APPENDIX: THEORETICAL MODEL

This appendix formalizes the Hughes-Turnbull argument that private regulation reduces neighborhood externality risk, applying the theory to the gated community environment to derive empirically testable implications regarding household attitude towards risk. Recall gated communities offer residents three different types of benefits: recreational or common area facilities (community goods or services); restrictions on neighborhood properties and their use (reduction in neighborhood externalities and externality risk); and greater privacy or safety (reduced accessibility to nonresidents). The stylized model presented here incorporates all three types of benefits.

Turnbull (1995) provides a review of the theoretical literature introducing consumption uncertainty into the urban housing market. Much of the literature reviewed there views either housing or non-housing consumption as uncertain, with ex post consumption realizations differing from ex ante consumption plans. In the case of neighborhood quality, however, different realized outcomes do not introduce uncertainty in housing and non-housing consumption per se, but instead affect ex post utility through a third channel—consumption of location-specific neighborhood attributes or amenities—which is influenced solely by the household’s choice of location. In this case, the choice of location comprises a gated or open neighborhood without loss of generality. It should be noted, however, that the uncertainty introduced through this third channel also leads to uncertainty over the ex post enjoyment from housing and other goods when consumption decisions are being made. How this source of uncertainty affects housing and other goods demands depends on how the potential variation in neighborhood amenities ex post affects the expected marginal rate of substitution between housing and non-housing consumption ex ante.

Household utility $u(x,y,q;r)$ is a strictly concave function of housing $x$, non-housing $y$, and neighborhood quality or amenity level $q$, where concavity reflects household risk aversion for all
types of consumption. The parameter $r$ indicates the household’s degree of risk aversion, as explained below. The household chooses $x$ and $y$ before the ex ante uncertain neighborhood quality $q$ is realized. The expected neighborhood quality, however, is distributed with mean $E[q] = m > 0$ and finite variance $V(q) > 0$. We separate realized neighborhood quality into the expected and risky components, $q = m + s\varepsilon$, where $s$ is an exogenous parameter and $\varepsilon$ is the stochastic term which is distributed with mean zero and finite positive variance, $E[\varepsilon] = 0$ and $V(\varepsilon) > 0$, respectively. In what follows it is useful to note $V(q) = s^2V(\varepsilon)$ so that $ds > 0$ denotes an increase in mean-preserving-spread such that $dE[q]/ds = 0$ and $dV(q)/ds = 2sV(\varepsilon) > 0$. For gated developments, the neighborhood amenity term $q$ includes recreational services provided by the HOA governing the gated community, infrastructure for which the community is responsible (e.g., roads and drainage), privacy and safety, as well as neighborhood externalities from surrounding structures and neighbors’ lifestyles. Homeowners in gated subdivisions each pay a portion of the costs of services provided by the subdivision, denoted $c(m,s)$, where $c_m > 0$ indicates that greater levels of expected neighborhood quality, which encompasses all of the listed aspects, require greater costs (including, the cost of gating itself) and $c_s < 0$ indicates that establishing and enforcing additional controls on structures and their use that lead to lower consumption risk ($ds < 0$) requires additional management inputs. Exogenous amenities or amenity risks not influenced by gated developments do not enter the HOA fee function $c(m,s)$.

The representative household’s money budget is $I$ and the price of housing at the subject location is $P$. The gated community parameters $\{m,s\}$ are exogenous to the individual household; they are set by the developer in anticipation of what households are willing and able to pay for the level of gated community benefits. Any modifications to $\{m,s\}$ are the result of collective decision
making by households in the community and so are outside the control of the individual household.

For a household choosing to live in the neighborhood, the household’s planned housing and non-housing consumption maximize expected utility $E[u(x,I-Px-c(m,s),m+s;\epsilon;r)]$. The household’s ex ante housing demand $x^* = x(P,I,m,s,r)$ is the implicit solution to the optimization problem.

The taste parameter $r$ in the ex post utility function captures the household’s degree of consumption risk aversion. This is a deep parameter that defines the slopes and curvatures of the utility function and is key to deriving comparative static results; see Turnbull (1995) for details on the applying the methodology, the underlying intuition and examples drawn from the literature. The parameter $r$ denotes the degree of risk aversion reflected by a specific ex post utility function such that greater $r$ signifies greater risk aversion. More specifically, greater $r$ indicates strictly concave ex post utility functions with stronger risk aversion, so that, using total and marginal risk premia to measure the degree of risk aversion associated with the expected utility function derived from the ex post utility function, greater $r$ indicates greater total and marginal risk premia. Imposing these restrictions on the relationships between $r$ and the total and marginal risk premia, the parameter $r$ provides an index of strength of risk aversion across households with different utility functions, all of which are strictly concave (so that all are risk averse to some degree). Since all functions in the family of functions being considered are strictly concave, the parametric restrictions derived below ensure that we are only considering utility functions that are strictly quasiconcave, that is, utility functions exhibiting diminishing marginal rates of substitution between any two goods ex post and similarly for the expected utility functions ex ante.

In order to derive the parametric restrictions that greater $r$ indicates greater total and marginal risk premia, ceteris paribus, define the total risk premium $\rho$ in the usual way, as the lump sum payment the household is willing and able to offer to be able to consume the expected
neighborhood amenities with less uncertainty instead of having to make housing and non-housing decisions facing greater q uncertainty. Denoting the reference risk level \( s_o \) (where \( s_o = 0 \) is possible), the risk premium \( \rho \) satisfies the condition:

\[
E[u(x,I-Px-c(m,s),m+s\epsilon;r)] = E[u(x,I-Px-c(m,s_o)-p,m+s_o\epsilon;r)]
\]  \hspace{1cm} (A.1)

Differentiating implicitly, this equation means that our definition of \( r \) as the parameter indicating greater risk aversion therefore requires for \( s > s_o \)

\[
dp/dr = \{ E[u_r]_o - E[u_r]_s \}/E[u_y]_o > 0
\]  \hspace{1cm} (A.2)

where we adopt the convention that expectations operator subscript \( o \) indicates the function inside the expectations operator is evaluated at \( s_o \) and expectations operator subscript \( s \) indicates it is evaluated at \( s > s_o \).

The risk premium for given household type \( r \) increases with greater mean-preserving neighborhood consumption risk, so that

\[
dp/ds = E[u_s m_s - u_q \epsilon],/E[u_y]_o > 0
\]  \hspace{1cm} (A.3)

Since \( m_s < 0 \) the first term in the numerator is negative. To evaluate the second term, use \( E[u_q\epsilon] = E[u_q]E[\epsilon] + COV[u_q,\epsilon] = COV[u_q,\epsilon] \) where the second equality follows from \( E[\epsilon] = 0 \). The covariance term takes the sign of \( \partial(u_q)/\partial \epsilon = su_{qq} < 0 \) using the strict concavity of the utility function (risk aversion). Thus \( E[u_q\epsilon] < 0 \) and the second additive term in the expectations operator in the
numerator is positive. Condition (A.3) requires that the positive second term in the expectations operator dominates the negative first term for risk averse households.

Greater $r$ also indicates stronger household risk aversion in the sense that the marginal risk premium increases with $r$

$$d(dp/ds)/dr = [E[u_0, m + u_q\epsilon], E[u_0] - (dp/ds)E[u_0, m + u_q\epsilon] / E[u_0] > 0$$

(A.4)

The intuition is that the greater household risk aversion, the more the household is willing to pay to avoid additional risk. Conditions (A.1)-(A.4) provide the functional form restrictions that ensure utility functions with greater $r$ exhibit stronger risk aversion.

Consider the marginal household type $r$ that is indifferent to living in the gated community and an alternative open neighborhood. To find the bid price of the marginal household type for a house in a neighborhood with attributes yielding $\{m, s\}$, substitute the housing demand into expected utility and set the resultant function equal to the utility level $u^o$ representing the household’s opportunity cost of living in the alternative neighborhood. The bid price condition is

$$E[u(x^*, I-Px^*-c(m,s), m+s\epsilon; r)] = u^o$$

(A.5)

This is the location equilibrium condition for the marginal type household: housing price $P$ is bid to the level that ensures the household is indifferent between the house in the subject neighborhood with attributes $\{m, s\}$ and fees $c(m, s)$ and an alternative neighborhood yielding expected utility $u'^o$. Differentiate (A.5) and rearrange to find the house price effects of the expected neighborhood attributes $m$ on house price as
\[ \frac{dP}{dm} = \left( E[u_q]_s/E[u_y]_s - c_m \right) / x^* \]

(A.6)

The first term in the numerator of (A.6) is the expected marginal rate of substitution between neighborhood attributes and non-housing consumption, a measure of how much spending on other goods the household is willing to give up ex ante to obtain an increment of the neighborhood attributes in all realized states. Houses located in gated developments that offer households greater expected benefits will sell at a premium only if the numerator of the above term is positive, that is, the additional benefits provided by the development are worth at least as much to households than the additional cost they must incur directly in the form of homeowner association and management fees. If not, household type \( r \) would not choose the gated development and would instead live at the site corresponding to the opportunity utility \( u^o \). The model therefore predicts (weakly) higher housing prices in gated subdivisions.

Turning to the lower consumption risk associated with gated developments, the effect of neighborhood amenity consumption risk on the bid price of household type \( r \) is

\[
\frac{dP}{ds} = \frac{E[u_q]_s - u_y c_2}{x^* E[u_y]_s} = - \left( \frac{dp}{ds} \right) \frac{E[u_y]_o}{x^* E[u_y]_s} < 0
\]

(A.7)

where the sign follows from (A.3). This establishes that houses in gated communities that successfully reduce consumption risk sell for higher prices, an intuitively appealing result. To see how household willingness-to-pay to live in this development varies with risk aversion, differentiate (A.7) with respect to household type \( r \) to find
\[ \frac{d(dP/ds)}{dr} = \frac{E[u_q\epsilon - u_yr_{cs}]}{xE[u_yr]} - \frac{E[u_q\epsilon - u_yr_{cs}]}{xE[u_y]} \cdot \frac{dP/ds}{xE[u_y]} \]

\[ = \frac{E[u_q\epsilon - u_yr_{cs}]}{xE[u_y]} - (dP/ds)E[u_yr]_{s} / xE[u_{ys}] \]

\[ = \frac{E[u_q\epsilon - u_yr_{cs}]}{xE[u_y]} + (dp/ds)E[u_{ys}]_{s} / xE[u_{ys}]^2 \]

\[ = - \frac{[d(dp/ds)/dr] / xE[u_{ys}]}{< 0} \quad (A.8) \]

substituting from (A.7) for the second line, (A.3) for the third line and (A.4) for the last line. The above result shows higher r households (more risk averse households) are willing and able to bid more for houses in lower risk neighborhoods. This means all households in a particular gated community who exhibit stronger risk aversion than the marginal household enjoy consumer surplus. Higher r-type households sort into lower risk neighborhoods while lower r-type households sort into higher risk neighborhoods.

Summarizing, gated subdivisions that offer higher expected neighborhood consumption and lower consumption risk have higher housing prices than open locations. Further, the risk relationships also imply that households with greater aversion to consumption risk (higher r types) will be willing and able to outbid lower r-type households for gated sites and so predominate in gated developments in equilibrium. Thus, houses in gated subdivisions have higher prices not only because the subdivisions offer higher levels of neighborhood amenities, but also because they offer lower neighborhood externality risk to their more risk averse residents—the essence of the alternative view of private regulation. Of course, if gated communities do not offer lower neighborhood amenity risk or if households do not care about the risk (i.e., the alternative view does not hold) then gated communities will not attract the more risk averse households in equilibrium.
Keeping this outcome in mind, we can now consider how exogenous neighborhood amenity risk affects housing prices inside and outside gated neighborhoods. Denote neighborhood attributes consumption \( q = m + s e + \theta e \) where \( \theta \) is an exogenous parameter, increases of which (from zero) indicate added exogenous risk. Substituting this revised neighborhood amenity consumption term into the bid price condition for the marginal household (A.5) and implicitly differentiating and evaluating at \( \theta = 0 \) yields

\[
\frac{dP}{d\theta} = E[u_q e] / x E[u_y] < 0
\]

(A.9)

where the sign follows from \( E[u_q e] < 0 \) as shown earlier. Greater exogenous neighborhood amenity risk lowers housing prices. In order to see how the strength of household risk aversion affects the housing price response to greater exogenous amenity risk, follow the procedure used to derive (A.8); differentiate (A.9) with respect to \( r \) and use the properties of the risk premium for \( \theta \) to show that a given increase in exogenous risk has a stronger effect on housing prices in neighborhoods populated with more risk averse households:

\[
\frac{d}{dr} \left( \frac{dP}{d\theta} \right) / x E[u_y] = - \left[ \frac{d(dP/d\theta)/dr}{x E[u_y]} \right] < 0
\]

(A.10)

Therefore, (A.10) yields a conceptually straightforward empirical test of the hypothesis that gated neighborhoods are populated by households with greater risk aversion. If true then an exogenous increase in risk for both gated and open neighborhoods will lower housing prices more in gated than in open neighborhoods. This is the test conducted in the paper.
REFERENCES

