

Table S1 Equations used to calculate strains induced due to cantilever bending

Cantilever Bending		
<i>Cartesian Strains</i>	<i>Cylindrical Strains</i>	<i>Surface Strains</i>
$\varepsilon_{xx} = \frac{\nu l F (L-Z) y}{E I}$	$\varepsilon_{rr} = \frac{\nu l F (L-Z) r \sin \theta}{E I}$	$\varepsilon_{x'x'} = \frac{-\nu l F (L-Z) y}{E I}$
$\varepsilon_{yy} = \frac{-\nu F (L-Z) y}{E I}$	$\varepsilon_{\theta\theta} = \frac{-\nu F (L-Z) r \sin \theta}{E I}$	$\varepsilon_{y'y'} = \frac{-\nu l F (L-Z) y}{E I}$
$\varepsilon_{zz} = \frac{-F (L-Z) y}{E I}$	$\varepsilon_{r\theta} = 0$	$\varepsilon_{x'y'} = 0$
$\varepsilon_{xy} = \varepsilon_{xz} = 0$	$\varepsilon_{\theta z} = \frac{F Q \cos \theta}{I G t}$	$\varepsilon_{IGt'zy} = \frac{F Q \cos \phi}{I G t'zy}$
$\varepsilon_{yz} = \frac{F Q}{I G t}$	$\varepsilon_{rz} = \frac{F Q \sin \theta}{r z I G t}$	$\varepsilon_{x'z'IGt} = \frac{F Q \sin \phi}{x'z'IGt}$
Four-Point and Three-Point Bending		
$\varepsilon_{xx} = \frac{-\nu l M y}{E I}$	$\varepsilon_{rr} = \frac{-\nu l M r \sin \theta}{E I}$	$\varepsilon_{x'x'} = \frac{-\nu l M y}{E I}$
$\varepsilon_{yy} = \frac{-\nu l M y}{E I}$	$\varepsilon_{\theta\theta} = \frac{-\nu l M r \sin \theta}{E I}$	$\varepsilon_{y'y'} = \frac{-\nu l M y}{E I}$
$\varepsilon_{zz} = \frac{M y}{E I}$	$\varepsilon_{zz} = \frac{M r \sin \theta}{E I}$	$\varepsilon_{x'y'} = \varepsilon_{x'z'} = \varepsilon_{y'z'} = 0$
$\varepsilon_{xy} = \varepsilon_{yz} = \varepsilon_{xz} = 0$	$\varepsilon_{r\theta} = \varepsilon_{rz} = \varepsilon_{\theta z} = 0$	

Table S2 Equations used to calculate cantilever bending induced strain gradients

Cartesian gradients	Cylindrical gradients	Surface gradients
$\frac{\partial \varepsilon_{xx}}{\partial x} = \frac{\partial \varepsilon_{yy}}{\partial x} = \frac{\partial \varepsilon_{zz}}{\partial x} = 0$	$\frac{\partial \varepsilon_{rr}}{\partial r} = \frac{\partial \varepsilon_{\theta\theta}}{\partial r} = \frac{\nu l F (L-z) \sin \theta}{E l I}$	$\frac{\partial \varepsilon_{x'x'}}{\partial x'} = \frac{\partial \varepsilon_{y'y'}}{\partial x'} = \frac{-\nu l F (L-z) \sin \phi}{E I}$
$\frac{\partial \varepsilon_{xy}}{\partial x} = \frac{\partial \varepsilon_{yz}}{\partial x} = \frac{\partial \varepsilon_{zx}}{\partial x} = 0$	$\frac{\partial \varepsilon_{zz}}{\partial r} = \frac{-F (L-z) \sin \theta}{E I}$	$\frac{\partial \varepsilon_{z'z'}}{\partial x'} = \frac{\partial \varepsilon_{x'y'}}{\partial x'} = 0$
$\frac{\partial \varepsilon_{xx}}{\partial y} = \frac{\partial \varepsilon_{yy}}{\partial y} = \frac{\nu l F (L-z) y}{E I}$	$\frac{\partial \varepsilon_{r\theta}}{\partial r} = 0$	$\frac{\partial \varepsilon_{y'z'}}{\partial x'} = \frac{-F y \sin 2\phi}{2GI}$
$\frac{\partial \varepsilon_{zz}}{\partial y} = \frac{-F (L-z) y}{E I}$	$\frac{\partial \varepsilon_{\theta z}}{\partial r} = \frac{-F y \sin 2\theta}{2GI}$	$\frac{\partial \varepsilon_{x'z}}{\partial x'} = \frac{-F y \sin 2\phi}{GI}$
$\frac{\partial \varepsilon_{xx}}{\partial z} = \frac{\partial \varepsilon_{yy}}{\partial z} = -\frac{\nu l F y}{E I}$	$\frac{\partial \varepsilon_{rz}}{\partial r} = \frac{-F y \sin 2\theta}{GI}$	$\frac{\partial \varepsilon_{x'x}}{\partial y'} = \frac{\partial \varepsilon_{y'y'}}{\partial y'} = \frac{\nu l F (L-Z)}{\cos \phi E l I}$
$\frac{\partial \varepsilon_{xy}}{\partial z} = \frac{\partial \varepsilon_{yz}}{\partial z} = \frac{\partial \varepsilon_{zx}}{\partial z} = 0$	$\frac{\partial \varepsilon_{rr}}{\partial r \partial \theta} = \frac{\partial \varepsilon_{\theta\theta}}{\partial \theta E l I r} = \frac{\nu l F (L-z) \cos \theta}{r \partial \theta}$	$\frac{\partial \varepsilon_{z'}}{\partial y'} = \frac{F y}{E l I}$
$\frac{\partial \varepsilon_z}{\partial z} = \frac{F y}{E l I}$	$\frac{\partial \varepsilon_z}{r \partial \theta} = \frac{-F (L-z) \cos \theta}{E I}$	$\frac{\partial \varepsilon_{x'y'}}{\partial y'} = 0$
	$\frac{\partial \varepsilon_{r\theta}}{\partial r \partial \theta} = 0$	$\frac{\partial \varepsilon_{y'z'}}{\partial y'} = \frac{-F(L-Z)IEI \cos^2 \phi}{\partial y'}$
	$\frac{\partial \varepsilon_{\theta z}}{\partial r \partial \theta} = \frac{-F y \cos^2 \theta}{GI} - \frac{\tau_{yz} \sin \theta}{rG}$	$\frac{\partial \varepsilon_{x'z'}}{\partial y'} = \frac{-F y \sin 2\phi}{2GI}$
	$\frac{\partial \varepsilon_{rz}}{\partial r \partial \theta} = \frac{-F y \sin 2\theta}{2GI} + \frac{\tau_{yz} \sin \theta}{rG}$	
	$\frac{\partial \varepsilon_{zz}}{\partial r \partial \theta} = \frac{F (L-z) \sin \theta}{E l I}$	

Table S3 Strain gradients induced by three-point and four-point bending

Cartesian gradients	Cylindrical gradients	Surface gradients
$\frac{\partial \varepsilon_{xx}}{\partial x} = \frac{\partial \varepsilon_{yy}}{\partial x} = \frac{\partial \varepsilon_{zz}}{\partial x} = 0$	$\frac{\partial \varepsilon_{rr}}{\partial r} = \frac{\partial \varepsilon_{\theta\theta}}{\partial r} = \frac{\nu l M \sin \theta}{EI}$	$\frac{\partial \varepsilon_{x'x'}}{\partial x'} = \frac{\partial \varepsilon_{y'y'}}{\partial y'} = \frac{-\nu M \sin \phi}{EI}$
$\frac{\partial \varepsilon_{xy}}{\partial x} = \frac{\partial \varepsilon_{yz}}{\partial x} = \frac{\partial \varepsilon_{zx}}{\partial x} = 0$	$\frac{\partial \varepsilon_{zz}}{\partial r} = \frac{M \sin \theta}{EI}$	$\frac{\partial \varepsilon_{x'y'}}{\partial x'} = \frac{\partial \varepsilon_{y'z'}}{\partial x'} = \frac{\partial \varepsilon_{z'x'}}{\partial x'} = 0$
$\frac{\partial \varepsilon_{xx}}{\partial y} = \frac{\partial \varepsilon_{yy}}{\partial y} = \frac{-\nu M}{EI}$	$\frac{\partial \varepsilon_{r\theta}}{\partial r} = \frac{\partial \varepsilon_{\theta r}}{\partial r} = \frac{\partial \varepsilon_{r\theta}}{\partial r} = 0$	$\frac{\partial \varepsilon_{z'z'}}{\partial x'} = 0$
$\frac{\partial \varepsilon_{xy}}{\partial y} = \frac{\partial \varepsilon_{yz}}{\partial y} = \frac{\partial \varepsilon_{zx}}{\partial y} = 0$	$\frac{\partial \varepsilon_{rr}}{r \partial \theta} = \frac{\partial \varepsilon_{\theta\theta}}{r \partial \theta} = \frac{\nu M \cos \theta}{EI}$	$\frac{\partial \varepsilon_{x'x'}}{\partial y'} = \frac{\partial \varepsilon_{y'y'}}{\partial y'} = \frac{-\nu M \cos \phi}{EI}$
$\frac{\partial \varepsilon_{zz}}{\partial y} = \frac{M}{EI}$	$\frac{\partial \varepsilon_{r\theta}}{r \partial \theta} = \frac{\partial \varepsilon_{\theta r}}{r \partial \theta} = \frac{\partial \varepsilon_{r\theta}}{r \partial \theta} = 0$	$\frac{\partial \varepsilon_{x'y'}}{\partial y'} = \frac{\partial \varepsilon_{y'z'}}{\partial y'} = \frac{\partial \varepsilon_{z'x'}}{\partial y'} = 0$
$\frac{\partial \varepsilon_{xx}}{\partial z} = \frac{\partial \varepsilon_{yy}}{\partial z} = \frac{\partial \varepsilon_{zz}}{\partial z} = 0$	$\frac{\partial \varepsilon_{zz}}{r \partial \theta} = \frac{M \cos \theta}{EI}$	$\frac{\partial \varepsilon_{x'x'}}{\partial z'} = \frac{\partial \varepsilon_{y'y'}}{\partial z'} = \frac{-\nu M \cos \phi}{EI}$
$\frac{\partial \varepsilon_{xy}}{\partial z} = \frac{\partial \varepsilon_{yz}}{\partial z} = \frac{\partial \varepsilon_{zx}}{\partial z} = 0$		$\frac{\partial \varepsilon_{x'y'}}{\partial z'} = \frac{\partial \varepsilon_{y'z'}}{\partial z'} = \frac{\partial \varepsilon_{z'x'}}{\partial z'} = 0$