1188q_1^5q_2+1146q_1^4q_2^2-546q_1^3q_2^3+129q_1^2q_2^4-12q_1q_2^5}\}$$

Since $\frac{\partial\pi_{M1}^{*}(s_1)}{\partial q_2}$ is continuous in s_1, it follows that there exists $\tilde{s}$ such that if $c_2 \in (\tilde{c}, \bar{c})$ and $s_1 < \tilde{s}$, then $\frac{\partial\pi_{M1}^{*}(s_1)}{\partial q_2} > 0$, i.e., manufacturer 1's profit increases in q_2. To show that the interval $(\tilde{c}, \bar{c})$ is non-empty, let us assume that $q_1 = 1$, $q_2 = 0.25$, $c_1 = 0.15$ and $s_1 = 0$. Then, one can show that $\tilde{c} \approx 0.126$ and $\bar{c} \approx 0.187$. ■

EXTENSION WITH RETAIL SERVICE.

Suppose that if retailer $i \in \{1,2\}$ provides service $s_i \geq 0$, then purchasing from retailer i will give the consumer utility $\theta q_i - p_i + s_i$. The retailer's fixed cost of providing the service is ks_i^2. The timing is as follows: first, the manufacturers set their wholesale prices, then the retailers choose their service levels and prices, and finally, consumers make their purchase decisions. For tractability, we assume that $q_1 = 1$, $q_2 = 0.5$ and $k = 2$.

We solve the game by backward induction. To ensure that in equilibrium each manufacturer has a positive market share, we assume that $c_2 \in (\underline{c}, \bar{c})$, where $\underline{c} \equiv \max\{0, \frac{16c_1-11}{10}\}$ and $\bar{c} \equiv \frac{1+c_1}{4}$. Given the wholesale prices w_1 and w_2, retailer $i \in \{1,2\}$ chooses its service s_i and price p_i to maximize its profit π_{Ri}, where $\pi_{R1} = (1 - \frac{p_1-s_1-p_2+s_2}{1-0.5})(p_1 - w_1) - 2s_1^2$ and $\pi_{R2} = (\frac{p_1-s_1-p_2+s_2}{1-0.5} - \frac{p_2-s_2}{0.5})(p_2 - w_2) - 2s_2^2$. To show that the Hessian matrices corresponding to π_{R1} and π_{R2} are negative definite, note that

$$H_{\pi_{R1}} = \begin{pmatrix} -4 & 2 \\ 2 & -4 \end{pmatrix} \text{ and } H_{\pi_{R2}} = \begin{pmatrix} -8 & 4 \\ 4 & -4 \end{pmatrix}$$

One can easily show that for any vector $z \neq 0$, we have $z' H_{\pi_{Ri}} z < 0$, i.e., the matrix $H_{\pi_{Ri}}$ is negative definite, and hence, the second-order conditions for the retailers' optimization problem are satisfied. Solving the first-order conditions ($\frac{\partial \pi_{Ri}}{\partial p_i} = 0$ and $\frac{\partial \pi_{Ri}}{\partial s_i} = 0$), we obtain the subgame equilibrium prices and service levels: $p_1^* = \frac{1+w_1+2w_2}{3}$, $p_2^* = \frac{1+4w_1+2w_2}{12}$, $s_1^* = \frac{1-2w_1+2w_2}{6}$ and $s_2^* = \frac{1+4w_1-10w_2}{12}$. Using these expressions, we can obtain the manufacturers' profits to be $\pi_{M1} = \frac{2(w_1-c_1)(1-2w_1+2w_2)}{3}$ and $\pi_{M2} = \frac{(w_2-c_2)(1+4w_1-10w_2)}{3}$. Solving the first-order conditions ($\frac{\partial \pi_{M1}}{\partial w_1} = 0$ and $\frac{\partial \pi_{M2}}{\partial w_2} = 0$), we obtain the manufacturers' equilibrium wholesale prices $w_1^* = \frac{11+20c_1+10c_2}{36}$ and

$w_2^* = \frac{1+c_1+5c_2}{9}$. Plugging the equilibrium prices into manufacturer 1's and retailer 1's profit functions, we obtain $\pi_{M1}^* = \frac{(11-16c_1+10c_2)^2}{972}$ and $\pi_{R1}^* = \frac{(11-16c_1+10c_2)^2}{1944}$.

Now, suppose that manufacturer 2 exits the market due to some exogenous shock. For a given wholesale price w_1, retailer 1 chooses s_1 and p_1 to maximize its profit $\pi_{R1} = (1 - \frac{p_1-s_1}{1})(p_1 - w_1) - 2s_1^2$. One can show that π_{R1}'s Hessian matrix is negative definite. Solving the first-order conditions ($\frac{\partial \pi_{R1}}{\partial p_1} = 0$ and $\frac{\partial \pi_{R1}}{\partial s_1} = 0$), we find that $p_1^{**} = \frac{4+3w_1}{7}$ and $s_1^{**} = \frac{1-w_1}{7}$. Manufacturer 1 correctly anticipates p_1^{**} and s_1^{**}, and chooses its price w_1 to maximize its profit $\pi_{M1} = \frac{4(w_1-c_1)(1-w_1)}{7}$. Using the first-order condition ($\frac{\partial \pi_{M1}}{\partial w_1} = 0$), we find manufacturer 1's equilibrium wholesale price $w_1^{**} = \frac{1+c_1}{2}$. Using the equilibrium retail and wholesale prices and retailer 1's service level, we can derive manufacturer 1's and retailer 1's equilibrium profits: $\pi_{M1}^{**} = \frac{(1-c_1)^2}{7}$ and $\pi_{R1}^{**} = \frac{(1-c_1)^2}{14}$.

Comparing manufacturer 1's and retailer 1's profits before and after manufacturer 2's exit from the market, we find that there exists $\hat{c} \equiv \frac{16c_1-11}{10} + \frac{9}{5}\sqrt{\frac{3(1-2c_1+c_1^2)}{7}}$ such that if $c_2 \in (\hat{c}, \bar{c})$, then manufacturer 2's exit makes both manufacturer 1 and retailer 1 worse off. To see that the interval $(\hat{c}, \bar{c})$ is non-empty, note that if $c_1 = 0.2$, then $\hat{c} \approx 0.16$ and $\bar{c} = 0.3$. ■

EXTENSION WITH CONSUMER HETEROGENEITY IN BOTH VERTICAL AND HORIZONTAL PREFERENCES.

We extend our core model by assuming that the manufacturers are also horizontally differentiated. We use a standard horizontal-differentiation model where manufacturer 1's product is "located" at 0 and manufacturer 2's product is at 1, and consumers are uniformly distributed on the line segment between 0 and 1. The total number of consumers is normalized to one. Each manufacturer sells through its exclusive retailer. If a consumer purchases manufacturer 1's product, she will obtain a utility $\theta q_1 - p_1 - tx$, and if she purchases manufacturer 2's product, her utility will be $\theta q_2 - p_2 - t(1-x)$, where q_i is manufacturer i's product quality, p_i is the retail price, $x \in [0,1]$ represents the consumer's horizontal preference and t measures the strength of the consumers' horizontal preferences. To be consistent with our core model, we assume that θ is uniformly distributed on $[0,1]$. All remaining parameters and variables are defined the same way as in the horizontal model above. One can show that the demand functions for products 1 and 2 are

$$D_1 = \begin{cases} 1 - \frac{2p_1+t}{2q_1} & \text{if } p_1 \le \frac{q_1}{q_2}p_2 - t \\ d_1 & \text{if } \frac{q_1}{q_2}p_2 - t < p_1 \le \frac{q_1}{q_2}(p_2+t) \\ 1 - \frac{p_1-p_2}{q_1-q_2} & \text{if } \frac{q_1}{q_2}(p_2+t) < p_1 \le q_1 - q_2 + p_2 - t \\ \frac{(q_1-q_2-p_1+p_2+t)^2}{4(q_1-q_2)t} & \text{if } q_1 - q_2 + p_2 - t < p_1 < q_1 - q_2 + p_2 + t \\ 0 & \text{if } p_1 \ge q_1 - q_2 + p_2 + t \end{cases} \tag{W21}$$

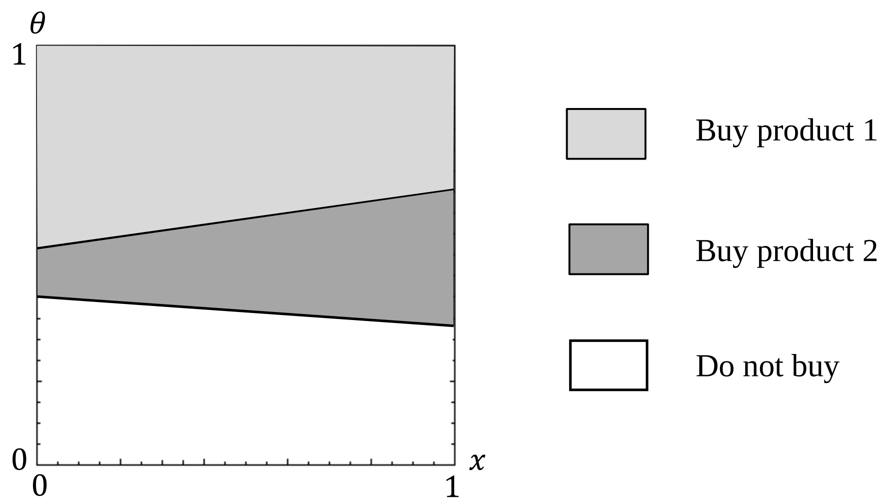
$$D_2 = \begin{cases} 1 - \frac{2p_2+t}{2q_2} & \text{if } p_1 \le q_2 - q_1 + p_1 - t \\ d_2 & \text{if } q_2 - q_1 + p_1 - t < p_2 \le q_2 - q_1 + p_1 + t \\ \frac{p_1-p_2}{q_1-q_2} - \frac{2p_2+t}{2q_2} & \text{if } q_2 - q_1 + p_1 + t < p_2 \le \frac{q_2}{q_1}p_1 - t \\ \frac{(p_2q_1-q_2(p_1+t))^2}{2(q_1-q_2)q_2(q_1+q_2)t} & \text{if } \frac{q_2}{q_1}p_1 - t < p_2 < \frac{q_2}{q_1}(p_1+t) \\ 0 & \text{if } p_2 \ge \frac{q_2}{q_1}(p_1+t) \end{cases} \tag{W22}$$

where $d_1 = \frac{q_1(2p_2q_2(p_1+t)+t(2q_1^2-2q_2^2-q_1t)-q_1(p_2^2-2p_1t))-p_1^2q_2^2}{2q_1(q_1-q_2)(q_1+q_2)t}$ and

$d_2 \equiv \frac{2p_1q_2(p_2+q_1-q_2+t)-q_2(p_1^2+p_2^2+q_1^2)-q_2^2(q_2-2q_1)+2q_1t(q_2-t)-q_2t(2q_2-t)-2p_2(q_1(q_2+2t)-q_2(q_2+t))}{4q_2(q_1-q_2)t}$.

We solve the game by backward induction. When t is not very large, for every $x \in [0,1]$, consumers with high θ will buy product 1, consumers with intermediate θ will buy product 2, and consumers with low θ will not buy any of the products. Figure W8 graphically illustrates the equilibrium market segmentation.

Figure W8 Equilibrium Market Segmentation for t Not Too High



Retailer 1's and retailer 2's profits are given by $\pi_{R1} = (1 - \frac{p_1-p_2}{q_1-q_2})(p_1 - w_1)$ and $\pi_{R2} = (\frac{p_1-p_2}{q_1-q_2} - \frac{2p_2+t}{2q_2})(p_2 - w_2)$, respectively. Retailer i will choose its price p_i to maximize its profit π_{Ri}. Solving the first-order conditions ($\frac{\partial \pi_{R1}}{\partial p_1} = 0$ and $\frac{\partial \pi_{R2}}{\partial p_2} = 0$), we find the subgame equilibrium retail prices $p_1^* = \frac{4q_1^2+q_2t-q_1(4q_2+t-4w_1-2w_2)}{8q_1-2q_2}$ and $p_2^* = \frac{q_2(-q_2+t+w_1)+q_1(q_2-t+2w_2)}{4q_1-q_2}$. The manufacturers correctly anticipate the retailers' subgame equilibrium prices, and manufacturer i chooses its wholesale price w_i to maximize its profit π_{Mi}, where $\pi_{M1} = \frac{(w_1-c_1)(4q_1^2+q_2(t+2w_1)-q_1(4q_2+t+4w_1-2w_2))}{2(4q_1^2-5q_1q_2+q_2^2)}$ and $\pi_{M2} = \frac{q_1(w_2-c_2)(q_1(q_2-t-2w_2)-q_2(q_2-t-w_1-w_2))}{(q_1-q_2)(4q_1-q_2)q_2}$.

Solving the first-order conditions ($\frac{\partial \pi_{M1}}{\partial w_1} = 0$, $\frac{\partial \pi_{M2}}{\partial w_2} = 0$), we find that $w_1^* = \frac{c_2 q_1(2q_1-q_2)+2c_1(2q_1-q_2)^2+(q_1-q_2)(8q_1^2+q_2 t-3q_1(q_2+t))}{16q_1^2-17q_1q_2+4q_2^2}$ and $w_2^* = \frac{2c_1(2q_1-q_2)q_2+4c_2(2q_1-q_2)^2+(q_1-q_2)(12q_1q_2-4q_2^2-8q_1t+3q_2t)}{32q_1^2-34q_1q_2+8q_2^2}$. Substituting the equilibrium wholesale and retail prices into the profit functions, we find that

$$\begin{aligned} \pi_{M1}^* &= \frac{(2q_1-q_2)(c_2q_1(2q_1-q_2)-c_1(8q_1^2-9q_1q_2+2q_2^2)+(q_1-q_2)(8q_1^2+q_2t-3q_1(q_2+t)))^2}{(4q_1^2-5q_1q_2+q_2^2)(16q_1^2-17q_1q_2+4q_2^2)^2} \\ \pi_{R1}^* &= \frac{(2q_1-q_2)^2(c_1(8q_1^2-9q_1q_2+2q_2^2)-c_2q_1(2q_1-q_2)-(q_1-q_2)(8q_1^2+q_2t-3q_1(q_2+t)))^2}{(q_1-q_2)(4q_1-q_2)^2(16q_1^2-17q_1q_2+4q_2^2)^2} \end{aligned} \tag{W23}$$

If the competitor exits the market, then demand for manufacturer 1's product becomes

$$\widetilde{D}_1 = \begin{cases} 1-\frac{2p_1+t}{2q_1} & \text{if } p_1 < q_1 - t \\ \frac{(q_1-p_1)^2}{2q_1 t} & \text{if } q_1 - t < p_1 < q_1 \\ 0 & \text{if } p_1 \geq q_1 \end{cases} \tag{W24}$$

Given manufacturer 1's wholesale price w_1, retailer 1 chooses p_1 to maximize its profit $\pi_{R1} = \widetilde{D}_1(p_1 - w_1)$. One can show that retailer 1's subgame equilibrium price is

$$p_1^{**} = \begin{cases} \frac{2q_1-t+2w_1}{4} & \text{if } w_1 < \frac{2q_1-3t}{2} \\ \frac{q_1+2w_1}{3} & \text{if } \frac{2q_1-3t}{2} < w_1 < q_1 \end{cases}$$

Manufacturer 1 correctly anticipates retailer 1's subgame equilibrium price and chooses its wholesale price w_1 to maximize its profit $\pi_{M1} = \widetilde{D}_1(p_1^{**}) \cdot (w_1 - c_1)$. One can show that manufacturer 1's optimal price is

$$w_1^{**} = \begin{cases} \frac{2q_1+2c_1-t}{4} & \text{if } t \leq \frac{2(q_1-c_1)}{5} \\ \frac{2q_1-3t}{2} & \text{if } \frac{2(q_1-c_1)}{5} < t < \frac{4(q_1-c_1)}{9} \\ \frac{q_1+2c_1}{3} & \text{if } t \geq \frac{4(q_1-c_1)}{9} \end{cases}$$

Using the equilibrium wholesale and retail prices, we can obtain manufacturer 1's and retailer 1's equilibrium profits:

$$\pi_{M1}^{**} = \begin{cases} \frac{(2q_1-2c_1-t)^2}{32q_1} & \text{if } t \le \frac{2(q_1-c_1)}{5} \\ \frac{(2q_1-2c_1-3t)t}{4q_1} & \text{if } \frac{2(q_1-c_1)}{5} < t < \frac{4(q_1-c_1)}{9} \\ \frac{8(q_1-c_1)^3}{243q_1 t} & \text{if } t \ge \frac{4(q_1-c_1)}{9} \end{cases}$$

$$\pi_{R1}^{**} = \begin{cases} \frac{(2q_1-2c_1-t)^2}{64q_1} & \text{if } t \le \frac{2(q_1-c_1)}{5} \\ \frac{t^2}{4q_1} & \text{if } \frac{2(q_1-c_1)}{5} < t < \frac{4(q_1-c_1)}{9} \\ \frac{16(q_1-c_1)^3}{729q_1 t} & \text{if } t \ge \frac{4(q_1-c_1)}{9} \end{cases} \quad \text{(W25)}$$

We will show that when t is not too large, then under some conditions $\pi_{M1}^{*} > \pi_{M1}^{**}$ and $\pi_{R1}^{*} > \pi_{R1}^{**}$, i.e., the competitor's presence makes manufacturer 1 and retailer 1 better off. Note that π_{M1}^{*}, π_{M1}^{**}, π_{R1}^{*} and π_{R1}^{**} in equations (W23) and (W25) are continuous in t. Moreover, one can easily verify that as $t \to 0$, the values of π_{M1}^{*}, π_{M1}^{**}, π_{R1}^{*} and π_{R1}^{**} converge to their values in our paper's core model. More specifically, as $t \to 0$, π_{M1}^{*} and π_{R1}^{*} converge to the corresponding expressions in equation (3) in the Appendix of the main paper, and π_{M1}^{**} and π_{R1}^{**} converge to the corresponding expressions in the main paper (i.e., they converge to $\pi_{M1}^{**} = \frac{(q_1-c_1)^2}{8q_1}$, and $\pi_{R1}^{**} = \frac{(q_1-c_1)^2}{16q_1}$, respectively). From Propositions 2 and 3 in the main paper, we know that if manufacturer 2 is not too competitive (i.e., its quality q_2 is not too high, and its cost c_2 is high enough), then $\lim_{t\to 0} \pi_{M1}^{*} > \lim_{t\to 0} \pi_{M1}^{**}$ and $\lim_{t\to 0} \pi_{R1}^{*} > \lim_{t\to 0} \pi_{R1}^{**}$. By continuity of π_{M1}^{*}, π_{M1}^{**}, π_{R1}^{*} and π_{R1}^{**} in t, it follows that there exists $\bar{t} > 0$ such that if $t < \bar{t}$ and manufacturer 2 is not too competitive, then $\pi_{M1}^{*} > \pi_{M1}^{**}$ and

$\pi^{*}_{R1} > \pi^{**}_{R1}$, i.e., the competitor's presence in the market makes manufacturer 1 and retailer 1 better off.[4]

Note that if we compare D_1 and $\widetilde{D}_1$ in (W21) and (W24), respectively, we can see that in the relevant price range (i.e., when $\frac{q_1}{q_2}(p_2 + t) < p_1 \leq q_1 - q_2 + p_2 - t$), D_1's intercept $(1 + \frac{p_2}{q_1 - q_2})$ is higher than $\widetilde{D}_1$'s intercept $(1 - \frac{t}{2q_1})$. Furthermore, $\frac{dD_1}{dp_1} < \frac{d\widetilde{D}_1}{dp_1}$, where $\frac{dD_1}{dp_1} = -\frac{1}{q_1 - q_2}$ and $\frac{d\widetilde{D}_1}{dp_1} = -\frac{1}{q_1}$, i.e., D_1 is steeper than $\widetilde{D}_1$. Hence, in the model where consumers are heterogeneous in both horizontal and vertical preferences, the qualitative effects of the competitor's exit on the firm's demand curve are consistent with the changes in our paper's main model, provided that the horizontal differentiation parameter t is not too large. ■

[4] A similar proof based on the continuity of π^{*}_{M1} in t can be used to show that if t is not very large and c_2 is large enough, then $\frac{\partial \pi^{*}_{M1}}{\partial q_2} > 0$, i.e., an increase in the competitor's perceived quality can benefit manufacturer 1. The intuition is similar to the one for Proposition 4 in the main paper.

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