

# Investigation of Size Effect on Control Performance of Tuned Liquid Dampers by using Real-Time Hybrid Simulation

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## ABSTRACT

Tuned liquid damper (TLD), as a low cost and maintenance energy-absorbing device, is an effective passive control technique to suppress structural vibration under wind and seismic loads. The dynamic characteristic of TLD is highly dependent on its geometric sizes as well as liquid volumes. Hence, the relationship between control performance and TLD sizes is critical to the optimization of TLD design. Due to its strongly nonlinear behaviour, experiments rather than pure numerical simulations may be more reasonable to evaluate dynamic responses of structure-TLD systems. In this paper, the real-time hybrid simulation (RTHS) is employed to investigate the size effect of TLD on control efficiency, in which TLD devices, with varied geometric sizes, are experimentally modelled as physical substructures, and controlled structures are numerically simulated as numerical substructures. A series of RTHSs are carried out with the premise of same controlled structures as well as mass ratio; and the variation of mass scale is also taken into account. Results show that the TLD performance is size-dependent; a shallow liquid in TLD with lower relative liquid depth may be more efficient on both peak and RMS responses control. Experiments with scaled TLD models will overestimate the control performance of prototype TLD devices, indicating that full-scale TLD experiments should be pursued.

**Keywords:** *Tuned liquid damper; Real-time hybrid simulation; Size effect; Control efficiency; Mass scale*

## 1 INTRODUCTION

Real-time hybrid simulation (RTHS) is a novel experimental technique to investigate dynamic behaviour of structures (e.g., [1-3]). The testing is carried out by partitioning the structure for emulation into physical and numerical substructures. Data are transmitted between the two substructures in real-time to maintain force equilibrium and displacement continuity.

Owing to the substructure technique in RTHS, nonlinear behaviour of local structures could be experimentally investigated with large-scale or even full-scale model tests. Chen and Ricles [4] studied single degree-of-freedom (SDOF) and multiple degree-of-freedom (MDOF) structures with a passive elastomeric damper subjected to earthquake ground motion. Chen and Ricles [2] used RTHS to study the two-story four-bay steel moment-resisting frame with large-scale magneto-rheological dampers in passive-on mode. Lee et al. [5] verified the accuracy of the RTHS by comparing responses of an idealized single-story and an idealized three-story structures installed with TLDs with those obtained from pure shaking table tests. Sorkhabi et al. [6] investigated the

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TLD efficiency for a single-story structure through a series of RTHSs. Mass and frequency ratios were adjusted by changing the characteristics of analytical substructures to perform parametric study. Malekghasemi et al. [7] utilized the experimental results of TLD on the basis of RTHS to assess the accuracy of a proposed numerical model (finite volume method/finite element method). This study extends the application of the RTHS technique. Zhu et al. [8] studied the application of both single tuned liquid column damper (TLCD) and multiple TLCD on seismic control of a SDOF steel frame.

Tuned liquid damper (TLD) is a commonly-used passive control technique which can dissipate structural energy effectively. TLD configuration is designed to have a frequency that is determined by the length and depth of the liquid contained and that is close to the fundamental frequency of structures [9-11]. Its inherent damping effect results from the force of sloshing liquid to the TLD tank walls and wave breaking. In the past decades, TLD has been developed and applied to control the wind- and seismic-induced vibration. In order to properly describe the nonlinear behaviour of liquid in TLD, numerous theoretical models are proposed. Sun et al. [9] presented a nonlinear model for rectangular TLD based on shallow-water wave theory. This model considered both liquid damping effects and wave breaking. Modi and Seto [10] proposed a numerical algorithm to determine the nonlinear property of TLD which includes the effects of wave dispersion, boundary-layers at the walls, floating particle interactions at the free surface, and wave-breaking. This algorithm was proven to be successfully predicting vortex resonance response. Yu et al. [11] proposed a model to simulate TLD as an equivalent tuned mass damper with nonlinear stiffness and damping (NSD). This model could capture the TLD behaviour under large amplitude excitations and during wave breaking. Other numerical methods, such as the smoothed particle hydrodynamics (SPH) [12], were also used to simulate the dynamic behaviour of liquid in the tank.

According to the linear wave theory, the frequency of TLD depends on the liquid length and depth of containers, indicating different TLDs can be designed to tune to a specific structural frequency. As a result, a practical question that, how to choose the size of TLD to capture the most optimal control performance, should be paid attention. Among the previous studies above, rare studies involve this discussion. Fujino et al. [13] actually studied the effect of container size of prototype-sized rectangular or circular containers on the additional damping. It provided a preliminary conclusion that bigger TLDs would be more effective in producing large additional damping under the condition of the same mass ratio. However, it should be noted that the frequencies of TLDs studied in Fujino et al. [13] were varied and not related to a fixed frequency of structure. From practical view, the determination of choosing a TLD with optimal geometric size for a specific structure control is more meaningful.

In this paper, taken the advantage of RTHS in large- or even full-scale TLD experiments and arbitrary adjustment of numerical substructures, investigations of size effect of TLD on its control performance are performed in detail. On the premise of the same parameters of structures and mass ratio, a group of TLD models with the same frequency but different sizes are designed; and a series of RTHSs considering varying mass scale are carried out. The conclusions obtained from these experiments are discussed.

## 2 RTHS FRAMEWORK FOR TLD STUDY

### 2.1 Size effect of TLD

As shown in Figure 1, a rectangular TLD is taken as the example. The frequency of the TLD is derived based on the linear water wave theory as

$$f_{TLD} = \frac{1}{2\pi} \sqrt{\frac{\pi g}{L} \tanh\left(\frac{\pi h_0}{L}\right)}. \quad (1)$$

where  $L$  is the length of TLD in the direction of excitation; and  $h_0$  is the liquid depth of TLD. Figure 2 shows the relationship among  $f_{TLD}$  and the liquid length and depth. Figure 2(a) presents changes in  $L$  according to the changes in  $h_0$  and  $f_{TLD}$ . It is shown that  $f_{TLD}$  increases with the increase of  $h_0$  when  $L$  is constant, and reaches to be a limitation under a relatively large  $h_0$ . This result indicates the TLD is suitable for controlling structures with low frequencies. Figure 2(b) presents changes in  $h_0$  according to the changes in  $L$  and  $f_{TLD}$ .  $f_{TLD}$  generally decreases with the increase of  $h_0$  when  $L$  is constant. A short length  $L$  and a large depth  $h_0$  will benefit for a higher natural frequency  $f_{TLD}$ . Since the liquid motion in TLD is strongly determinable for the damping effect, different control performance will be captured if different TLDs with different geometric sizes are applied.

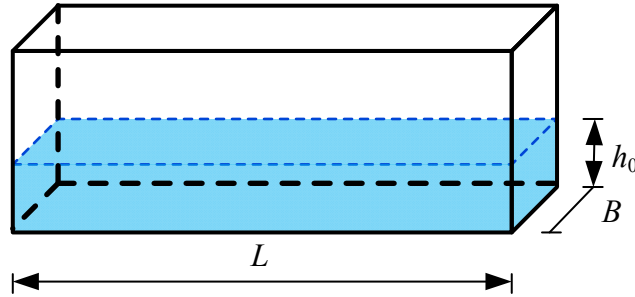
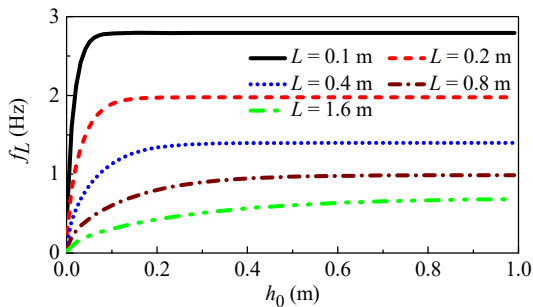
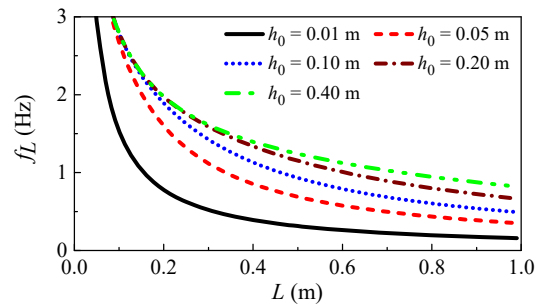


Figure 1 - Geometry of the rectangular TLD.



(a) TLD frequency vs. liquid depth



(b) TLD frequency vs. liquid length

Figure 2 - Parameters for the TLD frequency of a rectangular tank.

## 2.2 RTHS framework

As noted in Section 1, RTHS technique has been verified as a reasonable approach to investigate nonlinear dynamic behavior of structure-damper systems. Since the nonlinear behavior of TLD are extremely strong and difficult to be simulated accurately, full-scale experimental study can be considered to be a solution. The author of this paper has proposed a methodology for full-scale TLD experiments and studied the TLD performance through RTHS. More details can be referred on Reference [8].

The RTHS framework is displayed in Figure 3. The structure-TLD system is taken into account, in which the main structure is numerically simulated and the TLD device is experimentally tested in shaking table. It should be stressed that for the TLD system with multi-number TLD units with the same geometric sizes, if the mass ratio of the tested TLD model is not identical to the target mass ratio, the total TLD force can be obtained by multiplying the measured force of the tested TLD model with the ratio between the target mass ratio and the tested mass ratio. In this manner, the cost of model designed can be cut down.

The equation of motion of numerical substructure in RTHS can be expressed as

$$m_s \ddot{x}_s + c_s \dot{x}_s + c_s x_s = -m_s \ddot{x}_g + F_{TLD}. \quad (2)$$

where  $m_s$ ,  $c_s$ , and  $k_s$  are the mass, damping and stiffness, respectively;  $\ddot{x}_s$ ,  $\dot{x}_s$ , and  $x_s$  are the acceleration, velocity and displacement, respectively; the subscript  $s$  represents the controlled structure (numerical substructure);  $\ddot{x}_g$  is the earthquake acceleration;  $F_{TLD}$  is the interface force vector between the structure and TLD.

The force of TLD devices is measured by a force measurement system proposed by Wang et al. [14]. The TLD is installed over a ball bearing slide that is fixed on a shaking table, and an axial force sensor is used to connect the bottom of TLD and a fixed supporter in shaking table. In this manner, the TLD force is transferred to be the axial force, which can be conveniently measured by the axial force sensor. Wang et al. [14] and Zhu et al. [8] have verified the accuracy of this measurement system in RTHS.

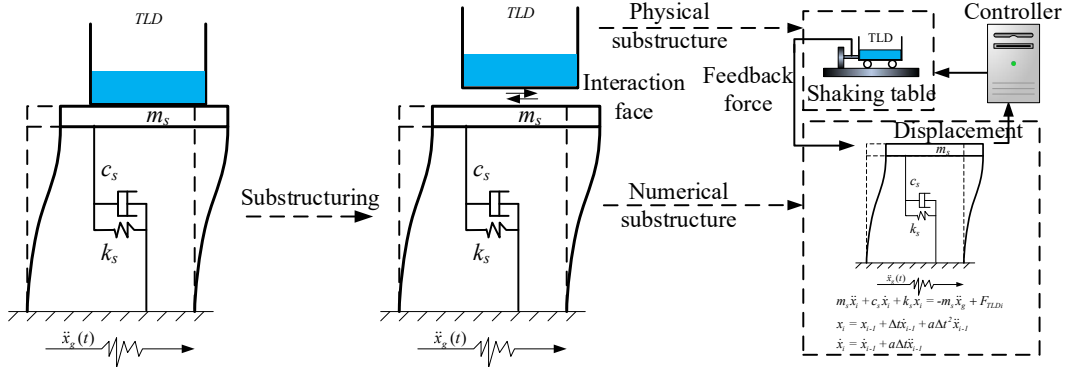


Figure 3 - Conceptual view of RTHS framework for structure-TLD system.

### 3 INVESTIGATIONS OF SIZE EFFECT OF TLD THROUGH RTHS

#### 3.1 Experimental structures

The controlled structure selected in this section is a five-story reinforced concrete building which was built in the campus of Tsinghua University. In numerical simulation, this structure is simplified by a mass-spring-damper system, and corresponding structural parameters are captured based on HAZUS program developed by Lu et al. [15]. The structural parameters of the five-story building is tabulated in Table 1. The fundamental frequency is around 1.53 Hz; and the damping ratio of the structure is selected to be 5%. According to the methodology for full scale TLD experiments in RTHS, the mass scale, selected to be  $8.068 \times 10^4$ , is only considered to obtain the scaled five-story building which will be numerically simulated. Then the scaled story stiffness and mass can be calculated. Finally, the mass of each story is 18.4 kg.

Table 1: Structural parameters of the five-story building

Name	Story	Classification	Story Stiffness (N/m)	Story Mass (kg)	Fundamental Frequency (Hz)
TTB-5	5	reinforced concrete	$1.686 \times 10^9$	$1.484 \times 10^6$	1.53

For the given structural frequency with 1.53 Hz, three TLD models, named TLD-1-1 to TLD-1-3, respectively, are designed with liquid length of 0.10 m, 0.15 m and 0.20 m, respectively, as shown in Table 2. The relative liquid depths  $h_0/L$  for the three TLDs are 0.100, 0.135 and 0.222, respectively. Since the mass ratio is determinable to the control effect of TLD, the mass ratio for the three TLDs are identical to 1.5%. Figure 4 gives the pictures of the three TLDs.

Table 2: Geometric sizes of TLDs

TLD	$L$ (m)	$h_0$ (m)	$B$ (m)	$f_{TLD}$ (Hz)	Number of units	$h_0/L$
TLD-1-1	0.10	0.010	0.230	1.53	6	0.100
TLD-1-2	0.15	0.023	0.400	1.53	1	0.135
TLD-1-3	0.20	0.044	0.157	1.53	1	0.222

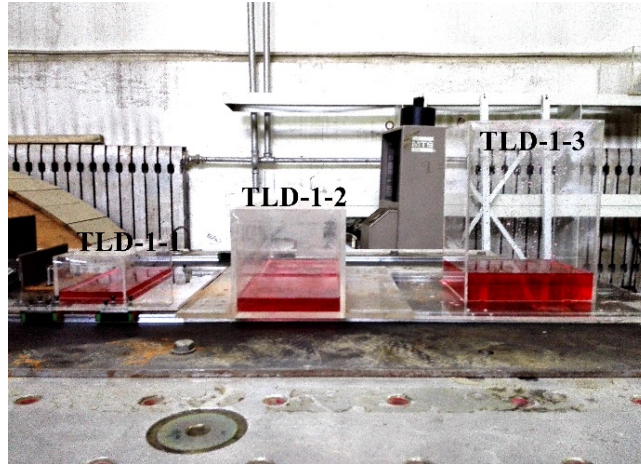


Figure 4 - Pictures of TLDs with different sizes.

Three commonly used ground motions, namely, Kobe, El Centro, and Taft, are applied to excite the emulated structure system. Three levels of peak ground acceleration (PGA) for the three ground motion are considered and scaled to 0.05 g, 0.1 g and 0.2 g, respectively. To evaluate reduction effectiveness of TLD, pure numerical simulations without TLD control are carried out, and corresponding results are treated as reference results. Gui- $\lambda$  method ( $\lambda = 11.5$ ) [16] with a time step of  $\Delta t = 1/2048$  s is employed as the integration algorithm for both RTHS and pure numerical simulation. To compare the seismic responses of the structure with and without TLD control, four factors are defined in terms of reducing peak and root-mean-square (RMS) responses of structures:

$$P_d = 1 - \frac{MAX(x_{withcontrol})}{MAX(x_{withoutcontrol})}, P_a = 1 - \frac{MAX(\ddot{x}_{withcontrol})}{MAX(\ddot{x}_{withoutcontrol})}. \quad (3)$$

$$R_d = 1 - \frac{RMS(x_{withcontrol})}{RMS(x_{withoutcontrol})}, R_a = 1 - \frac{RMS(\ddot{x}_{withcontrol})}{RMS(\ddot{x}_{withoutcontrol})}. \quad (4)$$

where the subscripts *withcontrol* and *withoutcontrol* represent the results obtained with and without TLD control, respectively.

### 3.2 Experimental results

Figures 5 presents displacement and acceleration responses of the top story in frequency domain under the three ground motion with peak acceleration of 0.1 g. All the three TLDs performs well on reducing both displacement and acceleration responses. It shows that TLD-1-1 is more effective than the other two TLDs on reducing frequency amplitudes, especially near the resonance frequency range.

Tables 3 and 4 provide the comparison of corresponding reduction ratios. In Table 3, it is observed that three TLDs reduce peak responses significantly under Kobe and Taft ground motions while play ineffective under El Centro ground motion. The reason can be explained according to the time histories in Figure 5 that the peak responses almost occur at the initial time range for El ground motion and TLD essentially takes no effect on this range.

In Table 4, the reduction ratios of TLD-1-1, TLD-1-2 and TLD-1-3 for RMS displacement ( $R_d$ ) are 19.86%, 20.85% and 19.78%, and 25.00% under Kobe ground motion, 32.26%, 24.19% and 22.03% for El Centro ground motion, and 32.57%, 29.27% and 24.65% under Taft ground motion. The values and tendency of reduction ratio for RMS acceleration ( $R_a$ ) under the three TLD control are quite similar as those for RMS displacement ( $R_d$ ). It is found that the control effect of TLDs under El Centro and Taft are significantly higher than that under Kobe; and TLD-1-1 generates the maximum reduction efficiency in most cases.

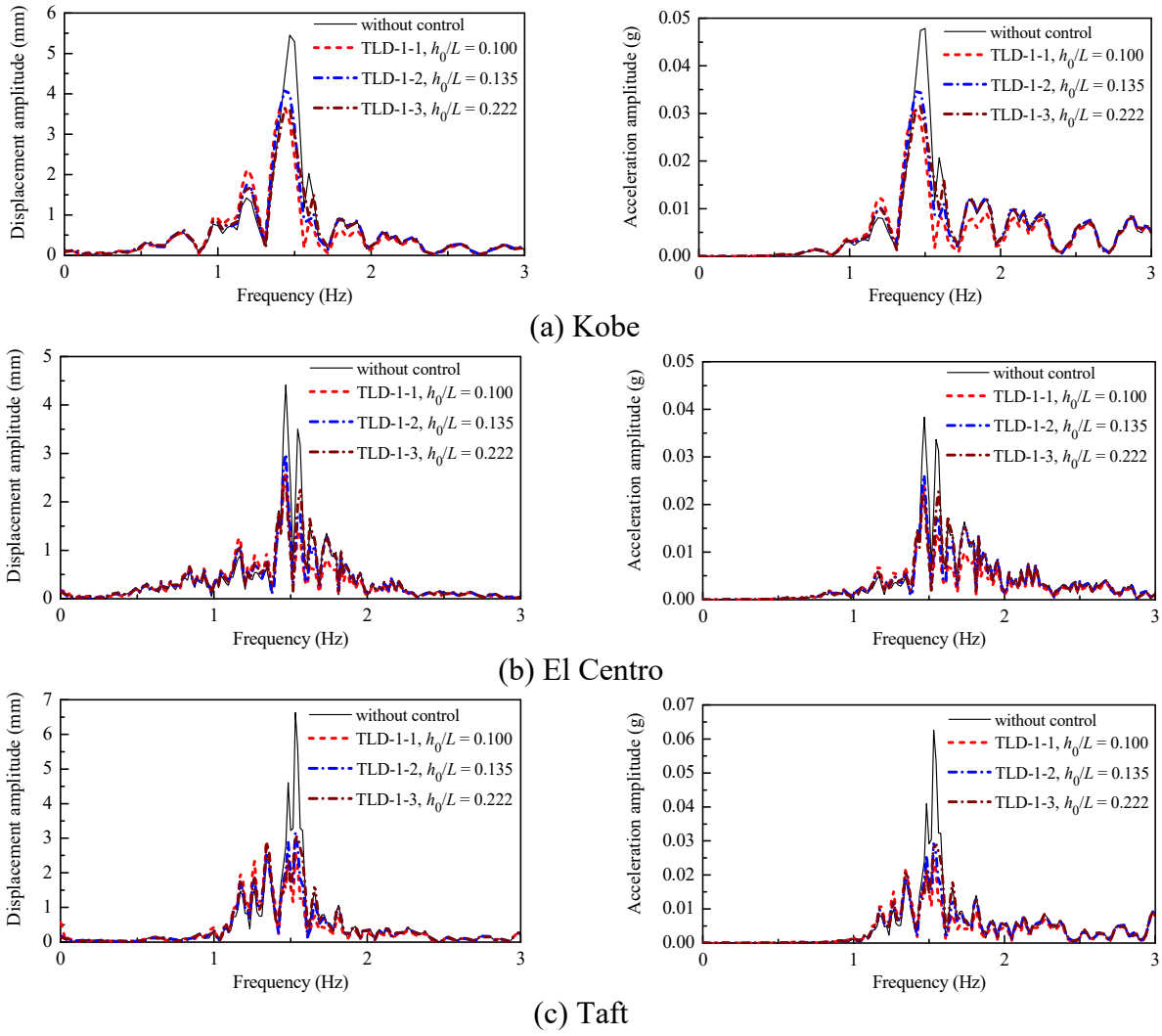


Figure 5 - Fourier spectrums of structural responses under the control of the three TLDs.

Table 3: Comparison of reduction on peak responses for the three TLDs

TLD	$P_d$ (%)			$P_a$ (%)		
	Kobe	El Centro	Taft	Kobe	El Centro	Taft
TLD-1-1	18.61	14.58	17.92	32.25	20.03	25.73
TLD-1-2	18.13	-1.69	13.58	25.12	-5.51	22.33
TLD-1-3	23.14	-2.83	12.47	16.58	-4.43	19.37

Table 4: Comparison of reduction on RMS responses for the three TLDs

TLD	$R_d$ (%)			$R_a$ (%)		
	Kobe	El Centro	Taft	Kobe	El Centro	Taft
TLD-1-1	19.86	32.26	32.57	25.00	37.52	37.18
TLD-1-2	20.85	24.19	29.27	15.11	22.31	28.55
TLD-1-3	19.78	22.03	24.65	16.24	19.17	22.71

To deeply investigate the relationship between the control performance and geometric size of TLD, a series of RTHS are carried out under the three ground motions with PGA ranging from 0.05 g to 0.2 g. The reduction ratio results are provided in Figures 6 and 7. It should be noted that the horizontal axis is represented by values of  $h_0/L$  which corresponds to TLD-1-1, TLD-1-2 and TLD-1-3, respectively. In Figure 6, the reductions of the three TLDs under Kobe and Taft ground motions are generally effective than those under El Centro ground motion. For a given PGA, the reduction ratios of both peak displacement and acceleration decrease with the increase of  $h_0/L$  from TLD-1-1 to TLD-1-3. In addition, for Kobe and Taft ground motions as shown in Figures 6(a) and

6(c), the reduction ratios of the three TLDs generally decrease with the increase of PGA; while for El Centro ground motion in Figure 6(b), an absolutely opposite phenomenon that the reduction ratio of TLD-1-1 increases with the increase of PGA can be found; and the reduction ratio of TLD-1-1 is greater than that of both TLD-1-2 and TLD-1-3. These results demonstrate that for the case of peak responses occurring at the initial time (under El Centro ground motion), a small  $h_0/L$  in TLD and an excitation with larger PGA benefit the liquid start to move easily and take effect.

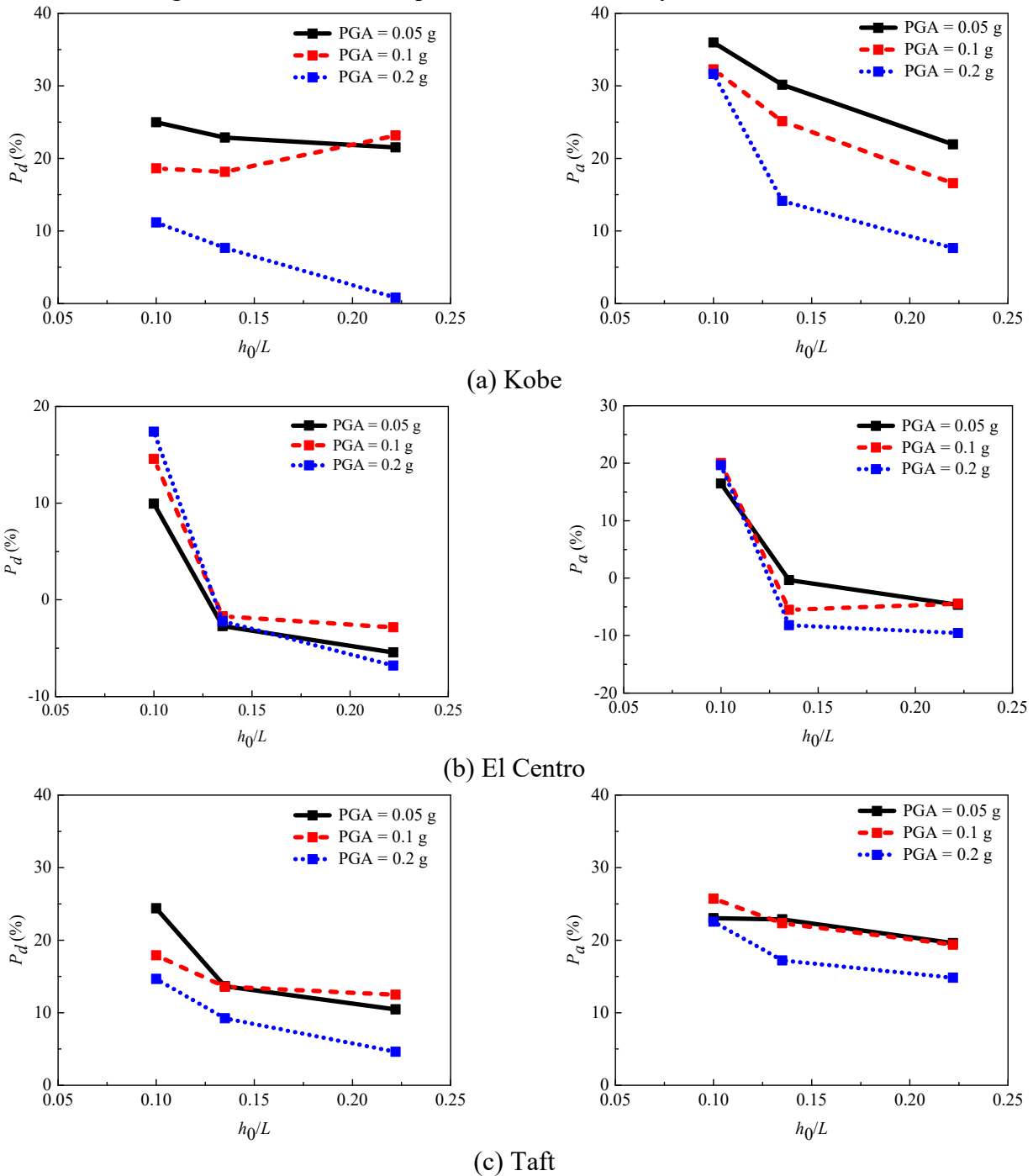


Figure 6 - Reduction efficiency of peak responses with the three TLDs under different PGA levels.

In Figure 7, for each PGA level, the reduction ratios on RMS responses control under the three ground motions decrease with the increase of  $h_0/L$ ; indicating that a shallow liquid in TLD with lower  $h_0/L$  may be more efficient on response control. This observation can be explained as follows: for a given frequency, a larger liquid length induces to a larger liquid depth; as a result, some liquid near the bottom would tend to be immobile and could be treated as additional mass in structure system which results in ineffective control.

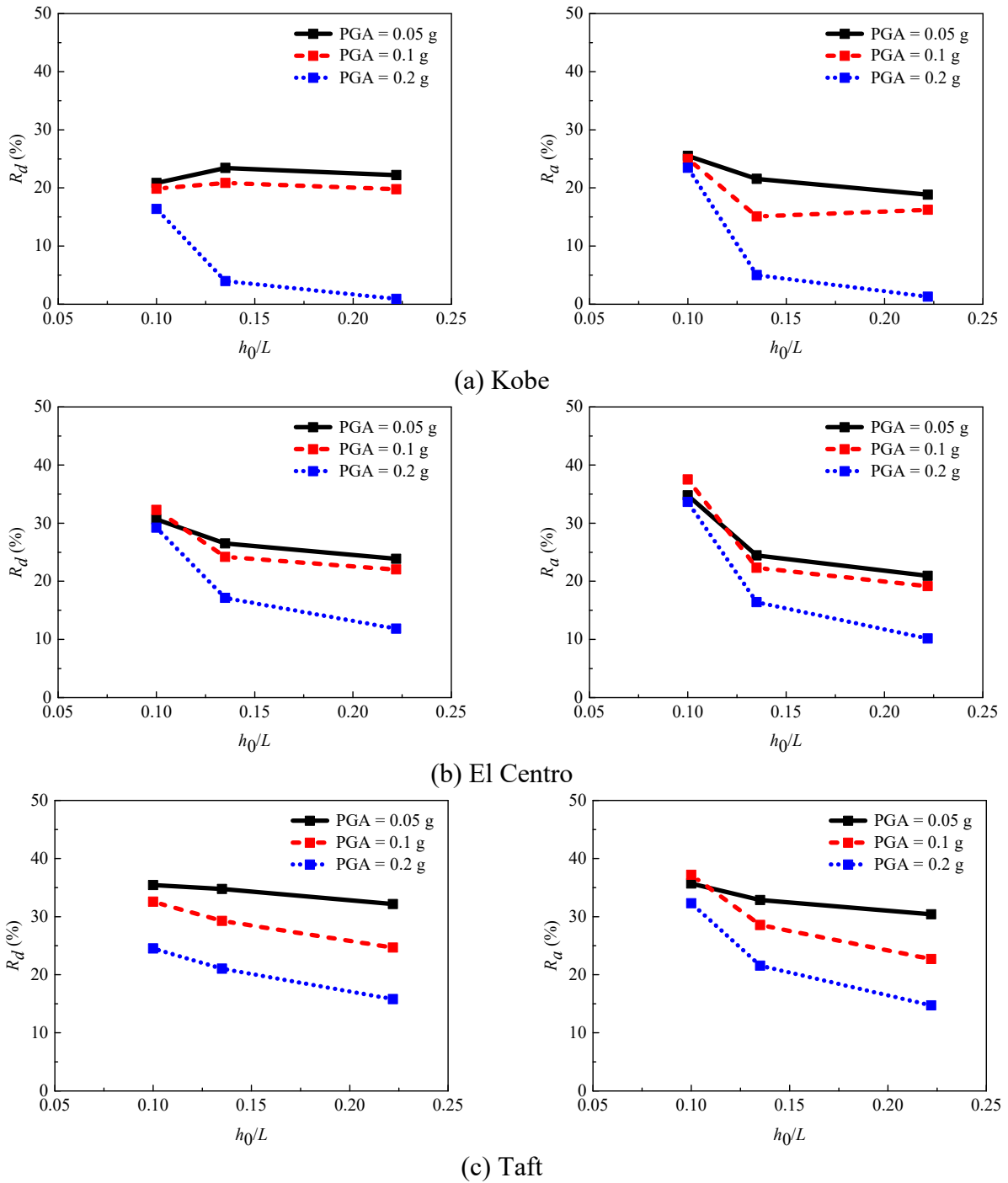


Figure 7 - Reduction efficiency of RMS responses with the three TLDs under different PGA levels.

#### 4 RTHS OF TLD CONSIDERING VARIATION OF MASS SCALE

In previous RTHS studies, the mass scale of numerical substructures is predetermined, then the TLD is designed under this mass scale condition. To study the mass scale effect on TLD control performance, a new structure, named ZJSB-7, is selected as the controlled structure. ZJSB-7 has 7 DOF and a fundamental frequency of 0.57 Hz; its damping ratio is assumed to be 2%. Wang et al. [14] finished RTHSs under the mass scale of  $C_m = 1.497 \times 10^4$ . In this section, two mass scale values are selected as  $C_m = 1.497 \times 10^4$  and  $1.497 \times 10^3$  to examine the effect of mass scale on TLD behavior. As a result, two new TLDs are designed under the condition of mass ratio  $\mu = 1.5\%$ . The parameters of scaled structure and TLD are listed in Table 6. Both TLD-2-1 and TLD-2-2 have relatively large liquid length with 0.48 m and 1.08 m, respectively, as shown in Figure 8.

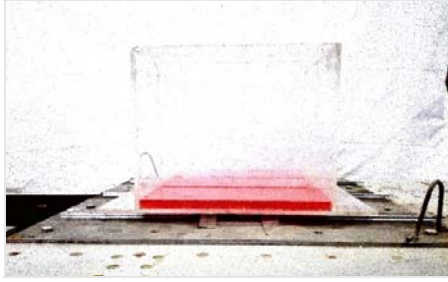


Table 5: Classification and structural parameters of the control structure

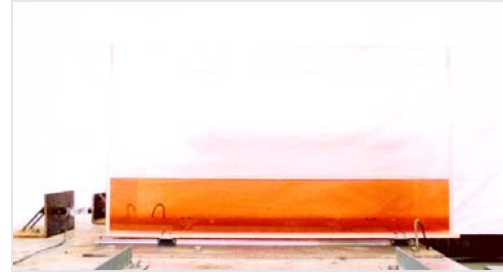
Name	Story	Classification	Story Stiffness (N/m)	Story Mass (kg)	Fundamental Frequency (Hz)
ZJSB-7	7	steel	$4.130 \times 10^8$	$1.400 \times 10^6$	0.57

Table 6: Parameters of TLDs and the controlled structure with different mass scale

TLD	$L$ (m)	$h_0$ (m)	$B$ (m)	$f_{TLD}$ (Hz)	Number of units	$h_0/L$	$C_m$	Story Stiffness (N/m)	Story Mass (kg)
TLD-2-1	0.48	0.031	0.66	0.57	1	0.065	$1.497 \times 10^4$	$2.759 \times 10^4$	93.5
TLD-2-2	1.08	0.168	0.54	0.57	1	0.156	$1.497 \times 10^3$	$2.759 \times 10^3$	935



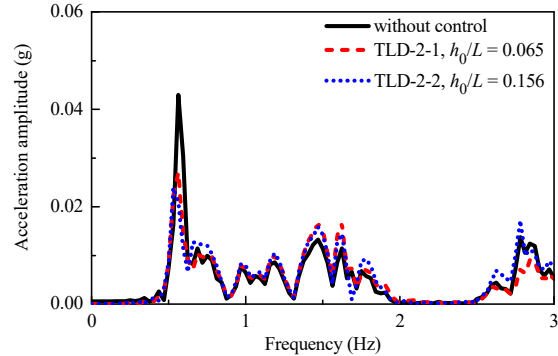
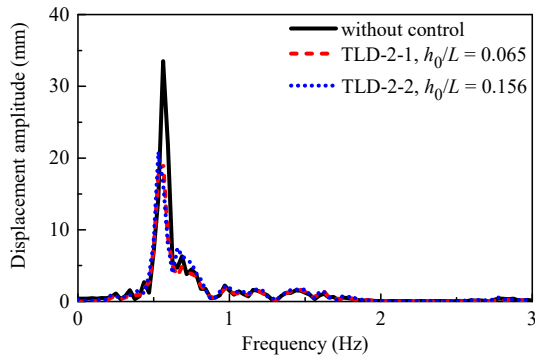
(a) TLD-2-1



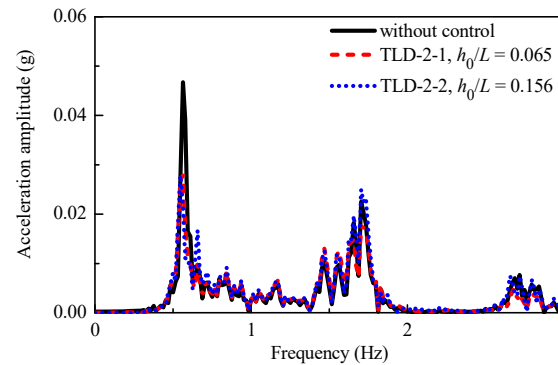
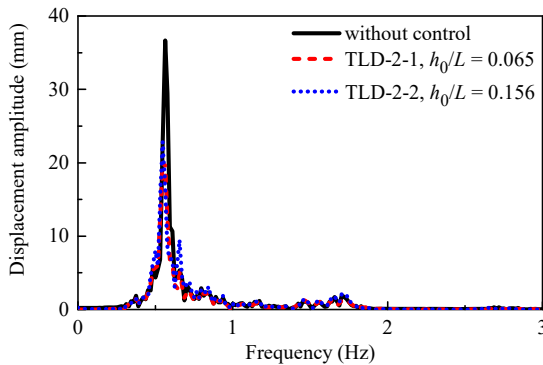
(b) TLD-2-2

Figure 8 - Pictures of TLDs with different sizes for mass scale study

The three ground motions with PGA of 0.1 g are used again to excite the structure. The reference results without control are obtained by pure numerical simulation with the same integration algorithm. Figure 9 displays the frequency responses of the top story. It is found that both TLD-2-1 and TLD-2-2 play superior performance on both displacement and acceleration response control; and the reduction on displacement is more effective than that on acceleration. In addition, the reduction efficiency of TLD-2-1 is generally better than TLD-2-2 on the first order modal response control.



(a) Kobe



(b) El Centro

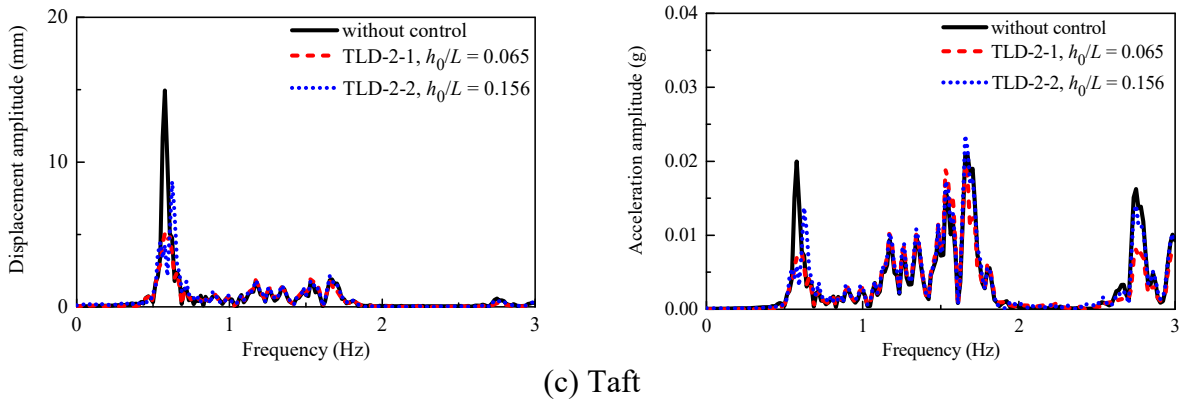


Figure 9 - Dynamic response of the structure under TLD control with different mass scales

Figure 10 presents the quantitative comparison of reduction ratio between TLD-2-1 and TLD-2-2. It is observed that the reduction ratio of TLD-2-1 is significantly higher than that of TLD-2-2 in all cases, which matches well with the conclusion in Section 3. Figure 11 shows the typical liquid motion in TLD-2-2 under the three ground motions. It is apparent that the most serve liquid motions are strong wave breaking for Kobe and El Centro ground motions, while are weak wave breaking for Taft ground motion.

Summarily, it is concluded that for a determined tuned frequency, a TLD with lower relative liquid depth  $h_0/L$ , working as a shallow liquid type, may be more effective. This result demonstrates that in conventional shaking table tests, experiments with scaled TLD models will overestimate control performance of prototype TLD devices. Since RTHS has the capability to perform full-scale experiments, RTHS on full-scale TLD models could be a new approach for TLD study.

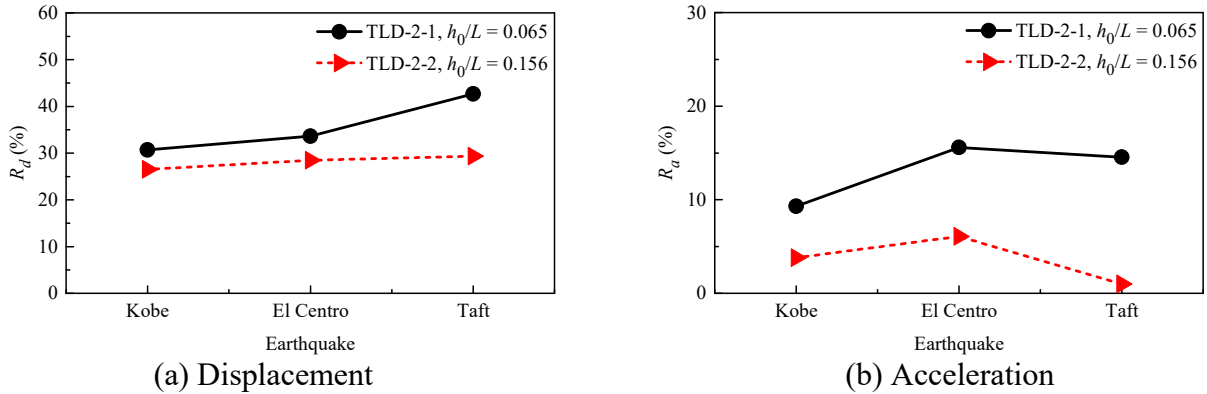
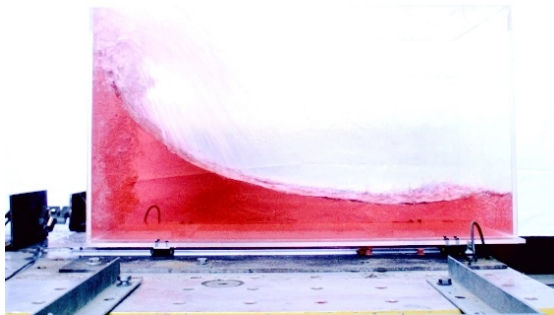


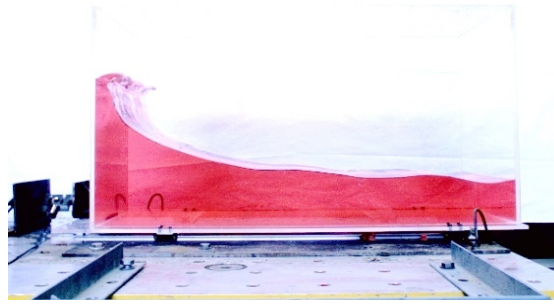
Figure 10 - Reduction ratios of TLD-2-1 and TLD-2-2



(a) Kobe



(b) El Centro



(c) Taft

Figure 11 - The most severe liquid motion in TLD-2-2 during the excitations

## 5 CONCLUSION

The effect of geometric sizes on TLD control performance is discussed in this paper by real-time hybrid simulations. Analysis is carried out by simulating controlled structures numerically while testing TLD devices experimentally. It is found that on the premise of the same mass ratio and structural parameters, the reduction effectiveness of TLDs decreases with the increase of relative liquid depth  $h_0/L$ , and also decreases with the increase of PGA, indicating that a shallow liquid in TLD with small  $h_0/L$  may be more efficient than that with large  $h_0/L$  in TLD. In addition, a small TLD benefits the reduction on peak responses when the peak responses occur at the initial time range. Hence, a TLD system with small TLD units rather than large TLD units is suggested in practical engineering. In addition, experiments with scaled TLD models will overestimate the control performance of prototype TLD devices, indicating that full-scale TLD experiments should be pursued.

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## REFERENCES

- [1] Nakashima, M., Masaoka, N. Real-time on-line test for MDOF systems. *Earthquake Engineering and Structural Dynamics*, 1999; **28**:393-420.
- [2] Chen, C., Ricles, J. M. Large-scale real-time hybrid simulation involving multiple experimental substructures and adaptive actuator delay compensation. *Earthquake Engineering and Structural Dynamics*, 2012; **41**:549-569.
- [3] Zhu, F., Wang, J. T., Jin F., Zhou M. X., Gui, Y. Simulation of large-scale numerical substructure in real-time dynamic hybrid testing. *Earthquake Engineering and Engineering Vibration*, 2014; **13**(4):599-609.
- [4] Chen, C., Ricles, J. M. Real-time hybrid testing using the unconditionally stable explicit CR integration algorithm. *Earthquake Engineering and Structural Dynamics*, 2009; **38**:23-44.

- [5] Lee, S. K., Park, E. C., Min, K. W., Lee, S. H., Chung, L., Park, J. H. Real-time hybrid shaking table testing method for the performance evaluation of a tuned liquid damper controlling seismic response of building structures. *Journal of Sound and Vibration*, 2007; **302**(3):596-612.
- [6] Sorkhabi, A. A., Malekghasemi, H., Mercan, O. Dynamic behaviour and performance evaluation of tuned liquid dampers (TLDs) using real-time hybrid simulation. *Structures Congress*, 2012; 2153-2162.
- [7] Malekghasemi, H., Sorkhabi, A. A., Ghaemmaghani, A. R., Mercan, O. Experimental and numerical investigations of the dynamic interaction of tuned liquid damper-structure, *Journal of Vibration and Control*, 2015; **21**(14):2707-2720.
- [8] Zhu, F., Wang, J. T., Jin, F., Lu, L. Q. Seismic performance of tuned liquid column dampers for structural control using real-time hybrid simulation. *Journal of Earthquake Engineering*, 2016; DOI:10.1080/13632469.2016.1138170.
- [9] Sun, L. M., Fujino, Y., Pacheco, B. M., Chaiseri, P. Modeling of tuned liquid damper (TLD). *Journal of Wind Engineering and Industrial Aerodynamics*, 1992; **43**(1-3):1883-1894.
- [10] Modi, V. J., Seto, M. L. Suppression of flow-induced oscillations using sloshing liquid dampers: analysis and experiments. *Journal of Wind Engineering and Industrial Aerodynamics*, 1997; **67-68**:611-625.
- [11] Yu, J. K., Wakahara, T., Reed, D. A. A non-linear numerical model of the tuned liquid damper. *Earthquake Engineering & Structural Dynamics*, 1999; **28**(6):671-686.
- [12] Bulian, G., Souto-Iglesias, A., Delorme, L., Botia-Vera, E. Smoothed particle hydrodynamics (SPH) simulation of a tuned liquid damper (TLD) with angular motion. *Journal of Hydraulic Research*, 2010; **48**:28-39.
- [13] Fujino, Y., Pacheco, B. M., Chaiseri, P., Sun, L. M. Parametric studies on tuned liquid damper (TLD) using circular containers by free-oscillation experiments. *Structural Engineering/Earthquake Engineering*, 1988; **5**(2):381-391.
- [14] Wang, J. T., Gui, Y., Zhu, F., Jin, F., Zhou, M. X. Real-time hybrid simulation of multi-story structures installed with tuned liquid damper, *Structural Control and Health Monitoring*, 2015; DOI: 10.1002/stc.1822.
- [15] Lu, X. Z., Han, B., Hori, M., Xiong, C., Xu, Z. A coarse-grained parallel approach for seismic damage simulations of urban areas based on refined models and GPU/CPU cooperative computing. *Advances in Engineering Software*, 2014; **70**:90-103.
- [16] Gui, Y., Wang, J. T., Jin, F., Chen, C., Zhou, M. X. Development of a family of explicit algorithms for structural dynamics with unconditional stability. *Nonlinear Dynamics*, 2014; **77**(4):1157-1170.