

INVESTIGATION INTO THE EFFECT OF DEVICE NONLINEARITY IN TUNED-INERTER-DAMPERS

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ABSTRACT. This paper investigates the impact of inerter nonlinearities on the performance of tuned-inerter-dampers (TID). The inerter completes the force-current analogy between mechanical and electrical networks, and it represents the mechanical element equivalent to a capacitor. The TID is a vibration suppression system designed to limit unwanted vibrations of civil engineering structures and has a layout similar to that of a tuned-mass-damper (TMD), where the mass element was replaced by an inerter, making it a two-terminal device. The TID offers significant advantages over TMDs, in terms of performance over a wider frequency band, installation and dimensions. Nevertheless, the study carried out to date has been mainly numerical, considering a linear, purely inertial model of the inerter, where the generated force was proportional to the relative acceleration between the devices terminals. In practice, the dynamics of an inerter include nonlinearities, especially at the low frequencies associated with civil engineering applications. Here, the identification of these nonlinearities is done experimentally, based on an off-the-shelf inerter. The structural system, a single-degree-of-freedom host structure with an attached TID, is tested using Real Time Dynamic Substructuring (RTDS). The inerter represents the physical substructure, while the remaining elements of the TID and the host structure form the numerical substructure. This split allows the optimisation of the TID parameters, since the values of the spring and the damper can be changed without altering the experimental setup. In addition, this configuration takes into account the inerters potentially complex dynamics by testing it experimentally together with the characteristics of the host structure. The experimental and numerical results show that with appropriate retuning of the stiffness and damping components of the nonlinear TID system, the performance of the linear TID can be regained.

KEYWORDS: inerter, dynamic substructuring, vibration suppression.

1 INTRODUCTION

The inerter is a two-terminal device with the property that the equal and opposite force applied at the nodes is proportional to the relative acceleration between the nodes and was introduced by Smith in the late 1990's [1]. This device represents the mechanical equivalent of a capacitor, thus completing the force-current analogy between mechanical and electrical networks. Many embodiments able to display this behaviour have been proposed since. Among these, we recall rack-and-pinion inerters [1], ball-screw inerters, fluid inerters [2] and semi active devices with variable inertance [3]. The inerter can also be synthesized in the electrical domain, as shown in [4].

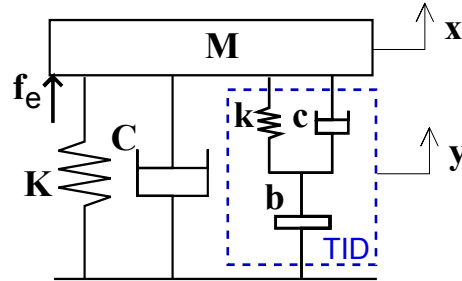


Figure 1: Two degree-of-freedom lumped mass system with an attached TID

The inerter has successfully been implemented in the automotive industry [5] and in the development of train suspensions [6, 7]. Recent studies looked into the use of inerters for civil engineering applications [8, 9, 10]. The TID, proposed in [8] and schematically shown in Figure 1, is an inerter-based vibration suppression device with a layout similar to that of traditional TMDs, where the mass is replaced by an inerter. It was shown that this device performs better than traditional TMD, when modelled inside base-excited single or multiple degree-of-freedom (DOF) systems. However, the TID studied so far assumed that the inerter is a linear and purely inertial device, generating a force proportional to the relative acceleration across its terminals. This assumption might not hold in practice and it is therefore important to look into the potential nonlinear characteristics of the inerter and assess their impact on the performance of TIDs.

Many nonlinear models, meant to capture the inerter's dynamics accurately, have been proposed in the literature [11, 12]. The experimental testing carried out focused mainly on ball-screw inerters. The main sources of nonlinearity were dry friction and the nonlinear compliance. Backlash does not play an important role, as the ball-screw is pretensioned. The impact of inerter nonlinearities on its performance has also been studied, mainly for vehicle suspension and steering instability applications. The general conclusion was that the deterioration is not significant and the nonlinear inerter-based suspension still performed better than a traditional passive suspension [13, 14].

However, we are interested in civil engineering applications, where the range of frequencies of interest is normally lower. At these low frequencies, the effects of nonlinearities are likely to be important and impact the performance of the device and host structure considerably. Our work focuses on assessing the impact of inerter's nonlinearities on the performance of TIDs and developing an efficient method for testing inerter-based system, using real-time dynamic substructuring (RTDS). This consists of splitting the system being studied into substructures, some represented numerically and the others physically built. These substructures must interact in real time, to allow the overall response to be studied.

The main advantage of using inerter-based devices over traditional vibration absorbers is given by their size and weight, especially in civil engineering applications. The inerter's apparent mass, the inductance, can be much larger than its physical mass. This is achieved through gearing. The off-the-shelf inerter involved in our tests has a mass of only 2kg and an inductance of 75kg.

Section 2 presents the tuning strategy for a linear TID, attached to a structure that is subjected to sinusoidal forcing, before the updated nonlinear model is introduced in Section 3. Section 4 looks into the optimisation of the nonlinear TID and Section 5 presents the experimental testing procedure and the results. Concluding remarks are included in Section 6.

2 OPTIMISATION OF LINEAR TUNED INERTER DAMPERS

It was shown in [8] that the TID can be optimised based on the fixed-points theory. This approach, introduced by Den Hartog for damped vibration absorbers, can be applied to find the optimal TID parameters, namely the damping and frequency ratios. Building on the results obtained in [8] for base-excited structures, a similar set of parameters is obtained here for the case when the external force is applied to the mass of the host structure.

The force exerted by a linear TID onto a single-degree-of-freedom (SDOF) host structure [8] is

$$F_{TID} = \frac{cs + k}{bs^2 + cs + k} bs^2 X = -T_{TID} X \quad (1)$$

where k , c , b represent the stiffness, damping and inertance of the TID and X is the host structure displacement, expressed in the Laplace domain.

If we consider that the structural damping is null and optimise the displacement of the host structure, the transfer function between the primary mass displacement, X , and the external force, F_e , can be written as

$$\frac{X}{F_e} = \frac{1}{Ms^2 + K - T_{TID}} \quad (2)$$

where T_{TID} can be obtained from Equation 1. This is then translated into the frequency domain, using $s = j\omega$, where $j = \sqrt{-1}$ and F_e is the Laplace transform of the external force. Following standard mathematical manipulation as shown in detail in [8], the optimal frequency ratio between the TID and the host structure is given by the non-trivial solution of

$$\bar{\omega}^4 - \frac{2(1 + \alpha + \alpha\mu)}{2 + \mu} \bar{\omega}^2 + \frac{2\alpha}{2 + \mu} = 0 \quad (3)$$

where $\mu = b/M$ is the inertance-to-mass ratio, α is a scalar coefficient representing the squared frequency ratio between the TID and host structure and $\bar{\omega} = \omega M/K$.

Equation 3 is identical to the equation obtained by Den Hartog [15] for TMDs. Therefore, it is not necessary to continue towards deriving the expressions of α and of the optimal damping ratio ξ and we can use the results obtained by Den Hartog namely,

$$\alpha = \frac{1}{(1 + \mu)^2}$$

and

$$\xi = \sqrt{\frac{3\mu}{8(1 + \mu)}} \quad (4)$$

The stiffness and damping parameters of the ideal (linear) TID can be calculated as

$$k = \mu\alpha K$$

and

$$c = 2\xi\sqrt{kb}. \quad (5)$$

Therefore, the tuning of linear TIDs can be done by choosing the desired mass ratio, μ , calculating the ratio between the squared TID and host structure fundamental frequencies, α and the damping ratio, ξ and then calculating the optimal TID stiffness, k , and damping, c .

However, this tuning methodology assumes the inerter is linear, which, especially for low-frequency structures, might not be a reasonable assumption, as described in the introductory section. A nonlinear model, based on experimental tests, is proposed in the following section. The tuning rules obtained in this section will be used for the optimisation of the nonlinear TID system, to assess their applicability and the potential performance deterioration caused by the TID non-idealised behaviour.

3 NONLINEAR TUNED INERTER DAMPER MODEL

The ball-screw type inerter used for the experimental testing is an off-the-shelf device, manufactured by Penske (model PS8760H.218C). The inerter's nonlinear behaviour is approximated by the inclusion of dry friction, in parallel to the inertance. A schematic representation is shown in Figure 2 (a), where b is the inertance and f_f is the dry friction.

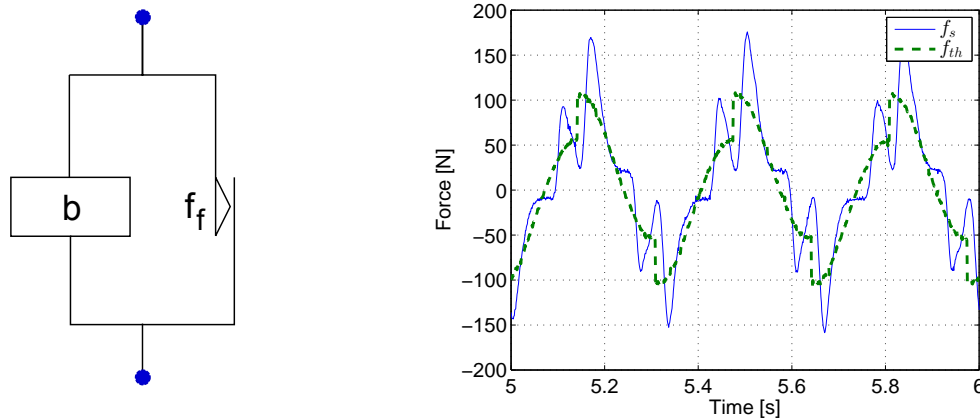


Figure 2: (a) Nonlinear inerter model; (b) Verification of the analytical model - Theoretical and measured inerter forces when the input is a 3 mm displacement sinewave at 3Hz.

The value of the inertance b was supplied by the manufacturer, while the dry friction was evaluated by running low frequency tests, for example at 0.5Hz. In that case, f_s , the measured force, is almost a square wave, with an amplitude of 20N. Several tests were then performed to verify the inertance and

dry friction values. Figure 2 (b) shows the measured f_s and the analytical force f_{th} when the input to the inerter is a sinusoidal displacement with an amplitude of 3 mm and frequency of 3Hz. Even though the device dynamics can only be partially captured by this simple model, this offers a good prediction of the inerter's behaviour. Other effects, such as the stiffness effect, backlash, linear damping and parasitic inertance may be included in a more complex model. However, this is unnecessary here as we wish to understand the broad effect of the device dynamics on the tuning before assessing it experimentally using RTDS.

4 NONLINEAR TUNED INERTER DAMPER OPTIMISATION

We will now apply the tuning methodology described in Section 3, to verify whether this may still be used for the nonlinear TIDs or a new strategy should be employed. Figure 3 shows a schematic representation of the SDOF structure with a nonlinear TID attached to it.

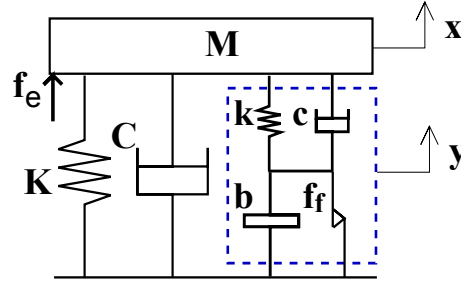


Figure 3: Host structure with a nonlinear TID device.

Starting from the identification work presented in Section 3, the parameters of the nonlinear TID are implemented in the numerical model in order to evaluate the effect of the real device's nonlinear behaviour on the performance. The the equations of motion characterising the new structure are

$$\begin{cases} M\ddot{x} + C\dot{x} + Kx + c(\dot{x} - \dot{y}) + k(x - y) = f_e, \\ b\ddot{y} + c(\dot{y} - \dot{x}) + k(y - x) + f_f \text{sign}(\dot{y}) = 0. \end{cases} \quad (6)$$

The equations are nonlinear and will be solved numerically.

4.1 Structural parameters

The host structure fundamental frequency is $\omega = 4\text{Hz}$ and the mass is taken to be $M = 1500\text{kg}$. Recalling that the inertance of the Penske inerter is $b = 75\text{kg}$ (using a 2kg device), this gives a mass ratio of $\mu = 0.05$. The resulting structural stiffness is $K = 947\text{kN/m}$. The structural damping is set to 2%, or $C = 1.5\text{kNs/m}$.

The parameters of the linear TID are $\alpha = 0.907$ and $\xi = 0.134$. It then follows that $k = 42.97\text{kN/m}$ and $c = 0.479\text{kNs/m}$.

4.2 Impact of the inerter nonlinearity

As previously mentioned, the external force acting on the host structure has the form $f_e(t) = A \sin(\omega t)$. The forcing amplitude is varied to assess the impact of the TID nonlinearity at different excitation levels. The forcing amplitude spans from 100N to 1000N.

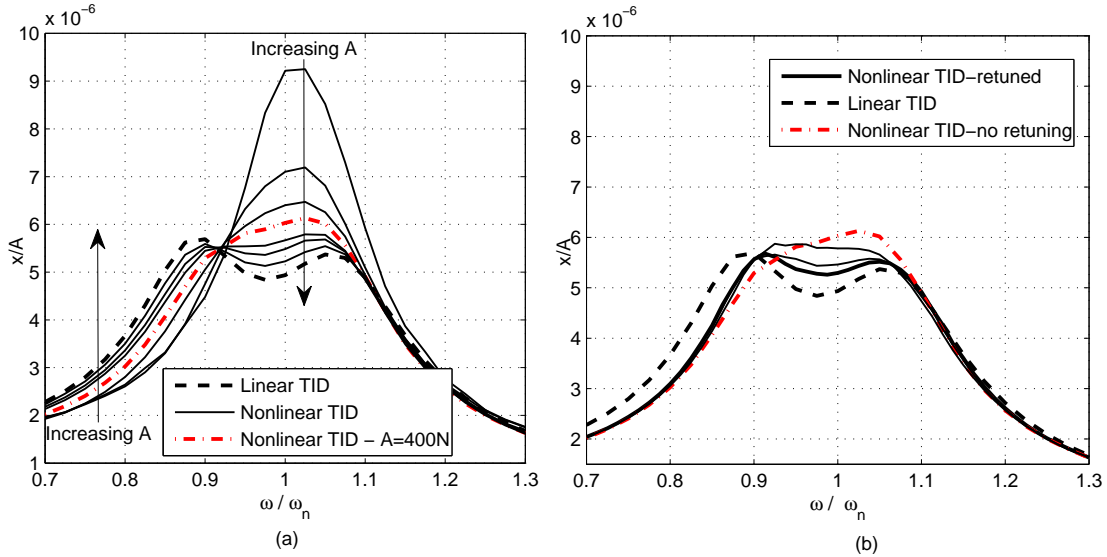


Figure 4: Frequency response of the structure equipped with a nonlinear TID device (a) Displacement of the main mass when linear optimal TID parameters are used (b) Retuning of the TID to accommodate nonlinear inerter effects; varying damping once the stiffness has been adjusted to equalise the amplitude of the fixed points for $A = 400\text{N}$ (updated optimal stiffness $k_n = 1.034k$).

Figure 4(a) shows how the maximum normalised displacement of the host structure, x , varies with the forcing frequency. The thin solid lines represent the nonlinear system response for a variety of forcing amplitudes. At low amplitudes, the normalised displacement is larger and only one peak is present. When the amplitude increases, the curves tend to that of the linear TID (thick dashed line). The reduced performance at low amplitudes suggests that, if the device is expected to perform at low forcing amplitudes, it would be beneficial to retune it by adjusting the stiffness and damping. The same variation applies to the inerter displacement, y . The red dash-dotted line represents case when the forcing amplitude is $A = 400\text{N}$. This is highlighted as it will be the focus of later discussion.

The device is then retuned to optimise its performance, by minimising the displacement response of the host structure. With the nonlinear effect included, the split peaks shift, resulting in the fixed points having different amplitudes. Therefore, it is necessary to adjust the TID stiffness accordingly. Once the fixed points reach equal amplitudes, the TID damping is tuned such that the response curve has a horizontal tangent in the fixed points, following Den Hartog's guidelines [15].

Figure 4(b) shows the retuning iterations for $A = 400\text{N}$. The stiffness has already been updated to obtained equal amplitude fixed points and only the damping variation is shown. The normalised response using a linear TID is added for reference (thick dashed line). At this amplitude, the structure displays a split peak detuned response. By varying the TID parameters it is found that the response becomes optimal when the TID damping is lowered. The optimal stiffness of the nonlinear TID is $k_n = 1.034k$. After fixing k_n , the damping is varied and the curves obtained are shown using the thin solid lines. The optimal response is obtained when the damping of the nonlinear TID is set to $c_n = 0.85c$ (thick solid line), compensating for the effect of the dry friction. The optimal retuned nonlinear TID performance is similar to that of the linear device. The red dash-dotted line representing the linearly tuned TID is included for reference,

As the forcing amplitude increases, the nonlinear TID performance, even without retuning, improves and converges to that of the linear device. This can be seen if the same retuning procedure is applied across the full amplitude range.

Following the approach presented in Section 3, the tuning of nonlinear TIDs can be done using the linear parameters as a starting guess and then evaluating the structural response for a range of TID stiffness and damping parameters situated in their vicinity. Once the split peaks are regained (this might not be the case for very low amplitude forcing), the k and c values are readjusted.

5 SUBSTRUCTURING TESTING AND EXPERIMENTAL RESULTS

The proposed tuning strategy for the TID was experimentally tested by developing Real Time Dynamic Substructuring (RTDS) Testing. RTDS is a testing technique developed to overcome limitations of more traditional methods, such as shaking table testing and pseudodynamic testing. The idea is to split the system being considered into two subsystems, one numerical and one physical, allowing full scale and real-time testing of a crucial element within the system [16]. These two subsystems interact in real time via actuators and sensors with the objective of achieving compatibility and equilibrium at the interface in real time. If this objective is met the subsystems will emulate the full system in real-time. RTDS testing is very effective when the nonlinearities or unmodelled dynamics of a system are concentrated or when new components need to be tested within the context of a whole assembly [16]. In the experiment presented here, the host structure and the stiffness and damper of the TID are defined as the numerical substructure while the inerter is the experimental substructure. This configuration was chosen to be able (i) to update the value of k and c following the proposed optimisation procedure (ii) to account for the nonlinearities in the behaviour of the inerter.

In a standard substructuring loop, the numerical model will calculate a displacement at the interface, y , under forcing f_e , this displacement is applied to the physical substructure by an actuator. The force generated at the interface due to the applied displacement is measured with load cell, f_s and fed back into the numerical model, the numerical model calculates a new displacement considering an input force $f_e - f_s$. This loop, represented in Figure 5(a), is repeated in real time until the end of the test.

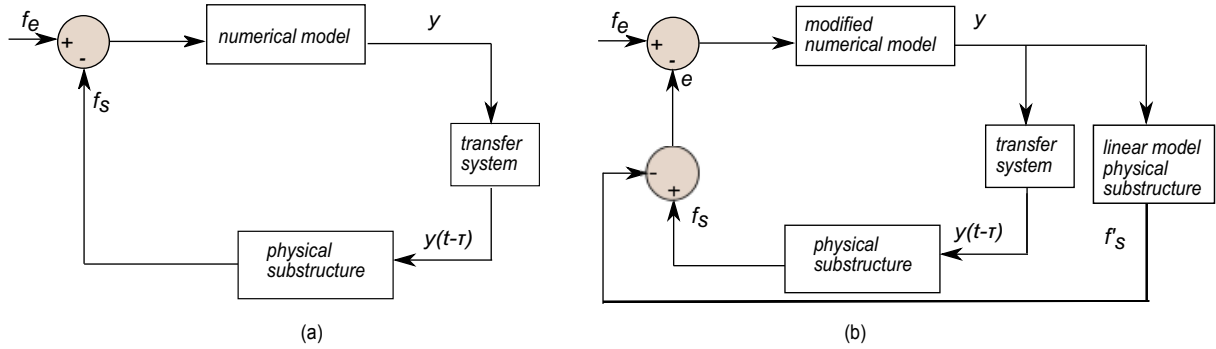


Figure 5: (a) Standard substructuring loop; (b) Modified substructuring loop.

In the experiment presented here, the standard loop was modified to avoid instability problems. The forces at the interface are inertial, fact known to cause instability, [17, 18]. Figure 5(b) shows the updated substructuring loop. With this strategy, instead of feeding back to the numerical model the actual measurement from the load cell at the interface, we feedback the deviation of actual measurement from a linear model of the physical substructure.

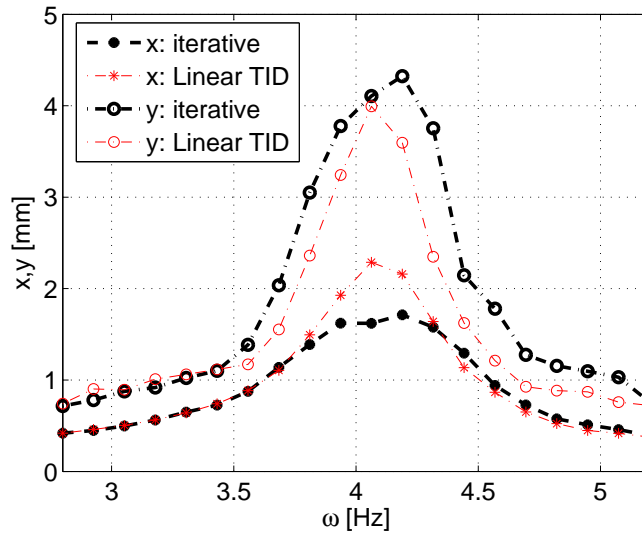


Figure 6: Experimental results for $f_e=200\text{N}$. Asterisk markers correspond to the host structure displacement, x ; circle markers correspond to the inerter displacement, y . Tuning using Den Hartog (red), retuned after two iterations (black).

Let us consider a numerical model consisting of an 1 DOF oscillator M, C, K and a nonlinear physical

substructure resisting with f_s the imposed displacement y . In the standard definition the numerical model is defined by $M\ddot{y} + C\dot{y} + Ky = f_e - f_s$. In the strategy proposed in this paper, a linear force at the interface f'_s is assumed and the numerical model is modified and defined as $M\ddot{y} + C\dot{y} + Ky + f'_s = f_e - e$ where $e = f_s - f'_s$.

For the tests, the inerter was mounted on a heavy testing bench. One of the terminals was clamped and the displacement at the interface y was applied by an electromechanical actuator. The displacement at the interface is monitored and controlled by a LVDT transducer and the force is measured by a load cell. The experiment was run in real time using Simunlink and DSPACE board. The numerical model is simulated using MATLAB Simulink, DSPACE allows for real-time communication between the numerical model and the physical substructure.

Figure 6 contains the experimental results corresponding to low forcing, $A = 200N$, when it is anticipated the nonlinearities will have a higher affect. The parameters are as follow: $b = 75kg$ (linear model of the physical substructure, $f'_s = by$), $\mu = 0.05$, $M = 1500kg$, $K = 9.5 \times 10^5 N/m$, $f = 4Hz$, $\zeta = 0.02$, where f and ζ are the resonant frequency and damping ratio of the host structure. The stiffness, k , and damping, c , of the TID, for the starting point are chosen as the optimal values derived for the linear TID, using $\alpha = \frac{1}{(1+\mu)^2}$ and $\xi = \sqrt{\frac{3\mu}{8(1+\mu)}}$. It can be seen that after iteratively retuning the TID parameters (as done numerically in Section 4), the split-peaks are regained.

6 CONCLUSION

This paper studied the impact of inerter nonlinearities on the performance of TIDs. Analytical and experimental work has been carried out in this respect. We first showed that the optimisation of a linear TID is equivalent to that of TMD when the external force is acting on the primary structure mass. The inerter model was updated by including the effect of dry friction. It was shown the linear TID retuning is only necessary for low amplitude loading scenarios, as the behaviour of the device converges to the ideal one for higher loads. The retuning consisted of a slight increase of the stiffness and decrease of damping. The numerical findings were then verified experimentally, using RTDS. A new approach for developing RTDS tests has been presented in order to overcome the instability issues. We conclude that, for the off-the-shelf inerter considered in this study, the effect of the inerter's nonlinearities is not dramatic and designing devices following linear TID optimisation results in near-optimal performance.

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