A new approach to identification of cracks in beams and experimental verification

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ABSTRACT

Cracking is a common type of damage in structural beams. Existing methods for the detection of cracks in beams are most commonly based on representing a crack by a reduction of the bending stiffness over a certain segment. While such representation may be acceptable for slender beams, it can be problematic for relatively thick beams which are typical in civil engineering structures. In the present study an explicit cracked beam element model is adopted, in which the effect of the crack is comprehensively described by a cracked stiffness matrix relating to the crack location and the crack depth. The cracked beam element model is implemented in a finite element model updating framework for the identification of the crack parameters. This paper provides an overview of this new crack identification approach and the verification of the effectiveness of the method from laboratory experiments. In the experimental verification, cracked beam specimens have been tested to extract the modal frequency and mode shape data, and these are compared with the predictions using the cracked beam element model. The measured modal data are also employed to carry out (inverse) crack identification to further verify the effectiveness of using the cracked beam element model for crack damage identification.

Keywords: *Thick beam; Cracked beam element; Damage identification; Finite element model updating; Aluminium beams*

1 INTRODUCTION

Modelling of cracks in a beam for the analysis of beam vibration is a classic problem and has been extensively studied. Generally there are four representative crack models for beams [1]. The first one describes the effect of crack by a reduction of the bending stiffness of the cracked beam segment. This model has been widely used in the crack damage detection of structures. The second approach assumes the stress and strain fields around the crack area; for example in Christides and Barr [2], the bending stress in the vicinity of the crack is assumed to decay exponentially from its maximum value at the cracked section to the uncracked value in a certain distance away from the crack. On this basis, the bending stiffness (flexural rigidity) distribution around the cracked section can be derived (e.g. [3]). The third model adopts a discrete spring to present the effect of a crack. The last approach stems from establishing the local flexibility of the cracked beam in relation to the strain energy release rate [4]. The strain energy release rate can be evaluated using the principles of the fracture mechanics.

In the present study we develop a crack identification approach using a cracked beam element model based on the local flexibility method. This paper provides an overview of the approach, followed by an experimental verification study. In the experimental verification, cracked

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beam specimens have been tested to extract the modal frequency and mode shape data, and these are compared with the predictions using the cracked beam element model. The measured modal data are also employed to carry out finite element model updating to further verify the effectiveness of using the cracked beam element model for crack damage identification.

2 OVERVIEW OF THEORETICAL FORMULATION

Fig. 1 shows a cracked beam element with the full 6 DOFs in the two-dimensional space. The crack is located at a distance of l_c from the left node and the crack depth is a. A crack depth ratio α is defined as the ratio of the crack depth a to the sectional depth h, $\alpha = a/h$. The width of the beam element is b.

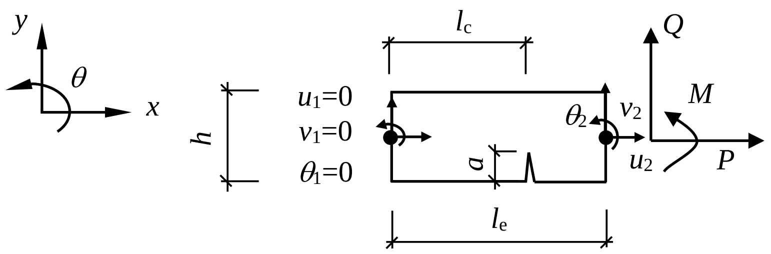


Figure 1 - Loading state of a cracked beam element.

The strain energy in the cracked beam element under a generalised load is equal to the strain energy of the intact beam element plus an additional strain energy brought by the crack. The additional strain energy due to the presence of crack can be evaluated by the fracture energy. According to fracture mechanics theory, the additional strain energy brought by the crack U_c for the beam element can be expressed as:

$$U_c = b\int_0^a G\mathrm{d}a \tag{1}$$

The energy release rate G can be expressed with the Stress Intensity Factors (SIFs) of the crack as [5]:

$$G = \frac{1}{E'}\left(K_{\mathrm{I}}^2 + K_{\mathrm{II}}^2 + \frac{1}{1-v}K_{\mathrm{III}}^2\right) \tag{2}$$

For the cracked element considered here, only the first two types of SIFs exist. Relationships between the SIFs and the applied loads are shown in Eq. (3) from the standard fracture mechanics theory:

$$K_{\mathrm{I1}} = \frac{P}{bh}\sqrt{\pi a}F_{\mathrm{I1}}(\alpha) \tag{3a}$$

$$K_{\mathrm{I2}} = \frac{6M}{bh^2}\sqrt{\pi a}F_{\mathrm{I2}}(\alpha) \tag{3b}$$

$$K_{\mathrm{II}} = \frac{Q}{bh}\sqrt{\pi a}F_{\mathrm{II}}(\alpha) \tag{3c}$$

where K_{I1} and K_{I2} takes into account the stress brought by axial force and moment, respectively, and K_{II} takes into account the stress brought by the shear force. $F_{I1}(\alpha)$, F_{I2} (α) and $F_{II}(\alpha)$ are dimensionless terms and can be found in [5].

With the additional strain energy, the additional flexibility brought by the crack can then be obtained by invoking Castigliano's theorem as:

$$c_{ij,c} = \frac{\partial U_c}{\partial F_i \partial F_j} \tag{4}$$

This yields:

$$c_{ij,c} = \frac{\partial^2}{\partial F_i \partial F_j} \int_0^a \frac{b}{E'} \left[\left(\frac{P}{bh} \sqrt{\pi a} F_{I1}(\alpha) + \frac{6(M + Ql_e - Ql_c)}{bh^2} \sqrt{\pi a} F_{I2}(\alpha) \right)^2 + \left(\frac{Q}{bh} \sqrt{\pi a} F_{II}(\alpha) \right)^2 \right] \mathrm{d}a \tag{5}$$

where, i, j = 1, 2, 3, and F_1=P, F_2=Q, F_3=M.

The above additional flexibility is added onto the standard flexibility matrix for a Timoshenko beam to form the cracked beam element flexibility.

It can be seen that there are 2 parameters representing the crack information in c_{ij}; crack depth a and crack location l_c. The complete 6×6 stiffness matrix for the element can be obtained by inverting the flexibility matrix and satisfying the force equilibrium in the elements, as follows:

$$K_c = T * C^{-1} * T^T \tag{6}$$

where $\boldsymbol{C}$ is the 3×3 flexibility matrix with c_{ij} as its elements. $\boldsymbol{T}$ is the transforming matrix,

$$T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -l_e & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

3 EXPERIMENTAL VERIFICATION WITH THICK ALUMINIUM BEAMS

3.1 Test specimens

Fig. 2 shows three test aluminium beams. The beams had a length of 600 mm, and a cross section of 50.8 × 50.8 mm. The three specimens reported here included an intact beam, a cracked beam with a single crack, and a beam with multiple cracks.

The cracks were created using saw cuts. The crack in the single-crack beam was at L_c = 375 mm from the left end and the crack depth ratio α was 0.5. The cracks in the multiple-crack beam were located at 125mm, 230mm, and 420mm from the left end, and the crack depth ratio was 0.35, 0.25, and 0.4, respectively.

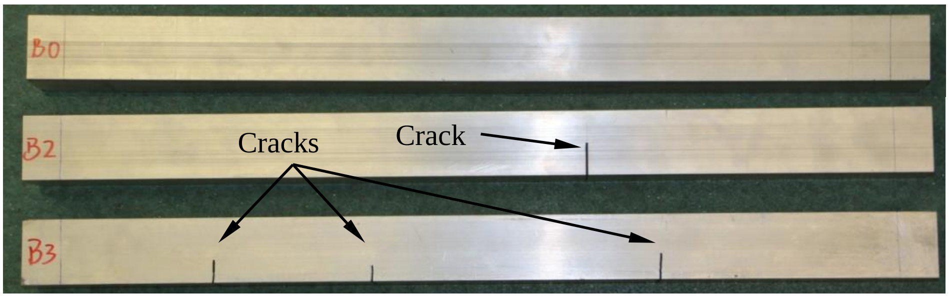
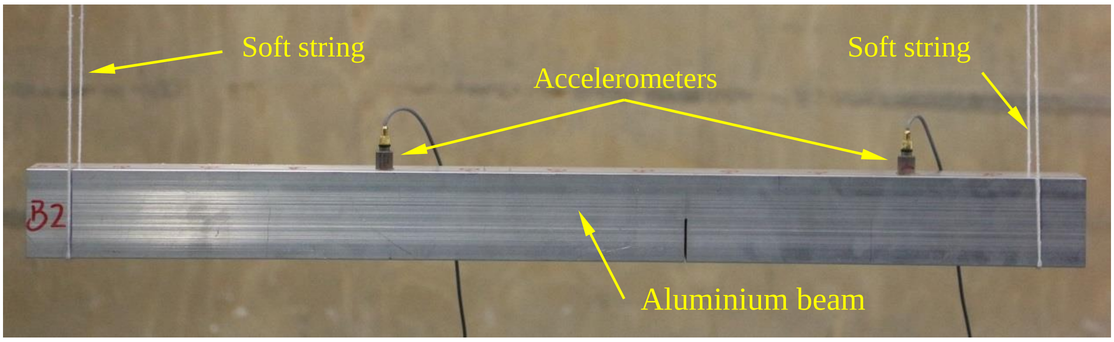


Figure 2 - Aluminium beam specimens with solid cross section.

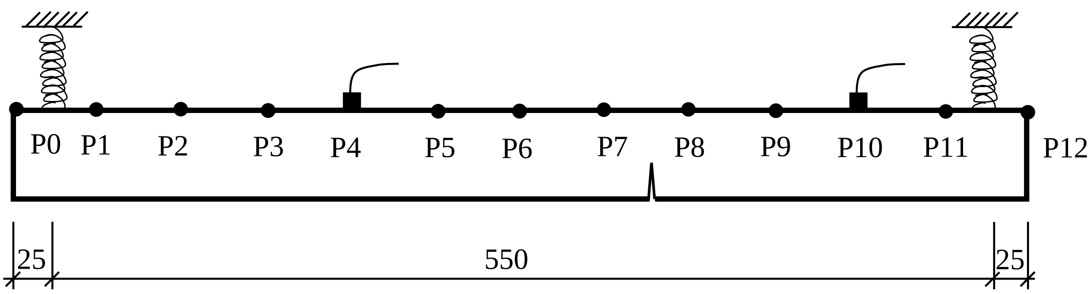
3.2 Modal testing setup and results

The beams were tested in free-free condition at the two ends, as shown in Fig. 3. A Brüel & Kjær 8206-002 impact hammer with an aluminium head was employed to excite the structure. Impact force and acceleration data were recorded with a data acquisition module (National Instruments 9234 system). The sampling rates for both the impact force and acceleration were set to be 25600 Hz. The use of such a high sampling rate was mainly to ensure adequate capture of the impact force in detail. The record duration of the signals was set to be 16 s. Both the natural frequencies and mode shapes of the beams were measured from the experiment.

For the measurement of the mode shapes, 11 uniformly distributed measurement locations were marked on the beam, as shown in Fig. 3. In the tests, two accelerometers were attached at point P4 and P10 while impact was applied at each measurement location from P1 to P11.



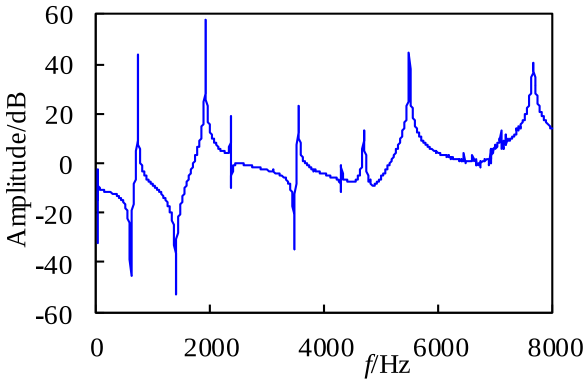
(a) Photo of setup



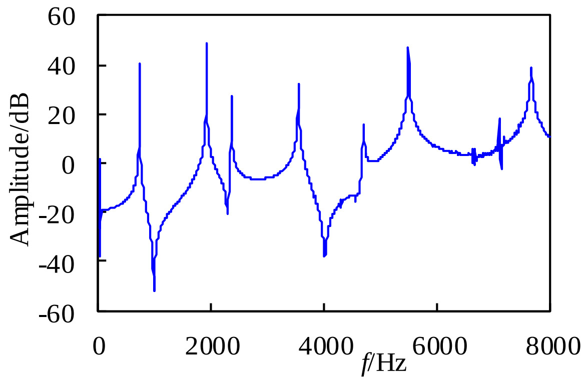
(b) Schematic view of setup (Unit: mm)

Figure 3 - Modal testing setup.

The frequency response function (FRF) curves were calculated from the Fourier transform of the acceleration and impact force signals. A force window was used to eliminate the noise contained in the blank area of impact force and 10 repetitive tests were performed for each excitation location, and the FRF curves of the repeated tests were averaged. Representative FRF curves of the intact beam are presented in Fig. 4. It shows very clear resonances and anti-resonances. The first five modes of natural frequencies and mode shapes of the beams can be extracted from the FRF curves conveniently using the peak-picking method.



(a) Driving FRF at P4



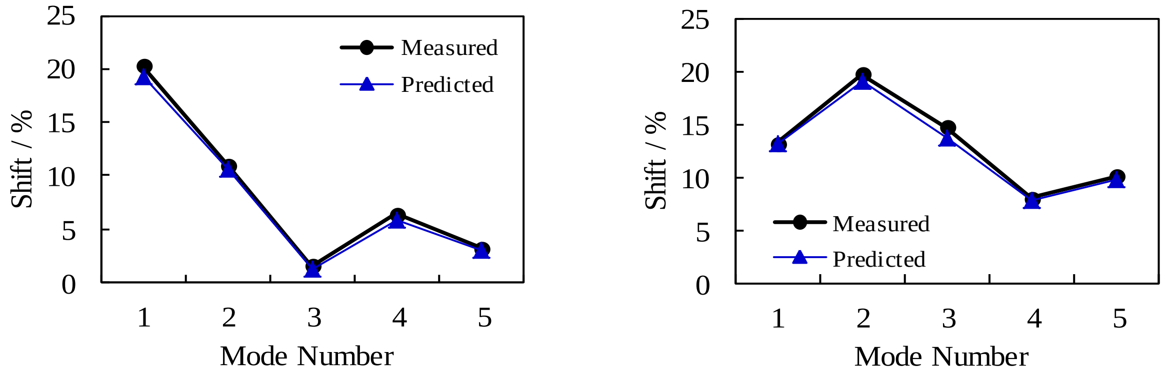
(b) Transfer FRF between P4 and P10

Figure 4 - Measured FRF curves.

3.3 Forward verification of the cracked beam element model

The cracked beam element model is verified against the measured modal testing results in this section. The intact and cracked beams are modelled with Timoshenko beam elements with high-accuracy cubic shape functions. Totally 12 beam elements with a uniform length of 50 mm are used in each model. By applying the crack parameters in the Timoshenko beam model, the first five modes of natural frequencies and mode shapes of the tested beams can be obtained.

Fig. 5 shows the comparison between measured and predicted frequency shifts brought by the cracks. It can be seen that the cracked beam element model is able to predict all the five modes with very high accuracy.



(a) Single-crack beam (b) Multiple-crack beam

Figure 5 - Comparison between measured and predicted frequency shifts.

The Modal Assurance Criterion (MAC) values between the measured and predicted mode shapes of the cracked beams, which are defined in Eq. (8), are used to verify the accuracy of the cracked beam element model concerning the mode shape calculation.

$$\mathrm{MAC}_i = \frac{\left(\phi_{\mathrm{dm}i}^{\mathrm{T}} \phi_{\mathrm{dc}i}^{\mathrm{T}}\right)^2}{\left(\phi_{\mathrm{dm}i}^{\mathrm{T}} \phi_{\mathrm{dm}i}\right)\left(\phi_{\mathrm{dc}i}^{\mathrm{T}} \phi_{\mathrm{dc}i}\right)} \tag{8}$$

where ϕ_i stands for the i^{th} mode shape vector of the beam, subscripts 'c' and 'm' stand for computed and the measured data, respectively.

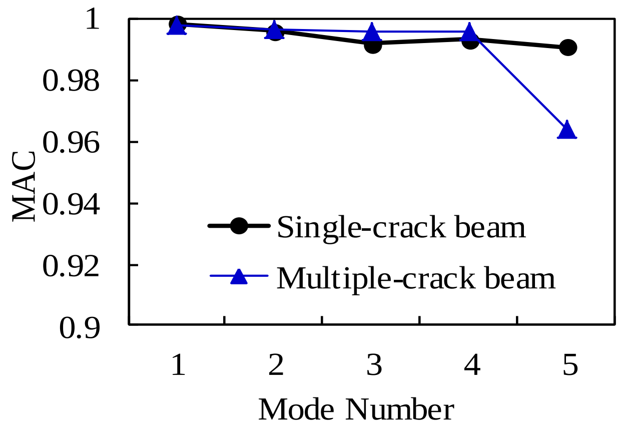


Figure 6 – MAC results between measured and predicted mode shapes.

MAC results are shown in Fig. 6. It can be seen that the MAC values are all higher than 0.964 and for the first 4 modes, the values are all higher than 0.993, indicating very good match between the measured and predicted mode shapes.

3.4 Model updating and crack damage identification with the cracked beam element model

The cracked beam element model is implemented for the identification of the crack damage with a finite element model updating procedure. In the Timoshenko beam finite element model for updating, each element is considered as a potential cracked element with the crack depth ratio (α)

and location (l_c) unknown. A non-zero α value would indicate a crack in the element while a close to zero value of α would indicate an intact element. Because of the free end boundary, the conditions in the two end elements are expected to have a very insensitive effect on the modal data. This implies that the damage in these element cannot be properly identified using the available modal information. As such they are not included (i.e., assumed to be intact) in the updating process. So there are 20 parameters to be updated with the cracked beam element model.

The objective function of the model updating is formed with the eigenvalue and mode shapes of the first five modes, as shown in Eq. (9),

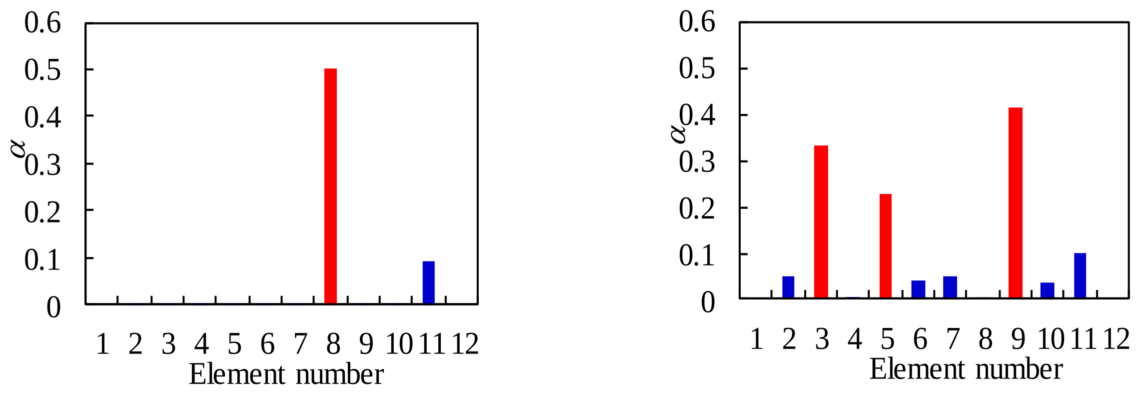
$$J = \frac{1}{N_{\mathrm{N}}}\sum_{i=1}^{N_{\mathrm{N}}} W_i \cdot \mathrm{abs}\left(\frac{f_{\mathrm{dNm}i}^2}{f_{0\mathrm{Nm}i}^2} - \frac{f_{\mathrm{dNc}i}^2}{f_{0\mathrm{Nc}i}^2}\right) + \frac{1}{N_{\mathrm{S}} N_{\mathrm{n}}}\sum_{i=1}^{N_{\mathrm{S}}} V_i \cdot \left(\sum_{j=1}^{N_{\mathrm{n}}} \mathrm{abs}\left(\varphi_{\mathrm{m}ji}^{\mathrm{d}} - \varphi_{\mathrm{c}ji}^{\mathrm{d}}\right)\right) \tag{9}$$

where J is the objective function to be minimised, f_{N} is the natural frequency, φ is the mode shape displacement, with the subscript 'm' indicating measured data and 'c' computed or predicted data, and the superscript 'd' indicating damaged (current) state and '0' the intact state. N_{N} (= 5) is the number of natural frequencies to be included and N_{S} (= 5) is the number of mode shapes to be included. N_{n} (= 11) is the number of nodes in the mode shapes. W_i and V_i are the weights for the i^{th} eigenvalue and mode shape, respectively. For simplicity and without losing generality in examining the performance of the cracked element model, the weights are set to be unity.

Genetic algorithm (GA) is employed to search the optimization solution for the objective function. The GA function in Matlab is used in conjunction with the beam model to carry out the updatings.

The results from the model updating using the cracked beam element model are shown in Fig. 7. For the single-crack beam, the correct cracked element should be the 8^{th} element with l_c = 25 mm, and for the multi-crack beam the cracks should be in the 3^{rd}, 5^{th} and 9^{th} elements with l_c = 25, 30, and 20mm, respectively.

It can be seen that all the cracked elements are identified and both the updated crack depth ratios and crack locations have relatively high accuracy. For the beams with a single crack, the error in the updated depth ratios (α) is smaller than 3%. For the beams with multiple cracks, the errors in the updated values are less than 10%. It should also be noted that a false crack is identified in the 11^{th} element of the beams. As has been explained, this should be attributable to the low sensitivity associated with the elements close to the free ends of the beam. The updated crack locations also exhibit very high accuracy.



(a) Single-crack beam (b) Multiple-crack beam

Figure 7 - Updated crack depth ratio (a) of the beams.

4 SUMMARY AND CONCLUSION

A new crack identification approach for beams has been developed using a cracked beam element model. The model is based on the additional flexibility brought by the crack which is formulated with the fracture mechanics principles. The shear deformation and rotational inertial effects, as well as the coupling between transverse and longitudinal vibrations, are considered in the model; thus the model is particularly suitable for the identification in thick beams.

The effectiveness of the crack model has been verified against experimental data. Modal testing results of relatively thick beams with both single and multiple-crack were obtained from the experiments. Verification results showed that the cracked beam element model is able to predict the first few modes of natural frequencies and mode shapes of the cracked beams with high accuracy. When applied in the (inverse) identification of the cracks with a model updating procedure, the cracked beam element model exhibits very good performance in that the locations (cracked element numbers) and the crack depths are all identified satisfactorily.

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