

## Simulation of the response of a lively footbridge under pedestrian loading with two tuned mass dampers for its two first modes (2.1Hz and 2.5Hz)

Norberto Ibán<sup>1</sup>, Javier Castaño<sup>1</sup>, Álvaro Magdaleno<sup>2</sup>, Mariano Cacho<sup>2</sup>, Alberto Fraile<sup>3</sup>,  
Antolín Lorenzana<sup>\*2</sup>

<sup>1</sup>Fundación CARTIF, Parque Tecnológico de Boecillo, Boecillo (Valladolid), Spain

<sup>2</sup>ITAP. EII. Universidad de Valladolid, Valladolid, Spain

<sup>3</sup>UPM, Escuela Técnica Superior de Ingenieros Industriales, Madrid, Spain

### ABSTRACT

Structures subjected to excitations like human induced vibrations may produce large accelerations and serviceability limit state problems. Passive, semi-active and active vibration controls have been proposed as possible solutions to reduce the vibration level at civil structures such as bridges, multi-storey buildings or slender floor structures, among others [1]. It is known that Tuned Mass Dampers (TMD) mitigates the vibration response of a structure by increasing its damping through the application of inertial forces generated in response to the movement of the structure [2]. Recently, different TMD implementations have been proposed in order to improve the tuning of mechanical parameters. In the case of structures with spatially distributed and closely spaced natural frequencies, the TMD design may not be obvious because Den Hartog's theory [3] may not be applied due to the existence of a coupling between the motions of the vibration modes of the structures and the used TMD's [4]. Alternative design techniques are applied for the case under study consisting on an arched bridge with a main span 40m long and several shorter access spans. The first two first modes are at 2.1Hz and 2.5Hz, both in the range prone to be excited by walking. Also the third one (at 3.18Hz) could be excited by runners.

For the simulation, firstly, a finite element model of the bridge is created in a commercial CAE software and static and modal response is numerically estimated. Then, experimental measurements using static loading test and ambient vibration tests are performed. Initial finite element model is adjusted to match with the static response by fitting some selected parameters. Modal parameters (natural frequencies, mode shapes and modal damping) are extracted and after that the current finite element model is updated. Once the numerical model is calibrated, TMDs are attached. The problem of finding the optimal location and tuning is not a simple one. For understanding the coupled response, several simulations are carried out, from the logical one (TMD located just in the middle of the main span and tuned at 2.1Hz) to others. The responses of the footbridge for different scenarios (depending on the number of TMDs installed and their position) are compared in order to extract some interesting conclusions.

**Keywords:** *Structural control, Passive vibration control, tuned mass damper, optimal control.*

### 1 INTRODUCTION

Although in the past, civil engineering sector made extensive use of approximate models to estimate the dynamic response of bridge type structures, nowadays is usual to model the structure using current CAE abilities. Simple discrete models have proved insufficient for the accurate modelling of slender footbridge structures as they cannot represent some effects as the closely spaced modes of vibration which frequently occur in practice. Additionally, modern footbridges become increasingly slender and prone to oscillate under pedestrian loading, so there is a much

greater need for vibrations to be considered at the design stage. Having the FE methods the capability for the accurate modelling of the dynamic behavior, and becoming CAE software more affordable, civil engineering practitioners do not hesitate in their use. However, with regard to lively structural design, there is a lack of expertise in FE modelling, particularly with regard to their vibration serviceability performance, being not rare that the model does not match with the real structure. The way forward for developing such expertise is by linking modal testing and FE analysis by the updating of the models of representative structures and extract general design guidelines. This type of approach is the usual in, for example, the aerospace engineering sectors [5, 6], but it is only recently that the civil engineering community has begun to adopt this advanced technology [7, 9].

The aim of the paper is to describe a procedure for the use of updated FE models with TMDs attached to estimate the response in terms of accelerations and evaluate the serviceability of the assembly.

## 2 STRUCTURE DESCRIPTION AND F.E. MODELING

The footbridge under study is an urban link with several minor access spans and one main 40m long arched central lively span. Most of the structural members are constructed using tubular steel profiles. An aerial photograph of the footbridge and 3D isometric view of its FE model is depicted in figure 1. More information about the structure can be found in [10]. Updated mode-shapes 1 and 2 are shown in figure 2. The structural damping (Rayleigh type) was set to 0.32%.



Figure 1. Footbridge under study: photograph and numerical FE model.

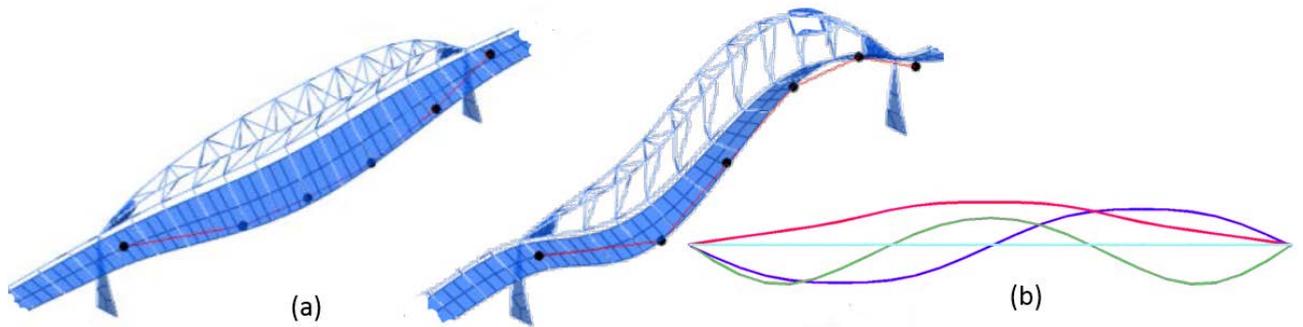


Figure 2. 3D F.E. model mode-shapes

Although it would be possible to applied the methodology using the 3D F.E. model, in order to make more efficient simulations, a less time consuming F.E. model has been created. In this case, just a 2D equivalent structure meshed in 12 elements and modeled using Euler beam elements has been used. Both models exhibit similar vertical modes (see figure 2.b), which are the interesting ones for the problem under study (pedestrian loading vertical response). Being  $L$  the length of the footbridge, note that the second mode-shape has a node at  $6L/12$  and the nodes of the third mode-shapes are located at  $4L/12$  and  $8L/12$ . In the next section, the dynamic problem is establish using the state space approach.

### 3 SPACE STATE MODELING

The dynamic equation to be solved is:

$$\mathbf{M} \ddot{\mathbf{q}} + \mathbf{C} \dot{\mathbf{q}} + \mathbf{K} \mathbf{q} = \mathbf{F}$$

Where  $\mathbf{M}$  is the mass matrix and  $\mathbf{K}$  is the stiffness matrix.  $\mathbf{C}$  is the damping matrix, evaluated through modal damping values  $\xi_i$  according to the following equations

$$\mathbf{V} = [\phi_1 | \phi_2 | \dots | \phi_n]$$

$$\tilde{\mathbf{C}} = \mathbf{V}^T \mathbf{C} \mathbf{V} \rightarrow \mathbf{C} = (\mathbf{V}^T)^{-1} \tilde{\mathbf{C}} \mathbf{V}^{-1} = (\mathbf{V} \tilde{\mathbf{C}}^{-1} \mathbf{V}^T)^{-1}$$

$$\tilde{\mathbf{C}} = \text{diag}(2 \xi_i \omega_i)$$

Only bending dof are considered (vertical deflection and angle, named generalized displacements):

$$\mathbf{q} = [v_1 \ \theta_1 \ v_2 \ \theta_2 \ v_3 \ \theta_3 \ \dots \ v_n \ \theta_n]^T$$

and standard procedures for meshing and assembling must be applied to all the matrices. Also boundary conditions (simple supported beam) must be included in the former formulation.

$\mathbf{F}$  is the input force, affecting to the dof considered.  $\mathbf{u}(t)$  is the harmonic function and  $\mathcal{F}$  is the vector containing the amplitudes of that force in the corresponding dof.

$$\mathbf{F} = \mathcal{F} \mathbf{u}(t)$$

Any TMD means an additional dof to be added in the matrix formulation. For that, when  $m$  TMDs are considered the displacement vector  $\mathbf{q}$  changes into  $\mathbf{q}'$ :

$$\mathbf{q}' = [\mathbf{q}^T | w_1 | w_2 | \dots | w_m]^T$$

And for any TMD the corresponding matrices to be assembled are:

$$\mathbf{M}_t = \begin{bmatrix} 0 & 0 \\ 0 & m_t \end{bmatrix} \quad \mathbf{C}_t = \begin{bmatrix} c_t & -c_t \\ -c_t & c_t \end{bmatrix} \quad \mathbf{K}_t = \begin{bmatrix} k_t & -k_t \\ -k_t & k_t \end{bmatrix}$$

Where  $m_t$ ,  $c_t$  and  $k_t$  are the moving mass of the TMD, its stiffness and its damping.

Considering as input the harmonic function ( $\mathbf{u}(t)$ ) and as output just the acceleration ( $\mathbf{y}$ ) in one selected dof, the dynamic equation of motion can be rewritten as:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u} \\ \mathbf{y} &= \mathbf{C} \mathbf{x} + \mathbf{D} \mathbf{u} \end{aligned}$$

Where  $\mathbf{x}$  is the state vector defined as

$$\mathbf{x} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}$$

$\mathbf{y}$  is the output vector  $\mathbf{y} = \ddot{\mathbf{q}}_i$ ,  $\mathbf{u}$  is the input  $\mathbf{u} = \mathbf{u}(t)$  and the new space state matrices are:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{n \times n} & \mathbf{I}_{n \times n} \\ -\mathbf{M}^{-1} \mathbf{K} & -\mathbf{M}^{-1} \mathbf{C} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} \mathbf{0}_{n \times 1} \\ \mathbf{M}^{-1} \mathcal{F} \end{bmatrix} \quad \mathbf{C} = \mathbf{A}(n+i, \text{all}) \quad \mathbf{D} = \mathbf{B}(n+i)$$

Where  $\mathbf{0}_{n \times n}$  is the square zero matrix  $\mathbf{I}_{n \times n}$  is the identity matrix,  $\mathbf{C} = \mathbf{A}(n+i, \text{all})$  is the row of the  $\mathbf{A}$  matrix with the acceleration of the  $i$ -th dof, and  $\mathbf{D} = \mathbf{B}(n+i)$  is the  $i$ -th element of the vector  $\mathbf{B}$ .

Once all the system is established, the transfer function is defined as

$$G_i(s) = \frac{\ddot{Q}_i(s)}{U(s)} \quad \ddot{Q}_i(s) = \mathcal{L}\{\ddot{q}_i(t)\} \quad U(s) = \mathcal{L}\{u(t)\}$$

where  $\mathcal{L}\{\}$  is the Laplace operator. After some manipulations the former equations become

$$G_i(s) = \frac{\ddot{Q}_i(s)}{U(s)} = \frac{Y(s)}{U(s)} = \mathbf{C} (s \mathbf{I}_{2n \times 2n} - \mathbf{A})^{-1} \mathbf{B} + \mathbf{D}$$

where  $\mathbf{I}_{2n \times 2n}$  is the identity matrix. The frequency response function (FRF) to be used in the following parts is just the magnitude of the Bode diagram of this transfer function, with is evaluated in Matlab using the following standard commands:

$\text{SYS} = \text{ss}(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}); \quad \mathbf{G} = \text{tf}(\text{SYS}); \quad \text{MAG} = \text{bode}(\mathbf{G}, \mathbf{W}); \quad \text{semilogy}(\mathbf{W}, \text{squeeze}(\text{MAG}))$
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### 4 FREQUENCY RESPONSE WITH TMDs AND SERVICEABILITY ESTIMATION

For all the studied cases, a tonne inertial moving mass is added (being the 2% of the total mass of the simplified model of the footbridge) in one or two TMDs (500kg each). The TMD may be tuned to mode 1 ( $w_1$ ) or mode 2 ( $w_2$ ) and located in some point ( $q$ ) along the beam. To

simplified the continuous problem, only 11 discrete locations are considered (as the structure is discretized in 12 beam elements) so point  $q$  is at  $jL/12$  from the left end of the structure. The notation will be  $\mathbf{TMDw}_i_{jL/12}$  which means that the TMD is tuned for the mode  $i$  ( $i=1,2$ ) and located in  $jL/12$  ( $j=1..11$ ).

All numerical values for the estimation of the response are obtained from the FRFs. In all the cases, the excitation point (force) and the response (acceleration) will be the same ( $p$ ) and the notation will be  $\mathbf{FRF}(p, \mathbf{TMDw}_i_{jL/12})$ . Also note that only the selected 11 discretized points  $p$  are going to be considered. In this way, the notation  $\mathbf{FRF}(3L/12, \mathbf{TMDw}_1_{5L/12})$  stands for the amplification factors in the point  $3L/12$  when the force is acting in  $3L/12$  and the TMD is located at  $5L/12$  and tuned for mode 1.

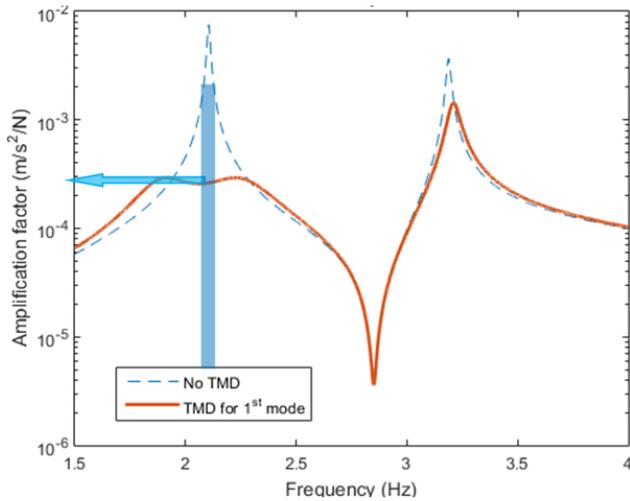


Figure 3.  $\mathbf{FRF}(6L/12, \mathbf{TMDw}_1_{6L/12})$

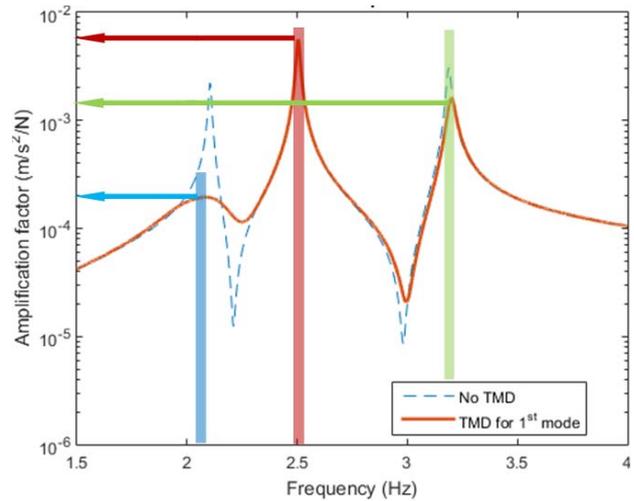


Figure 4.  $\mathbf{FRF}(3L/12, \mathbf{TMDw}_1_{6L/12})$

#### 4.1 TMD tuned at mode 1

To start with, a logical scenario consisting on one TMD tuned in the center of the structure and tuned to its first mode (2.044Hz,  $\xi=0.134$ ) is considered ( $\mathbf{TMDw}_1_{6L/12}$ ). The figure 3 shows the  $\mathbf{FRF}(6L/12, \mathbf{TMDw}_1_{6L/12})$ . It is noteworthy that the first peak has been flattened and the peak at  $w_3$  is almost not affected. In order to see the response for mode 2, a FRF out of  $6L/12$  must be evaluated as that position is a node of mode 2. Figure 4 shows  $\mathbf{FRF}(3L/12, \mathbf{TMDw}_1_{6L/12})$ , revealing, as expected, that the  $\mathbf{TMDw}_1_{6L/12}$  does not affect the response in mode 2.

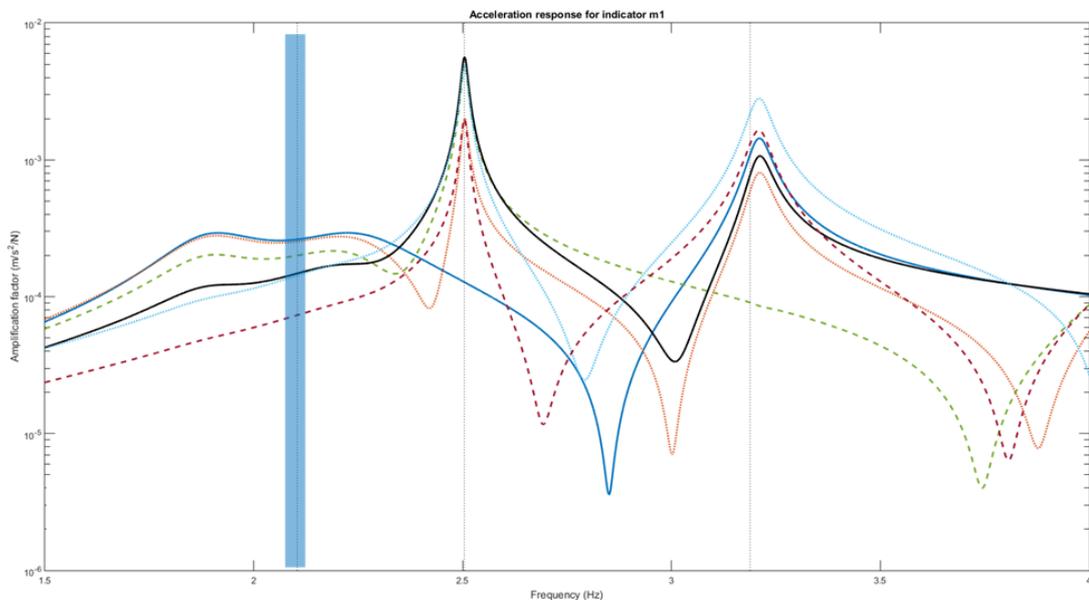


Figure 5.  $\mathbf{FRF}(iL/12, \mathbf{TMDw}_1_{6L/12})$ ,  $i=1, 2, 3, 4, 5$  and  $6$

If installing a TMDw1 is under consideration is due to the possibility of large vibrations when a pedestrian is crossing at a pace coincident with  $w_1$ . Although the pedestrian crossing is a transient problem, in order to estimate the vibration dose the auto FRFs at  $iL/12$  ( $i=1..11$ ) are going to be used. Note that during the crossing, the pedestrian is the one that excites the structure and perceives its vibration. Thus, at the middles ( $6L/12$ ) the amplification factor is 0.00029 (figure 3) and at  $3L/12$  it decreased to 0.00015 (blue arrow in figure 4). Following this logic, figure 5 shows how the amplification factor is increasing as the pedestrian approaches the center. Figure 6 shows a detail for frequencies around  $w_1$  and in Table 1 are the corresponding values. Table 1 also shows the values for the cases in which pedestrian pace is  $w_2$  and  $w_3$  (red and green arrows in figure 4). Note that these first 3 modes are prone to be excited by walking or running.

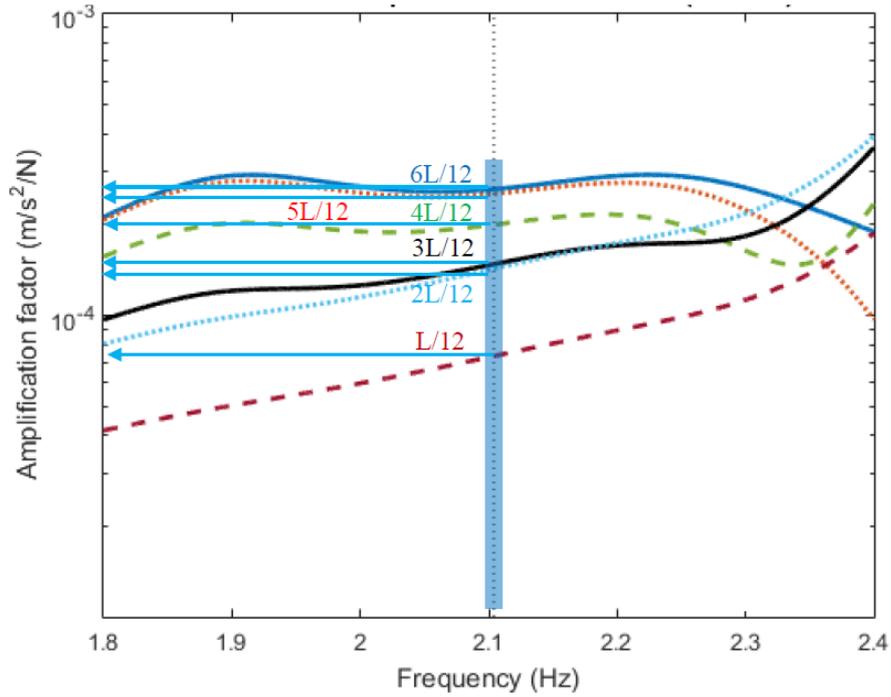


Figure 6. Detail for the FRF( $iL/12$ , TMDw1\_6L/12),  $i=1, 2, 3, 4, 5$  and 6

In addition Table 1 shows the cumulatively amplification factors for the crossing at  $w_1$ ,  $w_2$  and  $w_3$  pace. For a clearer display, information in Table 1 is shown graphically in figure 7. It can be seen how, in comparison with figure 8 (response without TMD), the installed TMDw1\_6L/12 is very effective for crossing at  $w_1$  and also affects significantly the response for crossing at  $w_3$  pace.

Table 1, Amplification factors for TMDw1\_ $jL/12$

TMDw1_	w1	w2	w3
L/12	0.00007	0.00198	0.00165
2L/12	0.00014	0.00493	0.00281
3L/12	0.00015	0.00563	0.00107
4L/12	0.00022	0.00495	0.00000
5L/12	0.00028	0.00201	0.00081
6L/12	0.00029	0.00000	0.00143
7L/12	0.00028	0.00201	0.00081
8L/12	0.00022	0.00495	0.00000
9L/12	0.00015	0.00563	0.00107
10L/12	0.00014	0.00493	0.00281
11L/12	0.00007	0.00198	0.00165
$\Sigma$	0.00199	0.03900	0.01411

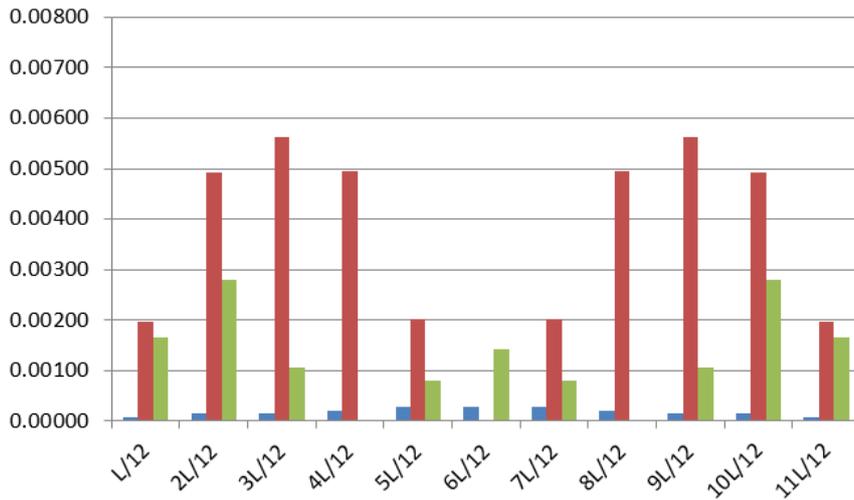


Figure 7. Amplification factors for TMDw1\_jL/12

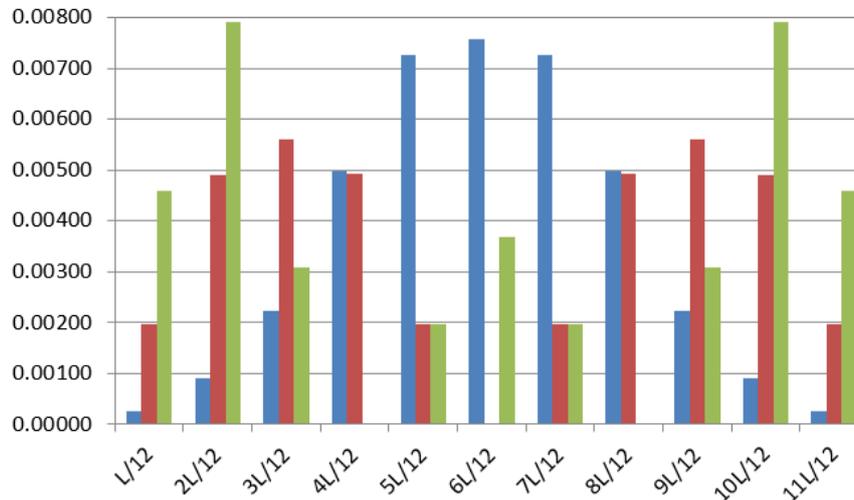


Figure 8. Amplification factors without TMD

It would be worthy to wonder about the response when TMDw1 were located at  $jL/12$  for  $j \neq 6$  (and tuned properly). Figure 9 shows the response for the case of  $j=3$ , resulting to be the optimum location for crossings at  $w_1$ ,  $m_2$  and  $m_3$  (averaged values assuming same number of crossings at each pace) as shown in figure 10.

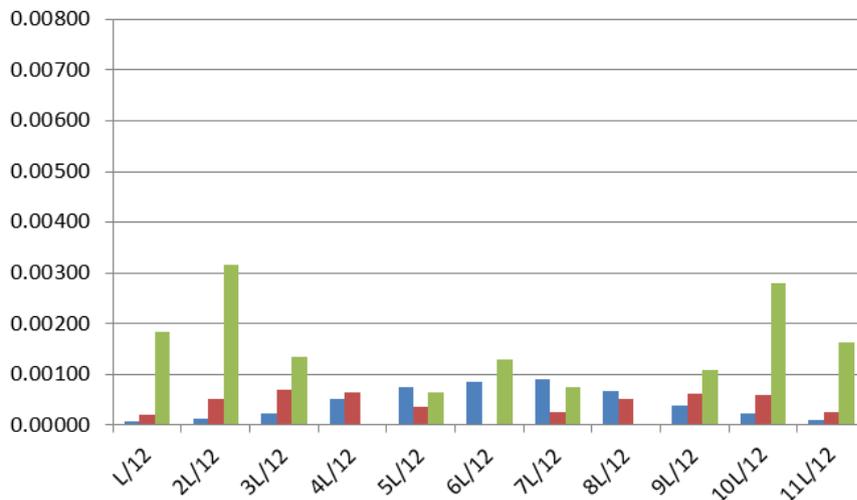


Figure 9. Amplification factors for TMDw1\_3L/12

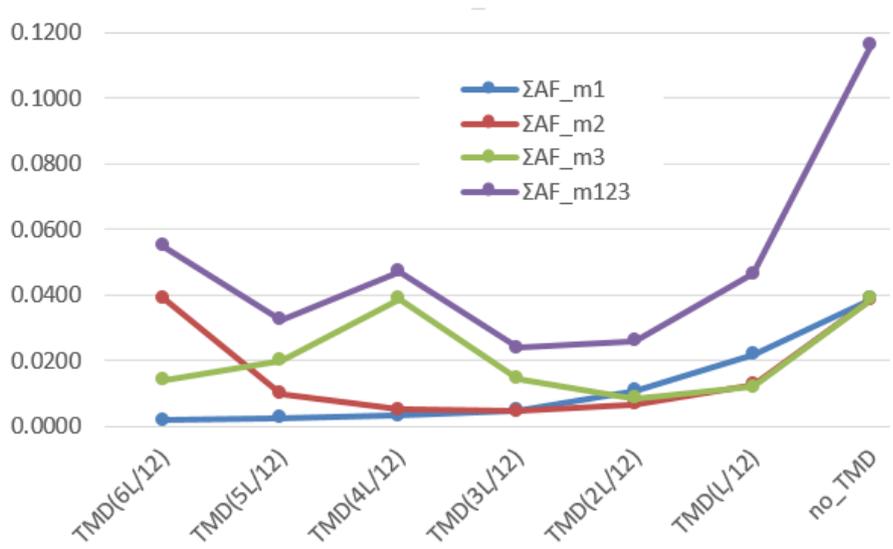


Figure 10. Cumulative amplification factors for crossing at  $w_1$ ,  $w_2$  and  $w_3$

#### 4.2 TMD tuned at mode 2

Now the problem is where to install a TMD tuned to mode 2 (TMDw2). When it is located at 3L/12 (TMDw2\_3L/12) and optimally tuned (2.4818Hz,  $\xi=0.10$ ) the response in 3L/12 is shown in figure 11

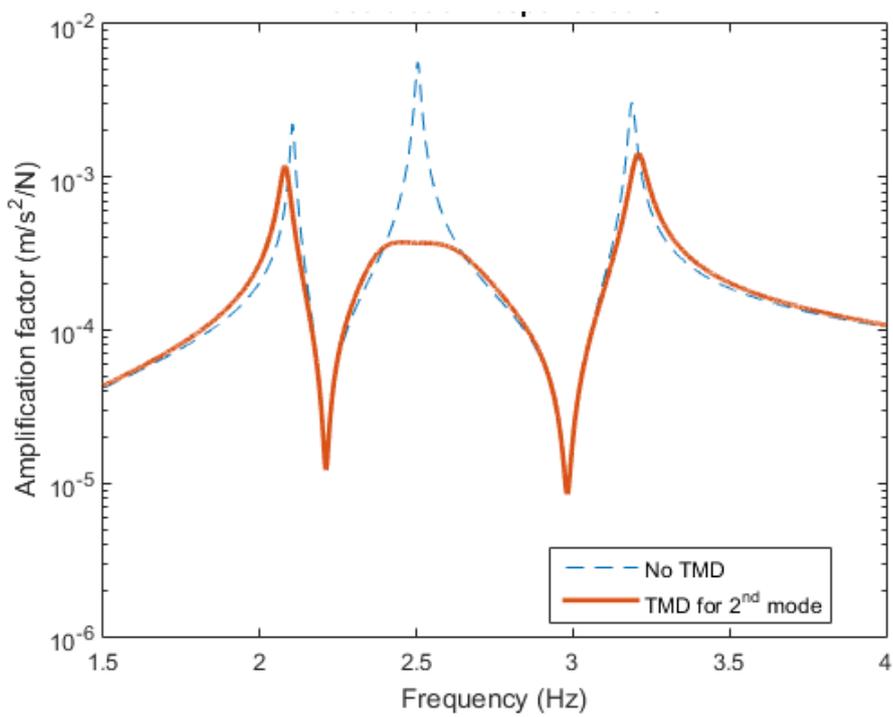


Figure 11. FRF(3L/12, TMDw2\_3L/12)

For comparison reasons, figure 12 shows FRF(TMDw1\_6L/12) and FRF(TMDw2\_6L/12) (figure 12.a) and also FRF(TMDw1\_3L/12) and FRF(TMDw2\_3L/12) (figure 12.b)

Similar to the previous case, the amplification factors in point 3L/12 are shown in figure 13. In this case, when the location of the TMDw2 is changed the results are shown in figure 14 revealing that if only crossings at  $w_2$  pace are considered, the best position is the logical one (3L/12) but for a cumulative response it is better to locate the TMDw2 at 2L/12

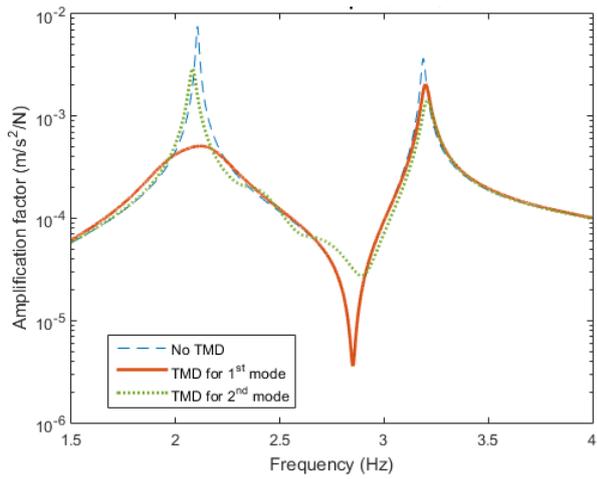


Figure 12.a.  $FRF(TMDw1_{6L/12})$  and  $FRF(TMDw2_{6L/12})$

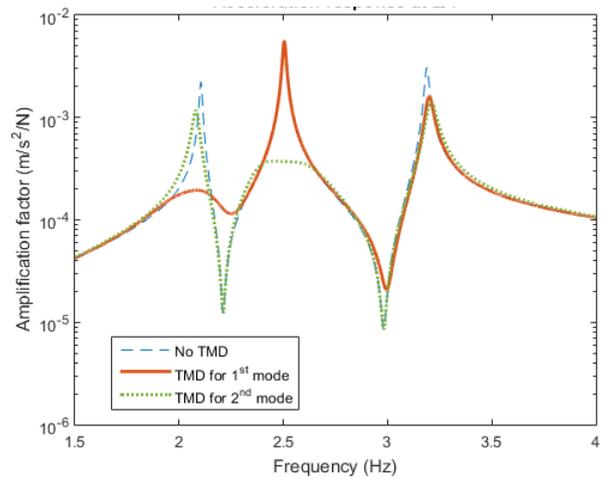


Figure 12.b.  $FRF(TMDw1_{3L/12})$  and  $FRF(TMDw2_{3L/12})$

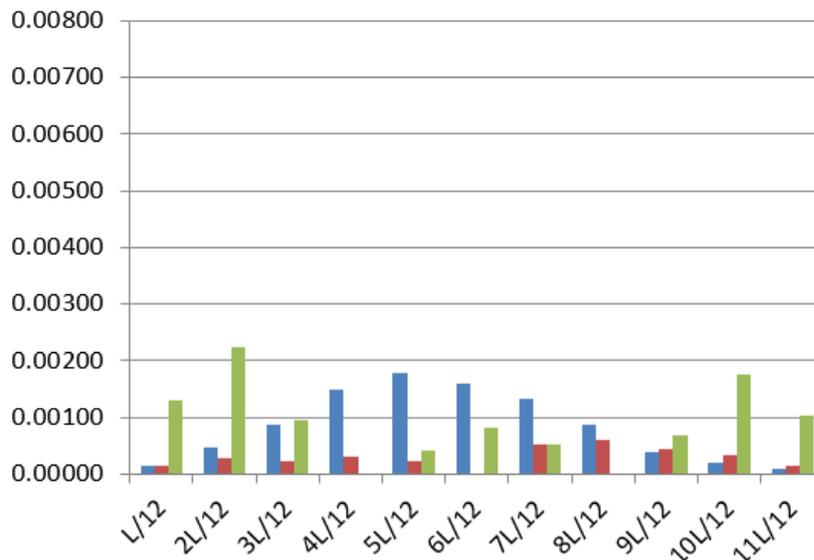


Figure 13. Amplification factors for  $TMDw2_{3L/12}$

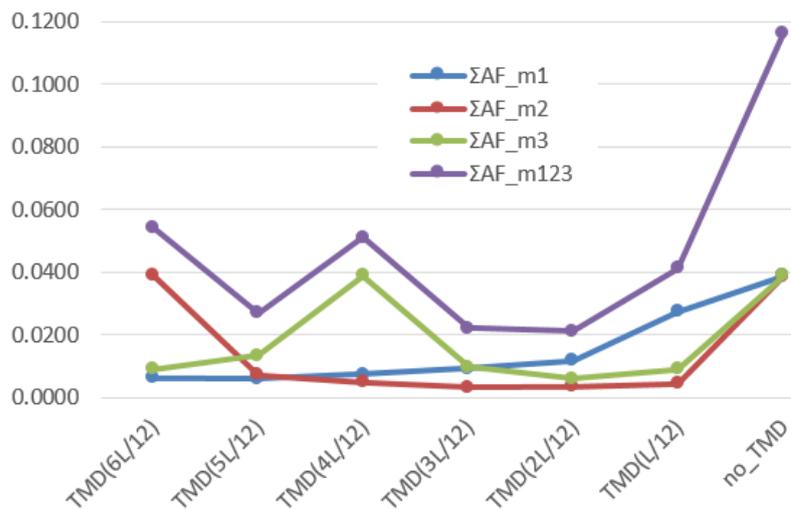


Figure 14 Cumulative amplification factors for crossing at  $w1$ ,  $w2$  and  $w3$

### 4.3 Two TMDs, tuned at mode 1 and mode 2

Another scenario, now with 2 TMDs, is considered. In this case, one TMDw1 with a moving mass of 500kg is located at  $6L/12$  and another one (TMDw2), same mass, located at  $3L/12$ . Both are optimally tuned (2.073Hz,  $\xi=0.10$  for TMDw1 and 2.496Hz,  $\xi=0.08$  for TMDw2). Corresponding FRF( $3L/12$ , TMDw1/TMDw2) is shown in figure 15 and the corresponding amplification factors in figure 16. Cumulative amplification factors for crossings at  $w_1$ ,  $w_2$  and  $w_3$  are 0.00267, 0.00382 y 0.0140, respectively, with an average value of 0.00682.

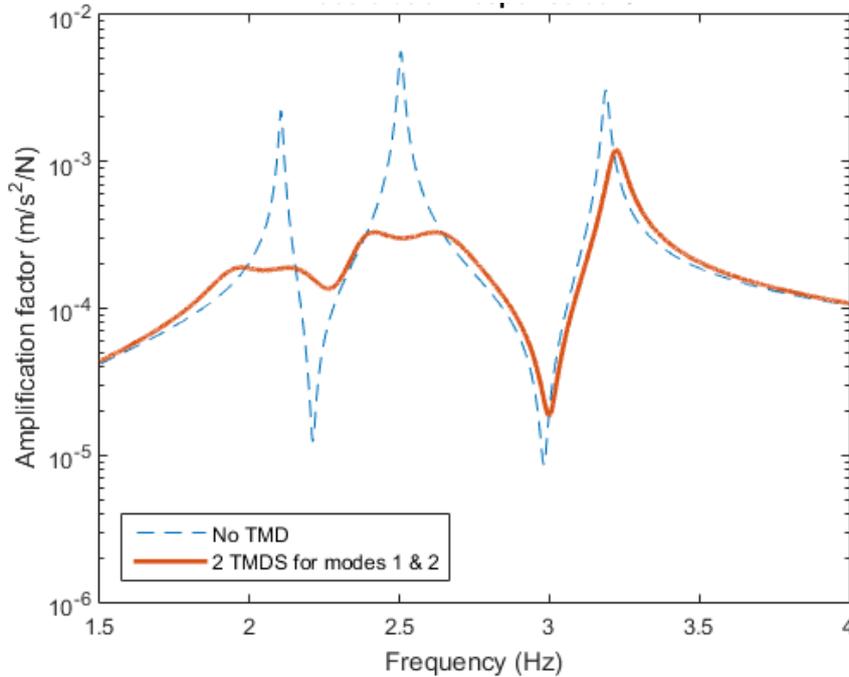


Figure 15. FRF( $3L/12$ , TMDw1\_6L/12/TMDw2\_3L/12)

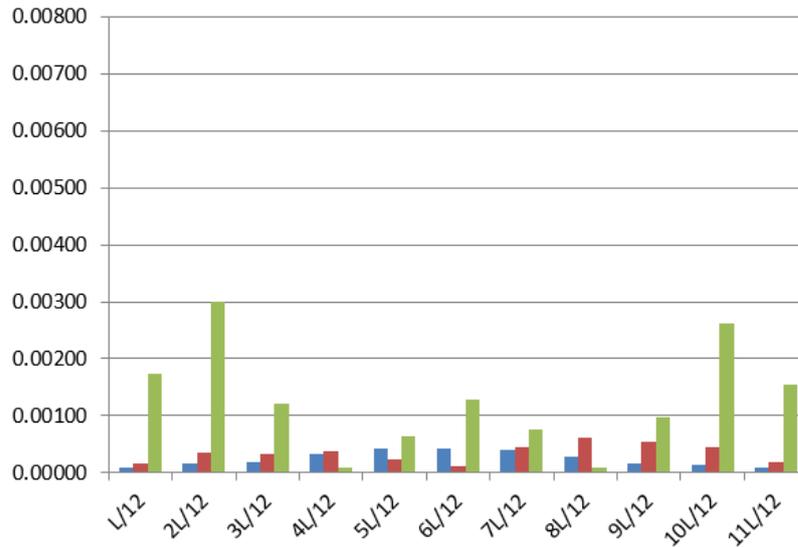


Figure 16. Amplification factors for TMDw1\_6L/12/TMDw2\_3L/12

Change the locations of these TMDs will be an interesting problem, as some reduction is expected. Nevertheless, for the discrete problem (only 11 possible locations) there are 55 events and a great computational effort should be paid in order to find the optimal location for TMDw1 and TMDw2 together.

## 5 CONCLUSIONS

According to former sections, some conclusions can be drawn:

- If only crossings at  $w_1$  pace are expected, the best TMD is TMDw1\_6L/12 (lower point of the red line in figure 17). When it is located at  $jL/12$  ( $j \neq 6$ ), the cumulative amplification factor is bigger although the TMD is optimally re-tuned. Good reductions are also expected when installing the TMD tuned to mode 2 (TMDw2, green line). Blue line is the cumulative amplification factor for the case TMDw1\_6L/12 and TMDw2\_3L/12, which is also a very good solution.

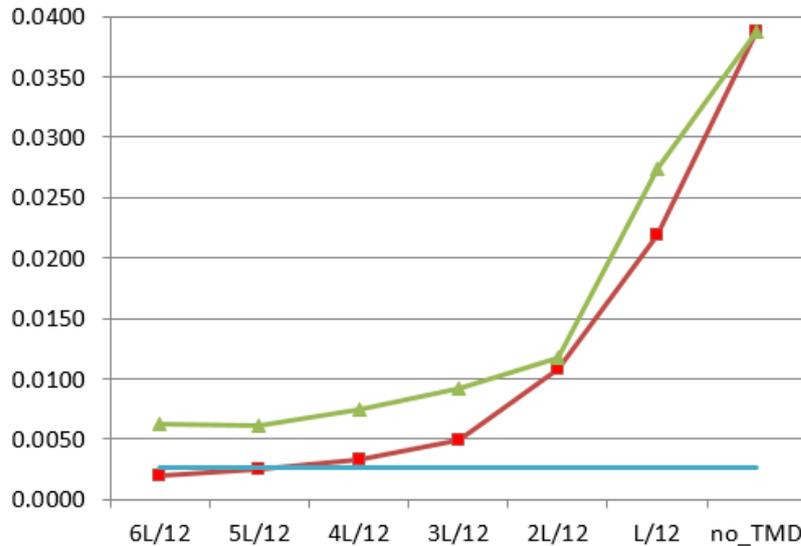


Figure 17. Walking at  $w_1$  pace.

- When crossing at  $w_2$  pace the best response is obtained for TMDw2\_3L/12 (green line in figure 18). It is also effective to instal TMDw1\_3L/12 (red line). The solution with 2 TMDs is also a good one.

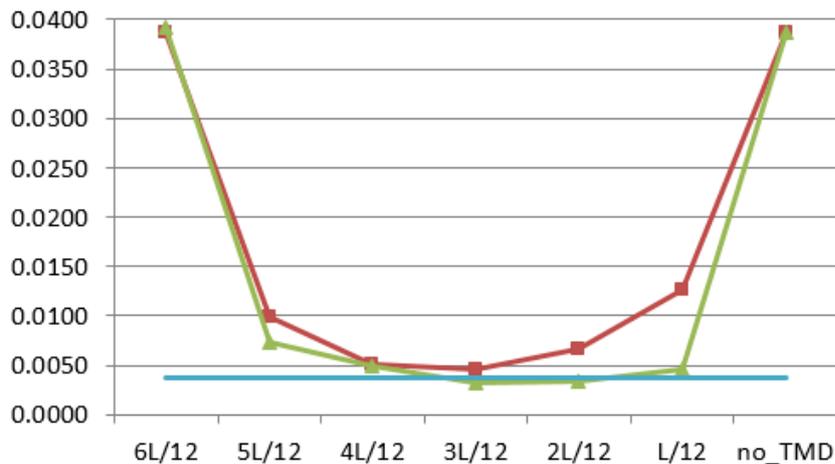


Figure 18. Walking at  $w_2$  pace.

- When crossing at  $w_3$  pace (figura 19), it is better to use TMDw2\_2L/12 (green line) although it is also effective TMDw1\_2L/12. The proposed solution with 2 TMDs is worst.

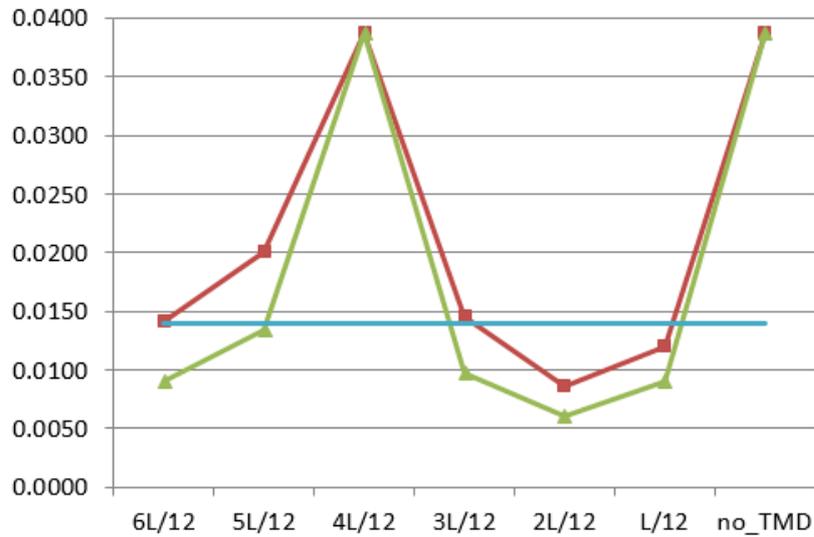


Figure 19. Walking at w3 pace.

- For evenly number of crossings at w1, w2 and w3 pace the best solution is to install TMDw2\_2L/12 (figure 20, green line), with is equivalen to the solution with 2 TMDs (blue line)

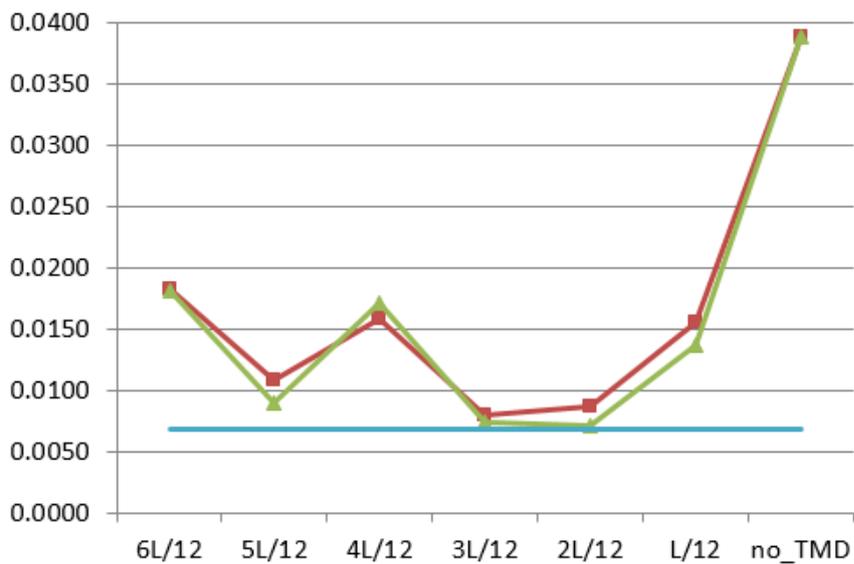


Figure 20. Average values (walking at w1, w2 and w3 pace).

There are a lot of possibilities for a single TMD (other locations or TMDw3) and for two TMDs (TMDw1\_TMDw2 for other locations than 6L/12 and 3L/12, or TMDw1\_TMDw3, or TMDw2\_TMDw3) or even for three TMDs (figure 21). The large number of alternatives leads to a optimization problem [11] to be studied in future coming studies.

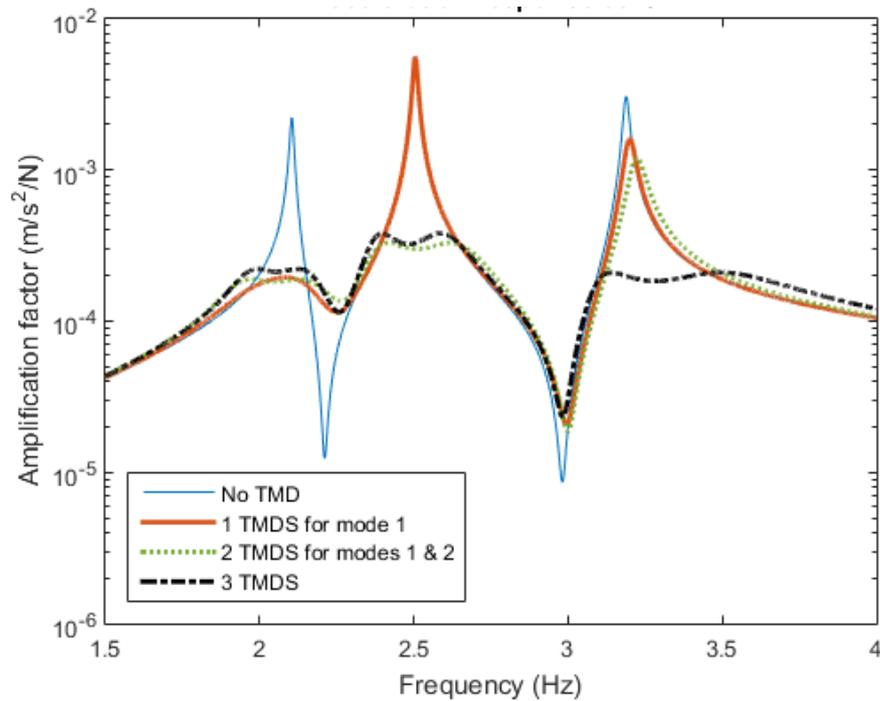


Figure 21:  $FRF(3L/12, TMDw1_{6L/12} \& TMDw2_{3L/12} \& TMDw4_{2L/12})$

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