

LQR–UKF SEMI–ACTIVE CONTROL OF UNCERTAIN STRUCTURES

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ABSTRACT. A novel semi-active control strategy for the effective vibration mitigation of structural systems of uncertain properties is introduced in this study. The implemented approach emerges through fusion of the Unscented Kalman filter (UKF), as a nonlinear observer, with the linear-quadratic regulator (LQR). The UKF is implemented towards the establishment of an adaptive joint state and parameter estimation problem, taking into account that numerical models of structural systems are often inadequate due to inherent uncertainties, such as noise and modeling errors, unknown system properties and influence of varying operational conditions. The improved state-space representation is accordingly fed into the LQR and a semi-active control scheme that utilizes clipping is applied. Since both estimation and control are executed within the same loop, particular attention is attributed to the derivation of the appropriate LQR strategy, pertaining to both the selection of optimal weight matrices and the real-time tuning of the control parameters. A simulated five-story shear-frame subject to earthquake motion serves as a case-study for validating the proposed methodology. The promising results encourage further investigation of the developed strategy, especially in regards to its real-time experimental implementation using semi-active actuators, such as magnetorheological dampers.

KEYWORDS: structural control, uncertain system properties, unscented Kalman Filter, real-time LQR, semi-active control.

1 INTRODUCTION

Over the last years, semi-active control methods [1, 2, 3] gain increasing attention in structural vibration mitigation [4, 5], as they interconnect the advantages of their passive [6] and active counterparts [7], offering the adaptability of the active class without requiring the corresponding power requirements [8]. To this end, many studies have investigated the implementation of semi-active control strategies, both on a simulation level [9] and through experimental validation [10, 11, 12].

A particular class of semi-active vibration control methods is devoted to uncertain, or time-varying structural systems [13]. Among other approaches, Guclu and Yazici [14] propose a fuzzy PID type controller to improve seismic control performance of a nonlinear structural system within the context however of an active system, namely a tuned mass damper. Bitaraf and Hurlebaus [15] employ a Simple Adaptive Control Method to control the response of a tall building exhibiting nonlinear behavior, by means of magnetorheological (MR) dampers. Cetin *et al.* [16] use Lyapunov-based design techniques for designing a nonlinear adaptive controller able to compensate parametric uncertainties related to both

the structural system and the MR damper. The proposed controller, relying on a Bayesian filter-based design, is experimentally validated on a six-degrees-of-freedom (DOFs) shear-frame structure installed on a shaking table. For a recent overview in semi-active vibration control the reader is referred to Casciati *et al.* [17].

Quite recently, an alternative semi-active control strategy explores the sequential implementation of the Unscented Kalman Filter (UKF) and the LQR [18] to the vibration mitigation of structural systems with inherent uncertainties. The UKF is employed for solving a joint state and parameter estimation problem, during which both the structural states (i.e., displacements and velocities) and the uncertain properties (i.e., stiffness, damping, strength) are considered as unknowns, inevitably leading to a non-linear identification problem. Accordingly, the LQR provides the necessary state feedback, while the semi-active character of the method is realized by means of clipping, in which the active control force components, i.e., the components that are contra-directional to the velocity of the DOF where the actuator is located, are eliminated.

The devised methodology has been successfully verified via numerical simulations using an MR damper [19, 18], taking into account the inherent dynamics of the actuator. Miah *et al.* [20] have further experimentally validated the method in hard real-time, i.e., when all required calculations are executed within the specified sampling period of the control loop. Yet, an important issue that has not thus far been addressed in this method pertains to the calculation of the LQR gain: in all the aforementioned studies, the gain is a priori calculated off-line using the currently available initial information for the uncertain parameters. This is clearly not an optimal strategy as it suppresses the effectiveness of the vibration mitigation scheme and limits its applicability range.

Under this setting, the aim of this study is to proceed one step forward and integrate an online method for the calculation of the corresponding state feedback gain matrix, which can be executed in hard real-time. Conventional LQR gain calculation methods utilize the solution of an eigenvalue problem that results from an invariant subspace formulation [21]. This strategy is, nevertheless, improper for online implementation due to the prohibitively high computational cost. To this end, the method described in Chowdhary and Lorens [22] is herein adopted, which formulates a recursive calculation of the associated Riccati matrix, under the assumption that the structure undergoes small changes among time steps, which in turn imply small changes in the optimum solution of the LQR problem. A simulated five-story shear building subject to harmonic earthquake excitation at the first structural frequency serves as a test case for validating the novel method.

The paper is organized as follows: Section 2 formulates the combined state and parameter estimation problem and justifies the use of the UKF, which is briefly reviewed in Section 3. Section 4 explains the adopted semi-active control strategy, including the online gain calculation and the clipping action. The numerical test case is presented in Section 5 and, finally, in Section 6 the results are discussed and suggestions for further research are offered.

2 PROBLEM FORMULATION

Consider a linear, time-invariant, viscously damped structural system of the form

$$\mathbf{M}(\boldsymbol{\alpha})\ddot{\mathbf{x}}(t) + \mathbf{C}(\boldsymbol{\alpha})\dot{\mathbf{x}}(t) + \mathbf{K}(\boldsymbol{\alpha})\mathbf{x}(t) = \mathbf{P}_1(\boldsymbol{\alpha})\mathbf{f}(t) + \mathbf{P}_2(\boldsymbol{\alpha})\mathbf{d}(t) \quad (1)$$

in which $\mathbf{x}(t)$ is the $[n \times 1]$ vibration displacement vector and $\mathbf{M}(\boldsymbol{\alpha})$, $\mathbf{C}(\boldsymbol{\alpha})$, $\mathbf{K}(\boldsymbol{\alpha})$ are the $[n \times n]$ mass, viscous damping and stiffness matrices, respectively, herein considered to be symmetric and positive definite $\forall \boldsymbol{\alpha}$. It is assumed that the structure is amenable to (i) a set of *measured* disturbance forces $\mathbf{d}(t)$ that excite certain DOFs, as matrix $\mathbf{P}_2(\boldsymbol{\alpha})$ indicates; (ii) a set of control forces $\mathbf{f}(t)$ that may be applied to certain DOFs, as matrix $\mathbf{P}_1(\boldsymbol{\alpha})$ indicates; (iii) that the structural system of Eq. 1 is *uncertain*, in the sense that all associated matrices depend on the parameter vector $\boldsymbol{\alpha}$, the values of which are unknown a-priori.

The objective herein is to mitigate the structural vibration induced by the disturbance forces using a semi-active control strategy that utilizes discrete state feedback in *hard real-time*. In order to accomplish this goal, Eq. 1 is firstly transformed to state-space as,

$$\dot{\boldsymbol{\xi}}(t) = \mathbf{A}_c(\boldsymbol{\alpha})\boldsymbol{\xi}(t) + \mathbf{B}_c(\boldsymbol{\alpha})\mathbf{f}(t) + \mathbf{L}_c(\boldsymbol{\alpha})\mathbf{d}(t) \quad (2a)$$

$$\mathbf{y}(t) = \mathbf{H}_c(\boldsymbol{\alpha})\boldsymbol{\xi}(t) + \mathbf{D}_c(\boldsymbol{\alpha})\mathbf{f}(t) + \mathbf{N}_c(\boldsymbol{\alpha})\mathbf{d}(t) \quad (2b)$$

where the matrices of the state equation (Eq. 2a) are given by

$$\begin{aligned} \mathbf{A}_c(\boldsymbol{\alpha}) &= \begin{bmatrix} \mathbf{O}_n & \mathbf{I}_n \\ -\mathbf{M}^{-1}(\boldsymbol{\alpha})\mathbf{K}(\boldsymbol{\alpha}) & -\mathbf{M}^{-1}(\boldsymbol{\alpha})\mathbf{C}(\boldsymbol{\alpha}) \end{bmatrix}, \\ \mathbf{B}_c(\boldsymbol{\alpha}) &= \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}(\boldsymbol{\alpha})\mathbf{P}_1(\boldsymbol{\alpha}) \end{bmatrix}, \text{ and} \\ \mathbf{L}_c(\boldsymbol{\alpha}) &= \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}(\boldsymbol{\alpha})\mathbf{P}_2(\boldsymbol{\alpha}) \end{bmatrix} \end{aligned} \quad (3)$$

and the matrices of the output equation (Eq. 2b) depend on the type of structural response available. For the case of vibration acceleration at specific DOFs it follows that

$$\begin{aligned} \mathbf{H}_c(\boldsymbol{\alpha}) &= \mathbf{T} \begin{bmatrix} -\mathbf{M}^{-1}(\boldsymbol{\alpha})\mathbf{K}(\boldsymbol{\alpha}) & -\mathbf{M}^{-1}(\boldsymbol{\alpha})\mathbf{C}(\boldsymbol{\alpha}) \end{bmatrix}, \\ \mathbf{D}_c(\boldsymbol{\alpha}) &= \mathbf{T}\mathbf{M}^{-1}(\boldsymbol{\alpha})\mathbf{P}_1(\boldsymbol{\alpha}), \text{ and} \\ \mathbf{N}_c(\boldsymbol{\alpha}) &= \mathbf{T}\mathbf{M}^{-1}(\boldsymbol{\alpha})\mathbf{P}_2(\boldsymbol{\alpha}) \end{aligned} \quad (4)$$

for some matrix \mathbf{T} that maps the states to the measured DOFs. Discretization of Eq. 2 at a sampling period T_s (s), assuming constant intersample behaviour of the control inputs and the applied disturbances, implies

$$\boldsymbol{\xi}_{k+1} = \mathbf{A}_d(\boldsymbol{\alpha})\boldsymbol{\xi}_k + \mathbf{B}_d(\boldsymbol{\alpha})\mathbf{f}_k + \mathbf{L}_d(\boldsymbol{\alpha})\mathbf{d}_k \quad (5a)$$

$$\mathbf{y}_k = \mathbf{H}_d(\boldsymbol{\alpha})\boldsymbol{\xi}_k + \mathbf{D}_d(\boldsymbol{\alpha})\mathbf{f}_k + \mathbf{N}_d(\boldsymbol{\alpha})\mathbf{d}_k \quad (5b)$$

for $\mathbf{H}_d(\boldsymbol{\alpha}) = \mathbf{H}_c(\boldsymbol{\alpha})$, $\mathbf{D}_d(\boldsymbol{\alpha}) = \mathbf{D}_c(\boldsymbol{\alpha})$, $\mathbf{N}_d(\boldsymbol{\alpha}) = \mathbf{N}_c(\boldsymbol{\alpha})$ and

$$\begin{aligned}\mathbf{A}_d(\boldsymbol{\alpha}) &= \mathbf{I}_n + T_s \mathbf{A}_c(\boldsymbol{\alpha}) + 0.5 T_s^2 \mathbf{A}_c^2(\boldsymbol{\alpha}), \\ \mathbf{B}_d(\boldsymbol{\alpha}) &= T_s \mathbf{B}_c(\boldsymbol{\alpha}), \text{ and} \\ \mathbf{L}_d(\boldsymbol{\alpha}) &= T_s \mathbf{L}_c(\boldsymbol{\alpha})\end{aligned}\quad (6)$$

It can be directly evidenced that the discrete-time representation of the initial structural equation, provided by Eq. 5, cannot be directly implemented to the solution of the addressed vibration mitigation problem. This is due to the lack of knowledge of the uncertain parameter vector $\boldsymbol{\alpha}$ and the state vector $\boldsymbol{\xi}[t]$, both of which must be estimated in hard real-time, prior to the control action. To this end, the UKF is used to address the combined state-parameter estimation problem.

3 JOINT STATE AND PARAMETER ESTIMATION

The derivation of the UKF initiates by introducing a fictitious equation for the unknown parameter vector

$$\boldsymbol{\alpha}_{k+1} = \boldsymbol{\alpha}_k + \mathbf{w}_{a,k} \quad (7)$$

with $\mathbf{w}_{a,k}$ denoting a zero mean Gaussian process noise of covariance matrix $\boldsymbol{\Sigma}_{aa}$. Accordingly, an augmented state vector is defined as $\bar{\boldsymbol{\xi}}_k = [\boldsymbol{\xi}_k \ \boldsymbol{\alpha}_k]^T$ and a corresponding state-space model is formulated from Eqs. 5, 7 as

$$\bar{\boldsymbol{\xi}}_{k+1} = \underbrace{\begin{bmatrix} \mathbf{A}_d(\boldsymbol{\alpha}) & \mathbf{O} \\ \mathbf{O} & \mathbf{I} \end{bmatrix}}_{\mathbf{A}(\boldsymbol{\alpha})} \bar{\boldsymbol{\xi}}_k + \underbrace{\begin{bmatrix} \mathbf{B}_d(\boldsymbol{\alpha}) \\ \mathbf{O} \end{bmatrix}}_{\mathbf{B}(\boldsymbol{\alpha})} \mathbf{f}_k + \underbrace{\begin{bmatrix} \mathbf{L}_d(\boldsymbol{\alpha}) \\ \mathbf{O} \end{bmatrix}}_{\mathbf{L}(\boldsymbol{\alpha})} \mathbf{d}_k + \underbrace{\begin{bmatrix} \mathbf{w}_{\xi,k} \\ \mathbf{w}_{a,k} \end{bmatrix}}_{\mathbf{p}_k} \quad (8a)$$

$$\mathbf{y}_k = \underbrace{\begin{bmatrix} \mathbf{C}_d(\boldsymbol{\alpha}) & \mathbf{O} \end{bmatrix}}_{\mathbf{C}(\boldsymbol{\alpha})} \bar{\boldsymbol{\xi}}_k + \underbrace{\mathbf{D}_d(\boldsymbol{\alpha})}_{\mathbf{D}(\boldsymbol{\alpha})} \mathbf{f}_k + \underbrace{\mathbf{N}_d(\boldsymbol{\alpha})}_{\mathbf{N}(\boldsymbol{\alpha})} \mathbf{d}_k + \mathbf{r}_k \quad (8b)$$

In Eq. 8a the quantity $\mathbf{w}_{\xi,k}$ denotes zero mean Gaussian process noise of covariance matrix $\boldsymbol{\Sigma}_{\xi\xi}$, uncorrelated with $\mathbf{w}_{a,k}$, that has been superimposed to Eq. 5a. The process noise \mathbf{p}_k of the augmented state equation is in turn characterized by zero mean and covariance matrix $\boldsymbol{\Sigma}_{pp} = \text{diag}\{\boldsymbol{\Sigma}_{\xi\xi}, \boldsymbol{\Sigma}_{aa}\}$. In Eq. 8b the quantity \mathbf{r}_k denotes zero mean Gaussian measurement noise of covariance matrix $\boldsymbol{\Sigma}_{rr}$.

Observe that, per definition of the augmented state vector, the state matrix of Eq. 8a includes bilinear products between the original state $\boldsymbol{\xi}_k$ and the unknown parameter vector. This results into a nonlinear problem treated through implementation of the UKF, the successive steps of which are indicated in Table 1 [23, 24, 20]. The functions $f(\cdot)$ and $g(\cdot)$, which appear on the table correspond to the state and the output parts of Eq. 8, respectively, that is

$$\bar{\boldsymbol{\xi}}_{k+1} \equiv f(\bar{\boldsymbol{\xi}}_k, \mathbf{f}_k, \mathbf{d}_k) + \mathbf{p}_k \quad (9a)$$

$$\mathbf{y}_k \equiv g(\bar{\boldsymbol{\xi}}_k, \mathbf{f}_k, \mathbf{d}_k) + \mathbf{r}_k \quad (9b)$$

Table 1: The initialization, measurement update and prediction steps of the UKF.

INITIALIZE	<ol style="list-style-type: none"> 1. Set initial values for the augmented state vector: $\bar{\boldsymbol{\xi}}_0$ and \mathbf{P}_0 2. Set UKF parameters (n_ξ the size of the augmented state vector) [25]: <ul style="list-style-type: none"> - $\alpha = 1, \beta = 2, \kappa = 0, c = \alpha^2(n_\xi + \kappa), \lambda = c - n_\xi,$ - $W_m^0 = \lambda/(n_\xi + \lambda), W_m^i = 1/2(n_\xi + \lambda), i = 1, 2, \dots, 2n_\xi$ - $W_c^0 = \lambda/(n_\xi + \lambda) + (1 - \alpha^2 + \beta), W_c^i = W_m^i, i = 1, 2, \dots, 2n_\xi$ - $\boldsymbol{\mu}_\xi = [W_m^0 \ \dots \ W_m^{2n_\xi}]^T$ - $\mathbf{W} = (\mathbf{I} - [\boldsymbol{\mu}_\xi \ \dots \ \boldsymbol{\mu}_\xi]) \times \text{diag}(W_c^0 \ \dots \ W_c^{2n_\xi}) \times (\mathbf{I} - [\boldsymbol{\mu}_\xi \ \dots \ \boldsymbol{\mu}_\xi])^T$
UPDATE	<p>at time k (when $\mathbf{y}_k, \mathbf{f}_k$ and \mathbf{d}_k are available)</p> <ol style="list-style-type: none"> 1. Calculate sigma points: $\bar{\boldsymbol{\xi}}_k^- = [\bar{\boldsymbol{\xi}}_k^- \ \dots \ \bar{\boldsymbol{\xi}}_k^-] + \sqrt{c}[\mathbf{0} \ \sqrt{\mathbf{P}_k^-} \ - \ \sqrt{\mathbf{P}_k^-}]$ 2. Propagate sigma points through the output equation: $\widehat{\mathbf{D}}_k^- = g(\bar{\boldsymbol{\xi}}_k^-, k)$ 3. Calculate output mean and covariance: $\hat{\mathbf{y}}_k = \widehat{\mathbf{D}}_k^- \boldsymbol{\mu}_\xi$ $\mathbf{P}_k^{yy} = \widehat{\mathbf{D}}_k^- \mathbf{W} \widehat{\mathbf{D}}_k^{-T} + \boldsymbol{\Sigma}_{rr}$ 4. Calculate cross covariance between state and output: $\mathbf{P}_k^{\xi y} = \bar{\boldsymbol{\xi}}_k^- \mathbf{W} \widehat{\mathbf{D}}_k^{-T}$ 5. Calculate state gain: $\mathbf{P}_k^{yy} \mathbf{K}_k = \mathbf{P}_k^{\xi y}$ 6. Update state mean and covariance: $\bar{\boldsymbol{\xi}}_k = \bar{\boldsymbol{\xi}}_k^- + \mathbf{K}_k [\mathbf{y}_k - \hat{\mathbf{y}}_k]$ $\mathbf{P}_k = \mathbf{P}_k^- + \mathbf{K}_k \mathbf{P}_k^{yy} \mathbf{K}_k^T$
PREDICT	<p>at time k</p> <ol style="list-style-type: none"> 1. Calculate sigma points: $\bar{\boldsymbol{\xi}}_k = [\bar{\boldsymbol{\xi}}_k \ \dots \ \bar{\boldsymbol{\xi}}_k] + \sqrt{c}[\mathbf{0} \ \sqrt{\mathbf{P}_k} \ - \ \sqrt{\mathbf{P}_k}]$ 2. Propagate sigma points through the state equation: $\widehat{\boldsymbol{\xi}}_k = f(\bar{\boldsymbol{\xi}}_k, k)$ 3. Predict state mean and covariance for $k+1$: $\bar{\boldsymbol{\xi}}_{k+1}^- = \widehat{\boldsymbol{\xi}}_k \boldsymbol{\mu}_\xi$ $\mathbf{P}_{k+1}^- = \widehat{\boldsymbol{\xi}}_k \mathbf{W} \widehat{\boldsymbol{\xi}}_k^T + \boldsymbol{\Sigma}_{pp}$

4 THE CONTROL STRATEGY

The calculation of the control action combines the use of LQR and clipping and it is has been designed for hard real-time execution. In more detail, the conventional LQR proceeds by solving the optimal control problem

$$J = \sum_{k=0}^{\infty} (\boldsymbol{\xi}_k^T \mathbf{Q} \boldsymbol{\xi}_k + \mathbf{f}_k^T \mathbf{R} \mathbf{f}_k), \quad \text{s.t. } \boldsymbol{\xi}_{k+1} = \mathbf{A} \boldsymbol{\xi}_k + \mathbf{B} \mathbf{f}_k, \quad \boldsymbol{\xi}_{k=0} = \boldsymbol{\xi}_0 \quad (10)$$

with \mathbf{Q} and \mathbf{R} designating positive definite weighting parameters [26, 18]. The control force that minimizes J is given by $\mathbf{f}_k = -\mathbf{G} \boldsymbol{\xi}_k$, where

$$(\mathbf{B}^T \mathbf{S} \mathbf{B} + \mathbf{R}) \mathbf{G} = \mathbf{B}^T \mathbf{S} \mathbf{A} \quad (11)$$

and \mathbf{S} corresponds to the solution of the discrete-time algebraic Riccati equation (DARE)

$$\mathbf{S} = \mathbf{A}^T \mathbf{S} \mathbf{A} + \mathbf{A}^T \mathbf{S} \mathbf{B} (\mathbf{B}^T \mathbf{S} \mathbf{B} + \mathbf{R})^{-1} \mathbf{B}^T \mathbf{S} \mathbf{A} + \mathbf{Q} \quad (12)$$

As Eqs. 10-12 imply, the conventional LQR formulation requires knowledge of the state and input matrices, which for the problem addressed herein is not possible, due to their dependence on the unknown parameter vector $\boldsymbol{\alpha}$ (Eq. 5a). In solving this problem, one way is to formulate the state and input matrices using the initial guess $\boldsymbol{\alpha}_0$ [20], which, clearly, is not an optimal approach. An alternative way is to solve the LQR problem in every iteration, as soon as estimates of $\boldsymbol{\alpha}_k$ become available by the UKF. Yet, corresponding non-iterative and numerically efficient algorithms involve the solution of an eigenvalue problem [21], which is prohibitively time-consuming for hard real-time operations.

A third option can be applied, if it is assumed that the structure undergoes small changes from time k to time $k+1$, which in turn implies small variations of the optimal solution of the LQR problem [22]. Then Eqs. 11-12 can be used in a recursive form for propagating the solution of the currently available Riccati matrix \mathbf{S} to the next time instant, namely

$$(\mathbf{B}_k^T \mathbf{S}_{k+1} \mathbf{B}_k + \mathbf{R}) \mathbf{G}_k = \mathbf{B}_k^T \mathbf{S}_{k+1} \mathbf{A}_k \quad (13)$$

for

$$\mathbf{S}_{k+1} = \mathbf{A}_k^T \mathbf{S}_k \mathbf{A}_k + \mathbf{A}_k^T \mathbf{S}_k \mathbf{B}_k (\mathbf{B}_k^T \mathbf{S}_k \mathbf{B}_k + \mathbf{R})^{-1} \mathbf{B}_k^T \mathbf{S}_k \mathbf{A}_k + \mathbf{Q} \quad (14)$$

Taking under consideration the advantages of semi-active control [4, 27, 19], it is herein assumed that the set of available actuators consists of semi-active devices. Thus, the aforementioned real-time LQR strategy must be accordingly adapted, in order to eliminate the active control forces. This is accomplished using clipping, which suppresses all components that are contra-directional to the velocity of the DOF, at which the actuator is located. This implies that the j th entry of the applied control force $\mathbf{f}_{semi,k}$ is calculated as

$$[\mathbf{f}_{semi,k}]_j = \begin{cases} [\mathbf{f}_k]_j & , \quad [\dot{\mathbf{x}}_k]_j [\mathbf{f}_k]_j \geq 0 \\ 0 & , \quad [\dot{\mathbf{x}}_k]_j [\mathbf{f}_k]_j < 0 \end{cases} \quad (15)$$

where $[\mathbf{f}_k]_j$ is the j th entry of the LQR calculated control force \mathbf{f}_k and $[\dot{\mathbf{x}}_k]_j$ the estimated velocity of the j th DOF.

Table 2: Adopted numerical values for the associated parameters of the numerical simulation.

Quantity	Value	Notes
m_i (kg)	[11.615, 10.926, 10.926, 10.926, 10.693]	storey mass
k_i (N)	[9308, 9435, 10677, 10045, 10721]	storey stiffness
f_i (Hz)	[1.32, 3.95, 6.24, 8.07, 9.33]	natural frequencies
ζ_i (%)	[0.83, 0.16, 0.13, 0.12, 0.22]	damping ratios
ξ_0	$\mathbf{0}_{1 \times 10}$	initial value for the state vector of Eq. 5a
$\bar{\xi}_0$	$\mathbf{0}_{1 \times 10}$	initial value for the state vector of Eq. 5a
ξ_0	$\mathbf{0}_{1 \times 10}$	initial value for the state vector of Eq. 5a
ξ_0	$\mathbf{0}_{1 \times 10}$	initial value for the state vector of Eq. 5a
α_0	$[11.5 \ 10.5 \ 10.3 \ 9.8 \ 11.9]^T \times 10^3$	initial value for the parameter vector α
$\bar{\xi}_0$	$[\xi_0, \alpha_0]^T$	initial value for the augmented state vector
$\mathbf{P}_{0,\xi}$	$10^{-15} \times \mathbf{I}_{10}$	initial values for the covariance matrix of ξ_0
$\mathbf{P}_{0,\alpha}$	$\text{diag}(30, 2, 1, 8, 2) \times 10^{-5}$	initial values for the covariance matrix of α_0
\mathbf{P}_0	$\text{diag}(\mathbf{P}_{0,\xi}, \mathbf{P}_{0,\alpha})$	initial values for the covariance matrix of $\bar{\xi}_0$
$\Sigma_{\xi\xi}$	$10^{-12} \times \mathbf{I}_{10}$	process noise covariance matrix associated with ξ
Σ_α	$1.49 \times 10^{-8} \times \mathbf{I}_5$	process noise covariance matrix associated with α
Σ_{pp}	$\text{diag}(\Sigma_{\xi\xi}, \Sigma_\alpha)$	augmented process noise covariance matrix
Σ_{rr}	$\text{diag}(40, 1, 1) \times 10^{-4}$	measurement noise covariance matrix

5 NUMERICAL APPLICATION

The proposed method is now applied and assessed on the simulation of a five-story shear structure that is amenable to earthquake-induced vibration and described by

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}(\alpha)\mathbf{u}(t) = \mathbf{P}_1\mathbf{f}(t) - \mathbf{M}\mathbf{1}\ddot{x}_g(t) \quad (16)$$

where $\mathbf{u}(t)$ corresponds to the DOFs' displacements relative to the base, $\ddot{x}_g(t)$ to the earthquake acceleration and $\mathbf{1} = [1, 1, 1, 1, 1]^T$. The structural and modal properties of the frame are shown in Tab. 1. The unknown parameter vector is selected to be the stiffness of every story and hence $\alpha = [k_1, k_2, k_3, k_4, k_5]^T$. A resonating harmonic earthquake of the form $\ddot{x}_g(t) = -0.2751 \sin(2\pi f_1 t)$ is selected as the disturbance source, where $f_1 = 1.32$ Hz corresponds to the first natural frequency of the frame (Tab. 1). A single actuator is assumed that is connected to the first story, thus $\mathbf{P}_1 = [1/m_1, 0, 0, 0, 0]^T$ and noise-corrupted vibration acceleration measurements from the first, the second and the fifth story are available. Under this setting, a 30s simulation is performed at a sampling period $T_s = 0.001$ s, using the numerical values of Tab. 2.

The results displayed in Figs. 1–4 confirm the effectiveness of the proposed approach. The controlled states have resulted on order of magnitude suppressed, in respect to the uncontrolled ones, while the presence of control force has introduced a slight phase shift. This is apparent in the 2s simulation window of Figs. 1–2 (right part). The robustness of the UKF is further validated by the performance of the

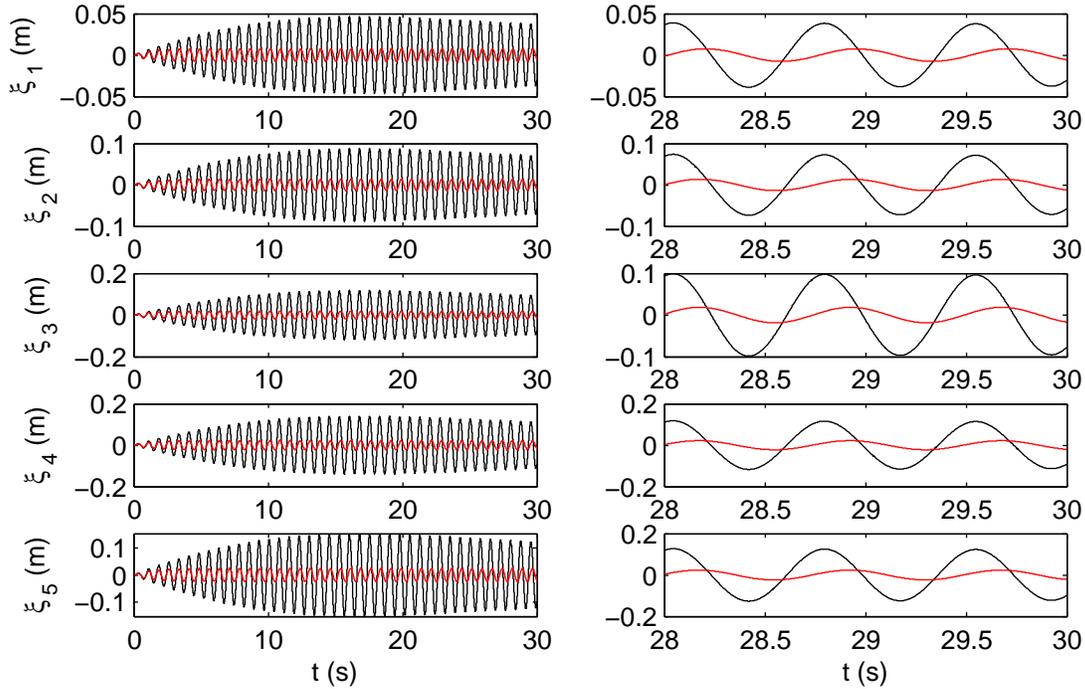


Figure 1: Uncontrolled (black) and controlled (red) relative displacements.

stiffness estimates, displayed in Fig. 3; all quantities have converged to the true values within the first 5s, except from the fifth one, which exhibits a slow convergence rate. However, this does not affect the accurate estimation of the states. In regards to the performance of the real-time calculation of the control gain, it is reflected to the behavior of the force-to-displacement and force-to-velocity curves displayed in Fig. 4, with clipping being more obvious in the latter curve.

6 CONCLUSIONS

A novel semi-active vibration mitigation method for structural systems with uncertainties is presented in this paper. Taking into account the partial knowledge of the structural dynamics, as well as the limited sensor information, the proposed scheme couples UKF, LQR and clipping, and requires considerably lower cost and specifications in comparison to active solutions. The illustrated simulation of the numerical shear frame under harmonic disturbance confirmed stability of the algorithm, especially in regards to the stable calculation of the states and the LQR gain matrix in every time step. The encouraging results suggest further investigation towards this direction. These include an in-depth investigation of the online calculation of the LQR gain, the examination of disturbances with diverse statistical characterization, as

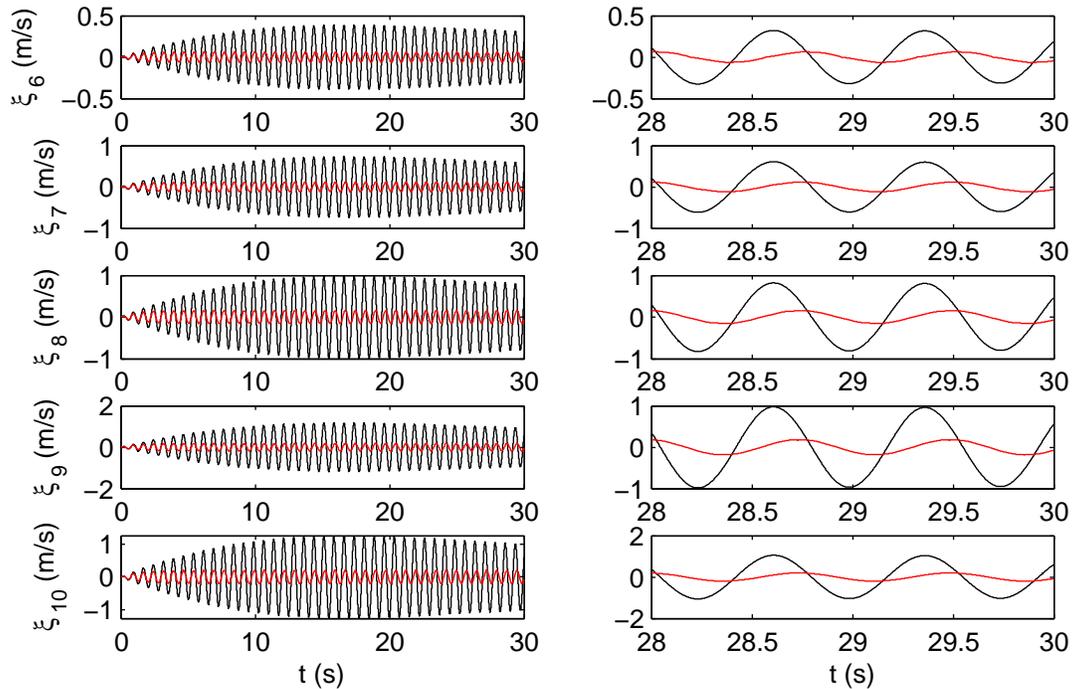


Figure 2: Uncontrolled (black) and controlled (red) relative velocities.

well as the experimental verification of the proposed methodology in structural control applications.

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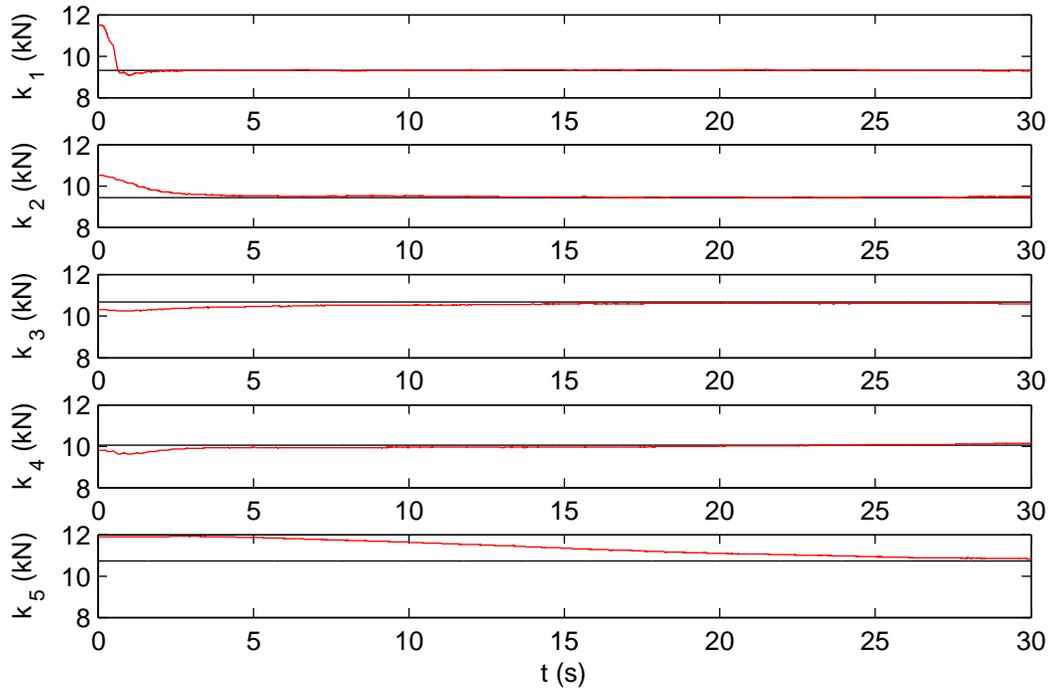


Figure 3: Actual (black) and identified (red) storey stiffness.

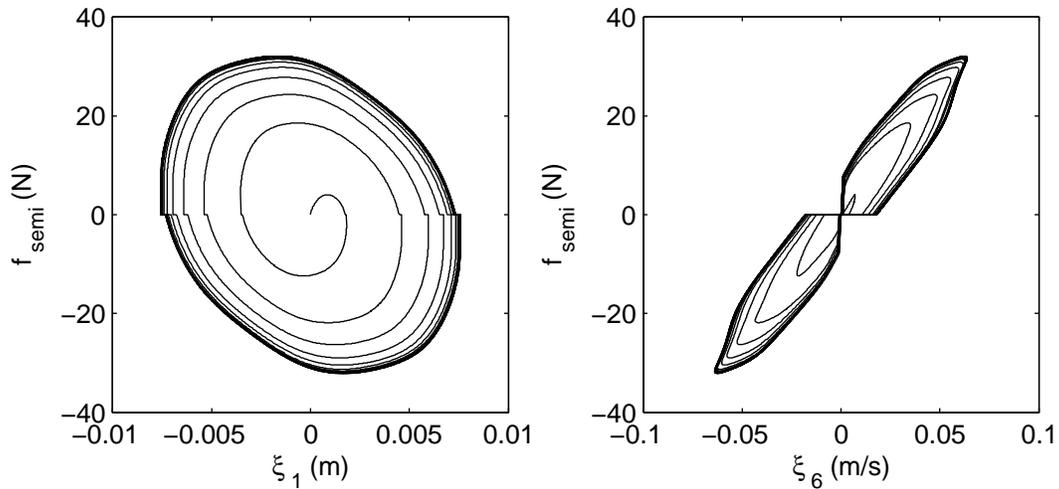


Figure 4: Force-to-displacement (left) and force-to-velocity (right) curves.

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