

## Mean Expected Bias

To evaluate how the bias affects estimation of treatment effects we considered the expected bias under the null hypotheses. The following tables include the results using the same scenarios than in the numerical evaluation study with the evaluation criterion *Mean Expected Bias*. As derived in the paper the expected treatment effect under the null hypotheses  $H_0 : \mu_E - \mu_C = 0$  for a particular allocation sequence  $\mathbf{z}$  equals

$$E\left(\sum_{j=1}^K w_j D_j\right) = \sum_{j=1}^K w_j (\tilde{\tau}_{jE}(\mathbf{z}) - \tilde{\tau}_{jC}(\mathbf{z})),$$

with the notation for  $\tilde{\tau}_{jE}(\mathbf{z}) = \tilde{\tau}_{jE}$  and  $\tilde{\tau}_{jC}(\mathbf{z}) = \tilde{\tau}_{jC}$  as given in the publication. We further used a common variance of  $\sigma = 1$ .

Because we simulate the distribution of the allocation sequences determined by the randomization procedure using Monte Carlo sampling of size  $L$  the bias of the expected treatment effect becomes

$$Bias = \frac{1}{L} \sum_{\ell=1}^L \sum_{j=1}^K w_j (\tilde{\tau}_{jE}(\mathbf{z}_\ell) - \tilde{\tau}_{jC}(\mathbf{z}_\ell))$$

with  $\mathbf{z}_\ell \in \Omega_{RP}, 1 \leq \ell \leq L$ .

Table 1: Mean Expected Bias of stratified and unstratified randomization procedures for BSD(9), CR, EBC(0.67) and PBR(4) depending on the amount of selection  $\eta = 0, 0.05$  and time trend bias  $\Theta = 0, 0.05$  for different allocation ratios and analysis using weighted ( $w_j^*$ ), unweighted ( $w_j = 1$ ) and unstratified ( $us$ ) t-test.

Alloc. Ratio	$\Theta$	$\eta$	Randomization Procedure	stratified Random.			unstratified Random.		
				$w_j^*$ test	$w_j = 1$ test	$us$ -test	$w_j^*$ test	$w_j = 1$ test	$us$ -test
20 : 60	0.05	0	BSD(9)	-0.0000	0.0000	0.0005	0.0000	0.0007	0.0022
			CR	-0.0000	-0.0007	-0.0020	0.0000	0.0003	0.0006
			EBC(0.67)	-0.0000	-0.0006	-0.0018	-0.0000	-0.0003	-0.0007
			PBR(4)	0.0000	0.0000	0.0000	-0.0000	0.0000	0.0000
	0.05	0.05	BSD(9)	0.0007	0.0116	0.0102	0.0001	0.0013	0.0054
			CR	-0.0003	0.0001	0.0013	0.0001	0.0012	0.0043
			EBC(0.67)	0.0051	0.0545	0.0604	-0.0000	0.0001	0.0656
			PBR(4)	0.0065	0.0695	0.0785	-0.0000	-0.0002	0.0785
	0	0.05	BSD(9)	0.0009	0.0129	0.0136	0.0000	0.0005	0.0022
			CR	-0.0002	0.0001	0.0013	-0.0000	-0.0003	0.0003
			EBC(0.67)	0.0051	0.0548	0.0613	-0.0000	-0.0002	0.0639
			PBR(4)	0.0065	0.0696	0.0787	0.0000	0.0001	0.0784
40 : 40	0.05	0	BSD(9)	-0.0000	-0.0001	-0.0006	0.0001	0.0009	0.0038
			CR	-0.0001	-0.0004	0.0002	0.0001	0.0009	0.0020
			EBC(0.67)	-0.0000	-0.0003	-0.0002	0.0000	0.0001	0.0006
			PBR(4)	-0.0000	-0.0001	-0.0001	0.0000	0.0000	0.0001
	0.05	0.05	BSD(9)	0.0011	0.0107	0.0162	0.0000	-0.0001	0.0053
			CR	-0.0002	-0.0006	-0.0018	-0.0001	-0.0009	-0.0026
			EBC(0.67)	0.0053	0.0532	0.0645	-0.0000	-0.0000	0.0632
			PBR(4)	0.0067	0.0672	0.0784	-0.0001	-0.0005	0.0786
	0	0.05	BSD(9)	0.0010	0.0100	0.0129	0.0001	0.0006	0.0026
			CR	-0.0002	0.0000	-0.0003	-0.0001	-0.0009	0.0002
			EBC(0.67)	0.0052	0.0523	0.0617	0.0000	0.0001	0.0634
			PBR(4)	0.0067	0.0673	0.0784	-0.0000	-0.0001	0.0782
8 × 10	0.05	0	BSD(2)	0.0000	0.0000	0.0003	0.0000	0.0000	-0.0007
			CR	0.0000	0.0000	0.0025	-0.0000	-0.0000	-0.0014
			EBC(0.67)	0.0000	0.0000	-0.0001	0.0000	0.0000	0.0010
			PBR(2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.05	0.05	BSD(2)	0.0136	0.0334	0.0599	-0.0001	-0.0002	0.0072
			CR	-0.0041	-0.0001	0.0016	-0.0001	-0.0003	0.0000
			EBC(0.67)	0.0122	0.0308	0.0529	-0.0001	-0.0001	0.0634
			PBR(2)	0.0229	0.0572	0.0905	0.0000	0.0000	0.0785
	0	0.05	BSD(2)	0.0136	0.0333	0.0597	-0.0001	-0.0002	0.0035
			CR	-0.0041	0.0001	0.0001	0.0001	0.0002	0.0004
			EBC(0.67)	0.0122	0.0309	0.0530	0.0000	-0.0000	0.0642
			PBR(2)	0.0229	0.0572	0.0904	0.0001	0.0002	0.0784

Table 2: Mean Expected Bias of stratified and unstratified randomization procedures for BSD(9), CR, EBC(0.67) and PBR(4) depending on the amount of selection  $\eta = 0, 0.05$  and time trend bias  $\Theta = 0, 0.05$  for different allocation ratios and analysis using weighted ( $w_j^*$ ), unweighted ( $w_j = 1$ ) and unstratified ( $us$ ) t-test.

Alloc. Ratio	$\Theta$	$\eta$	Randomization Procedure	stratified Random.			unstratified Random.		
				$w_j^*$ test	$w_j = 1$ test	$us$ -test	$w_j^*$ test	$w_j = 1$ test	$us$ -test
20 : 60	0.2	0	BSD(9)	-0.0001	-0.0019	-0.0063	-0.0003	-0.0042	-0.0163
			CR	0.0000	0.0030	0.0089	0.0001	-0.0004	-0.0007
			EBC(0.67)	-0.0001	-0.0017	-0.0037	-0.0000	-0.0011	-0.0041
			PBR(4)	-0.0000	-0.0001	-0.0002	0.0000	-0.0000	-0.0000
	0.2	0.2	BSD(9)	0.0030	0.0479	0.0412	0.0001	-0.0021	-0.0029
			CR	-0.0008	0.0035	0.0134	0.0005	0.0079	0.0168
			EBC(0.67)	0.0202	0.2202	0.2476	0.0001	0.0009	0.2538
			PBR(4)	0.0261	0.2785	0.3135	0.0001	0.0009	0.3135
	0	0.2	BSD(9)	0.0031	0.0497	0.0458	0.0001	0.0019	0.0099
			CR	-0.0014	-0.0011	-0.0025	0.0001	0.0012	-0.0017
			EBC(0.67)	0.0202	0.2194	0.2450	0.0000	0.0003	0.2549
			PBR(4)	0.0261	0.2781	0.3133	-0.0001	-0.0018	0.3125
40 : 40	0.2	0	BSD(9)	-0.0002	-0.0017	-0.0081	-0.0002	-0.0020	-0.0198
			CR	0.0002	0.0023	0.0092	-0.0000	-0.0004	0.0029
			EBC(0.67)	-0.0002	-0.0019	-0.0081	0.0001	0.0012	0.0020
			PBR(4)	-0.0000	-0.0001	-0.0001	0.0000	0.0001	0.0002
	0.2	0.2	BSD(9)	0.0039	0.0392	0.0559	-0.0002	-0.0015	0.0131
			CR	-0.0010	-0.0025	-0.0009	0.0005	0.0049	0.0047
			EBC(0.67)	0.0209	0.2083	0.2430	0.0002	0.0016	0.2565
			PBR(4)	0.0268	0.2684	0.3124	0.0000	0.0002	0.3144
	0	0.2	BSD(9)	0.0040	0.0403	0.0587	0.0002	0.0018	0.0115
			CR	-0.0008	-0.0007	-0.0005	-0.0001	-0.0007	-0.0058
			EBC(0.67)	0.0211	0.2101	0.2488	0.0001	0.0012	0.2528
			PBR(4)	0.0269	0.2690	0.3132	0.0001	0.0007	0.3136
8 × 10	0.2	0	BSD(2)	0.0000	0.0000	-0.0028	0.0001	0.0002	0.0112
			CR	0.0002	0.0002	0.0106	-0.0001	-0.0002	-0.0159
			EBC(0.67)	0.0000	0.0000	-0.0026	-0.0000	-0.0000	0.0024
			PBR(2)	0.0000	0.0000	0.0002	0.0000	0.0000	-0.0002
	0.2	0.2	BSD(2)	0.0544	0.1336	0.2420	0.0002	0.0007	0.0150
			CR	-0.0165	-0.0005	0.0054	-0.0006	-0.0010	-0.0111
			EBC(0.67)	0.0485	0.1229	0.2140	0.0000	-0.0000	0.2522
			PBR(2)	0.0915	0.2288	0.3617	-0.0001	-0.0003	0.3134
	0	0.2	BSD(2)	0.0543	0.1334	0.2384	0.0001	0.0011	0.0139
			CR	-0.0161	0.0003	-0.0013	0.0002	0.0002	-0.0011
			EBC(0.67)	0.0489	0.1242	0.2113	0.0001	0.0004	0.2547
			PBR(2)	0.0915	0.2288	0.3618	0.0002	0.0005	0.3137