

Unscented Kalman filter for simultaneous identification of structural parameters and unknown excitations of a building structure

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ABSTRACT.

Identification of structural parameters under unknown excitations is an important task in structural health monitoring. This paper presents an efficient algorithm for simultaneously identifying structural parameters of a building structure and unknown excitations based on unscented Kalman filter (UKF). To utilize the UKF method directly, the observation equation of the structural system with unknown excitations is firstly transformed from a multiple linear regression equation to a simple linear regression equation by projecting on to the column space of influence matrix of unknown excitations. Then, an analytical recursive UKF algorithm is developed for identifying unknown excitations and structural parameters such as stiffness and damping. The feasibility and accuracy of the proposed algorithm are finally demonstrated in terms of an example shear building, in which the measurement noise is included. The results clearly exhibit that the proposed algorithm can simultaneously identify unknown excitations and structural parameters efficiently and accurately.

Keywords: Structural parameters identification; unknown excitations; simultaneous identification; unscented Kalman filter

1 INTRODUCTION

The simultaneous identification of structural parameters and unknown excitations is one of the most important tasks of structural health monitoring. A great deal of studies has been conducted for structural parameter identification in the last few decades, including the methods of Hilbert transform-based methods [1-3], wavelet transform-based methods [4-6], neural network-based methods [7] and adaptive tracking-based methods [8-10].

Most of these methods mentioned above require that all the external excitations be known or measured. However, in view of practice, it is difficult to measure all external excitations accurately because of the measurement technique and cost. To identify the structural parameters without information of external excitations, several studies have been performed in the time domain, such as statistical average algorithm[11] and random decrement technique[12]. Nevertheless, these time domain techniques have special requirements such as zero mean input excitations or free-decay responses. To abolish these restrictions on input and output, numerical recursive methods based on least-squares estimation methods and extended Kalman filter methods have been utilized for system identification of structural parameters without any input information [13-15]. Wang and Haldar [16] proposed an iterative algorithm for identifying constant structural parameters based on the traditional least-squares estimation method. Because this method does not require input information, there is no limit for the type of external excitations. Wang and Haldar [17] then combined the proposed method with the extended Kalman filter for improving the accuracy of system identification. Unlike the iterative algorithms mentioned above, Yang et al. [18-20] presented an

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analytical recursive solution based on the least-squares estimation method by placing the unknown input excitation into an extended unknown vector, and then they combined the proposed method with an adaptive tracking technique for the case in which the structural parameters are varying.

The extended Kalman filter method has been widely used for nonlinear system identification by linearizing the nonlinear state transition function and observation function with Taylor series expansions. However, the derivation of the Jacobian matrices can be nontrivial and the linearization approximations can lead to poor performance. Recently, another filtering technique named unscented Kalman filter method was proposed by Julier [21] and further enhanced by Wan and Van [22]. The unscented Kalman filter is able to handle any functional nonlinearity by approximating the posterior probability density by a Gaussian density which is represented by a set of sample points.

This paper presents an efficient algorithm for identifying structural parameters of a building structure with unknown excitations based on the unscented Kalman filter method. By projecting on to the column space of influence matrix of unknown excitations, the observation equation of the structural system with unknown excitations is transformed from a multiple regression equation to a simple regression equation. An analytical recursive unscented Kalman filter algorithm is then developed for identifying unknown excitations and structural parameters such as stiffness and damping. The feasibility of the proposed algorithm is finally demonstrated in terms of an example shear building with the measurement noise included.

2 PROPOSED ALGORITHM

2.1 Equation of motion

In this study, the structural mass matrix M is assumed to be known. For generality, external excitations are separated into two parts: measured excitations and unknown excitations. Consequently, the second-order differential equation of motion of a building structure with n DOF is given by

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = \varphi^* f^*(t) + \varphi f(t) \quad (1)$$

where M , C and K represent the $n \times n$ mass, damping and stiffness matrices of the building structure, respectively; $\ddot{X}(t)$, $\dot{X}(t)$ and $X(t)$ are the $n \times 1$ structural acceleration, velocity and displacement response vectors, respectively; $f^*(t)$ is the $r \times 1$ measured excitation vector with the influence matrix φ^* ($n \times r$); and $f(t)$ is the $s \times 1$ unknown excitation vector with the influence matrix φ ($n \times s$).

2.2 Transformation of observation equation

The extended state vector consists of structural displacement, velocity and the unknown structural parameters can be written as

$$Z(t) = [X(t)^T \dot{X}(t)^T \theta^T]^T \quad (2)$$

in which θ is an $m \times 1$ unknown structural parameter vector include structural stiffness and damping coefficients. Because the structural parameters are constants, the differential of θ is zero. The nonlinear state equation can be obtained as

$$\frac{dZ(t)}{dt} = \begin{bmatrix} \dot{X}(t) \\ M^{-1}(-C\dot{X} - KX + \varphi^* f^*(t) + \varphi f(t)) \\ 0 \end{bmatrix} = f(Z(t), f(t), f^*(t)) \quad (3)$$

By substituting $\frac{dZ(t)}{dt} = \frac{Z_{k+1} - Z_k}{\Delta t} = f(Z(t), f(t), f^*(t))$, the state space equation can be expressed in discrete form as

$$\mathbf{Z}_{k+1} = \mathbf{Z}_k + \int_{k\Delta t}^{(k+1)\Delta t} f(\mathbf{Z}_k, \mathbf{f}_k, \mathbf{f}_k^*) dt = F(\mathbf{Z}_k, \mathbf{f}_k, \mathbf{f}_k^*) \quad (4)$$

where \mathbf{Z}_{k+1} and \mathbf{Z}_k are the values of \mathbf{Z} at time $t = (k+1)\Delta t$ and $t = k\Delta t$ respectively.

The observation (measurement) equation associated with Eq. (1) at time $t = k\Delta t$ can be described as

$$\mathbf{y}_k = \mathbf{H}(\mathbf{Z}_k) - \boldsymbol{\varphi} \mathbf{f}_k + \mathbf{v}_k \quad (5)$$

where $\mathbf{y}_k = -\mathbf{M}\ddot{\mathbf{X}}_k + \boldsymbol{\varphi}^* \mathbf{f}_k^*$ is a $n \times 1$ measurement vector which can be obtained by the measured structural acceleration responses $\ddot{\mathbf{X}}_k$ and the measured excitations \mathbf{f}_k^* ; $\mathbf{H}(\mathbf{Z}_k) = \mathbf{C}\dot{\mathbf{X}}_k + \mathbf{K}\mathbf{X}_k$ is the nonlinear function of the state vector \mathbf{Z}_k ; \mathbf{v}_k represents a $n \times 1$ noise vector, taking into consideration the model uncertainty of the structure and the measurement noise. The noise vector can be assumed as a white noise with normal probability distribution. The subscript k represents the values of matrices or vectors at time $t = k\Delta t$.

In this study, a straightforward way in terms of the projection matrix is used to transform the multiple regression problem described by Eq. (5) to a simple regression problem.

From Eq. (5), one can obtain:

$$\boldsymbol{\varphi} \mathbf{f}_k = -\mathbf{y}_k + \mathbf{H}(\mathbf{Z}_k) + \mathbf{v}_k \quad (6)$$

The closest solution of \mathbf{f}_k in Eq. (6) can be given by the following projection matrix.

$$\mathbf{f}_{k,LS} = (\boldsymbol{\varphi}^T \boldsymbol{\varphi})^{-1} \boldsymbol{\varphi}^T (-\mathbf{y}_k + \mathbf{H}(\mathbf{Z}_k) + \mathbf{v}_k) \quad (7)$$

The error of the solution from Eq. (7) is given by:

$$\begin{aligned} \text{err} &= \boldsymbol{\varphi} \mathbf{f}_k - \boldsymbol{\varphi} \mathbf{f}_{k,LS} \\ &= (-\mathbf{y}_k + \mathbf{H}(\mathbf{Z}_k) + \mathbf{v}_k) - \boldsymbol{\varphi} (\boldsymbol{\varphi}^T \boldsymbol{\varphi})^{-1} \boldsymbol{\varphi}^T (-\mathbf{y}_k + \mathbf{H}(\mathbf{Z}_k) + \mathbf{v}_k) \\ &= (\mathbf{I}_n - \boldsymbol{\varphi} (\boldsymbol{\varphi}^T \boldsymbol{\varphi})^{-1} \boldsymbol{\varphi}^T) (-\mathbf{y}_k + \mathbf{H}(\mathbf{Z}_k) + \mathbf{v}_k) \end{aligned} \quad (8)$$

in which \mathbf{I}_n is the $n \times n$ identity matrix; the matrix $\boldsymbol{\varphi} (\boldsymbol{\varphi}^T \boldsymbol{\varphi})^{-1} \boldsymbol{\varphi}^T$ is the projection matrix that projects the vector $-\mathbf{y}_k + \mathbf{H}(\mathbf{Z}_k) + \mathbf{v}_k$ on to the space spanned by the columns of $\boldsymbol{\varphi}$. As a limit, the error in Eq. (8) tends to be zero, leading to

$$\boldsymbol{\Phi} \mathbf{y}_k = \boldsymbol{\Phi} \mathbf{H}(\mathbf{Z}_k) + \boldsymbol{\Phi} \mathbf{v}_k \quad (9)$$

where $\boldsymbol{\Phi} = \mathbf{I}_n - \boldsymbol{\varphi} (\boldsymbol{\varphi}^T \boldsymbol{\varphi})^{-1} \boldsymbol{\varphi}^T$ for simplicity of presentation. It is noted that the projection matrix $\boldsymbol{\Phi}$ naturally has the two properties: (1) $\boldsymbol{\Phi}$ is a symmetric matrix, $\boldsymbol{\Phi}^T = \boldsymbol{\Phi}$; and (2) $\boldsymbol{\Phi}^2 = \boldsymbol{\Phi}$. As a result, the multiple regression equation expressed by Eq. (5) is transformed into the simple regression equation expressed by Eq. (9).

2.3 Recursive unscented kalman filter algorithm

To implement unscented Kalman filter, redefined the state vector as

$$\mathbf{Z}_k^a = [\mathbf{Z}_k^T \quad \mathbf{v}_k^T]^T \quad (10)$$

The initialized expression of the identification process can be shown as

$$\hat{\mathbf{Z}}_0^a = [\hat{\mathbf{Z}}_0^T \quad \mathbf{0}]^T, \mathbf{P}_0^a = \begin{bmatrix} \mathbf{P}_0 & \\ & \mathbf{R} \end{bmatrix} \quad (11)$$

where $\hat{\mathbf{Z}}_0$ is the initialized expression of the extended state vector \mathbf{Z}_k ; \mathbf{P}_0 is the initialized expression of error covariance matrix; \mathbf{R} is the error covariance of the measurement noise vector.

2L+1 sigma points around the current state vector at k th time step named $\boldsymbol{\chi}_k^a$ are generated as

$$\boldsymbol{\chi}_k^a = [\hat{\mathbf{Z}}_k^a \quad \hat{\mathbf{Z}}_k^a + \sqrt{\mathbf{P}_{i,k}^a (L + \lambda)} \quad \hat{\mathbf{Z}}_k^a - \sqrt{\mathbf{P}_{i,k}^a (L + \lambda)}] \quad (12)$$

where

$$\lambda = \varphi^2 (L + \gamma) - L \quad (13)$$

in which i indicates the i th column of the corresponding matrix; L is the dimension of the redefined state vector; φ determines the spread of the sigma points and γ is a secondary scaling parameter. The value of γ is usually set to $(3-L)$ or 0, and φ is a small positive number between 0 to 1 which controls the size of the distribution of the sigma point.

The predicted state vector $\hat{\mathbf{Z}}_{k+1|k}$ can be calculated as

$$\mathbf{x}_{k+1|k}^Z = \mathbf{F}(\mathbf{x}_k^a, \hat{\mathbf{f}}_k, \mathbf{f}_k^*) \quad (14)$$

$$\hat{\mathbf{Z}}_{k+1|k} = \sum_{i=0}^{2L} W_i^{(m)} \mathbf{x}_{i,k+1|k}^Z \quad (15)$$

and the predicted error covariance matrix $\mathbf{P}_{k+1|k}$ can be obtained as

$$\mathbf{P}_{k+1|k} = \sum_{i=0}^{2L} W_i^{(c)} [\mathbf{x}_{i,k+1|k}^Z - \hat{\mathbf{Z}}_{k+1|k}] [\mathbf{x}_{i,k+1|k}^Z - \hat{\mathbf{Z}}_{k+1|k}]^T \quad (16)$$

The predicted measurement vector $\Phi \hat{\mathbf{y}}_{k+1|k}$ and the predicted covariance matrix \mathbf{P}_{k+1}^{YY} can be calculated as follows

$$\hat{\boldsymbol{\rho}}_{k+1|k} = \Phi \mathbf{H}(\mathbf{x}_{k+1|k}^Z, \mathbf{x}_k^v) \quad (17)$$

$$\Phi \hat{\mathbf{y}}_{k+1|k} = \sum_{i=0}^{2L} W_i^{(m)} \boldsymbol{\rho}_{i,k+1|k} \quad (18)$$

$$\mathbf{P}_{k+1}^{YY} = \sum_{i=0}^{2L} W_i^{(c)} [\boldsymbol{\rho}_{i,k+1|k}^Z - \Phi \hat{\mathbf{y}}_{k+1|k}] [\boldsymbol{\rho}_{i,k+1|k}^Z - \Phi \hat{\mathbf{y}}_{k+1|k}]^T \quad (19)$$

The measurement update step is as follows:

$$\mathbf{P}_{k+1}^{XY} = \sum_{i=0}^{2L} W_i^{(c)} [\mathbf{x}_{i,k+1|k}^Z - \hat{\mathbf{Z}}_{k+1|k}] [\boldsymbol{\rho}_{i,k+1|k}^Z - \Phi \hat{\mathbf{y}}_{k+1|k}]^T \quad (20)$$

$$\mathbf{Kal}_{k+1} = \mathbf{P}_{k+1}^{XY} (\mathbf{P}_{k+1}^{YY})^{-1} \quad (21)$$

$$\hat{\mathbf{Z}}_{k+1} = \hat{\mathbf{Z}}_{k+1|k} + \mathbf{Kal}_{k+1} [\Phi \mathbf{y}_{k+1} - \Phi \hat{\mathbf{y}}_{k+1|k}] \quad (22)$$

$$\mathbf{P}_{k+1} = \mathbf{P}_{k+1|k} - \mathbf{Kal}_{k+1} \mathbf{P}_{k+1}^{YY} \mathbf{Kal}_{k+1}^T \quad (23)$$

$$\hat{\mathbf{f}}_{k+1} = (\Phi^T \Phi)^{-1} \Phi^T (-\mathbf{y}_{k+1} + \mathbf{H}(\hat{\mathbf{Z}}_{k+1})) \quad (24)$$

The weights $W_i^{(m)}$ and $W_i^{(c)}$ are given by

$$W_0^{(m)} = \lambda / (L + \lambda) \quad (25)$$

$$W_0^{(c)} = \lambda / (L + \lambda) + (1 - \varphi^2 + \beta) \quad (26)$$

$$W_i^{(m)} = W_i^{(c)} = 1 / \{2(L + \lambda)\} \quad (27)$$

in which β is a constant which can be set to 2 optimally.

3 NUMERICAL INVESTIGATIONS

3.1 Description of example of building structure

A simple five-story shear building is chosen as the example building structure. It has the identical story height of 3 meters (see Fig. 1). The building structure has uniform mass $m = 800\text{kg}$ and uniform horizontal story (shear) stiffness $k = 1.2 \times 10^6 \text{ N/m}$ for all five stories. The five natural frequencies of the building structures are calculated as 1.755, 5.121, 8.073, 10.371, and 11.829 Hz. The acceleration responses of the building structure are measured by five accelerometers with one on each floor of the building structure. From a practical viewpoint, the white noise of 2% intensity is added to the calculated acceleration response. The noise intensity is defined as the ratio of the

root mean square of the noise to the root mean square of the acceleration response. Since the highest natural frequency of the building is 11.829 Hz, a low-pass filter with a cut off frequency of 30 Hz is applied to the noise-polluted acceleration responses. The displacement and velocity responses of the building are obtained from the measured noise-polluted acceleration responses through numerical integrations.

The building structure is subjected to earthquake-induced ground acceleration. The ground acceleration is taken as the EI Centro earthquake with scaled peak ground acceleration (PGA) of 4.0 m/s². The structural damping matrix is assumed to be the Rayleigh damping matrix $C = \alpha M + \beta K$ where α and β are the two constant coefficients decided by the first and second modal damping ratios of 2%.

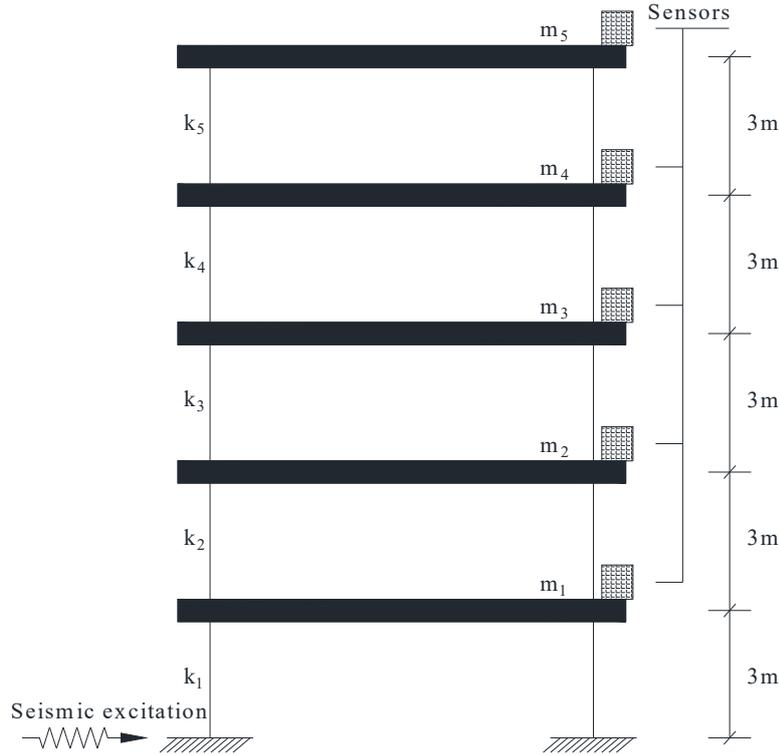


Figure 1: The example shear-building subject to seismic excitation

3.2 Results of identification

The unknown parameter vector at time $t = k\Delta t$ can be written as $\mathbf{Z}_k = [\mathbf{K}_\theta \quad \alpha \quad \beta]_k^T$, where $\mathbf{K}_\theta = [k_1 \ k_2 \ \dots \ k_n]$ is the stiffness coefficients and $n=5$.

The sampling interval Δt is set as 0.002s, and accordingly the sampling frequency is 500 Hz for all the measurement responses. The initial values of story stiffness coefficients are taken as 0.8 times the real one. The initial values for the two constant coefficients α and β are unit but the actual values are 0.3283 and 9.2636×10^{-4} respectively. As a result, the initial estimated unknown parametric vector can be written as $\hat{\mathbf{Z}}_0 = [0.8\mathbf{K}_\theta \quad 1 \quad 1]^T$. The initial estimation error covariance matrix is set as $\mathbf{P}_0 = 10^6 \times \mathbf{I}_{n+2}$, where \mathbf{I}_{n+2} represents an $(n+2) \times (n+2)$ identity matrix. The building structure is subjected to earthquake-induced ground acceleration $\mathbf{f}(t) = \ddot{\mathbf{x}}_g(t)$, the

influence matrix of the unknown excitations can be set as $\varphi = -\mathbf{M}[1 \ 1 \ 1 \ 1 \ 1]^T$. The influence matrix of the known inputs φ^* is a null matrix.

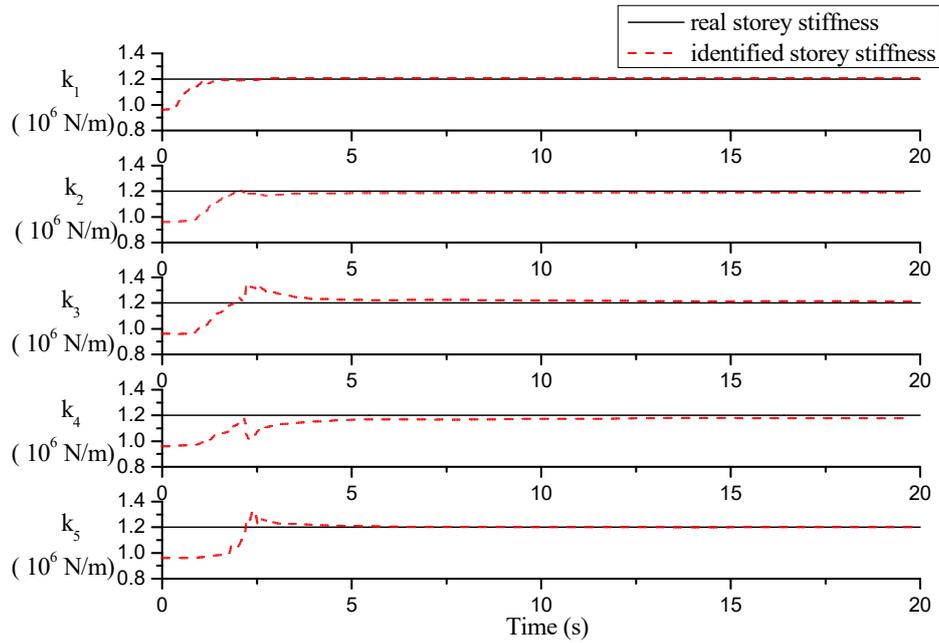


Figure 2: Identified results of story stiffness

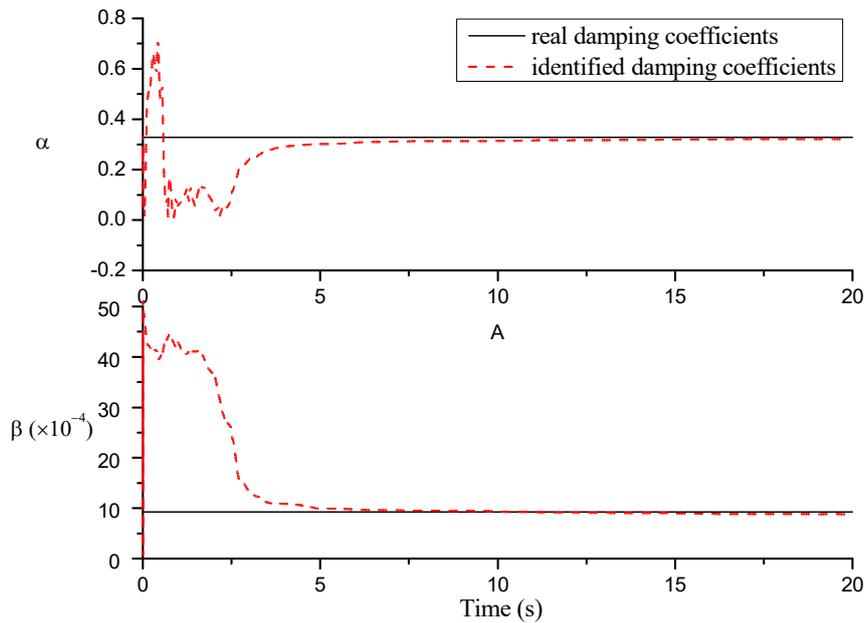


Figure 3: Identified results of Rayleigh damping coefficients

Fig. 2 presents the identified results of stiffness coefficients of the five stories of the building. Fig. 3 shows the identified results of two coefficients α and β . In Figs. 2 and 3, the identified results are presented as dash lines but the real values are presented as solid lines for comparison. It can be seen that after a very short time period (less than 1.25s), the identified results converge to the actual ones. The proposed algorithm can identify constant structural stiffness and damping coefficients accurately. The identified results of the unknown earthquake-induced ground

acceleration are presented in Fig. 4 with dash lines and compared with the actual ones with solid lines. Clearly, the proposed algorithm can identify the unknown excitation very well.

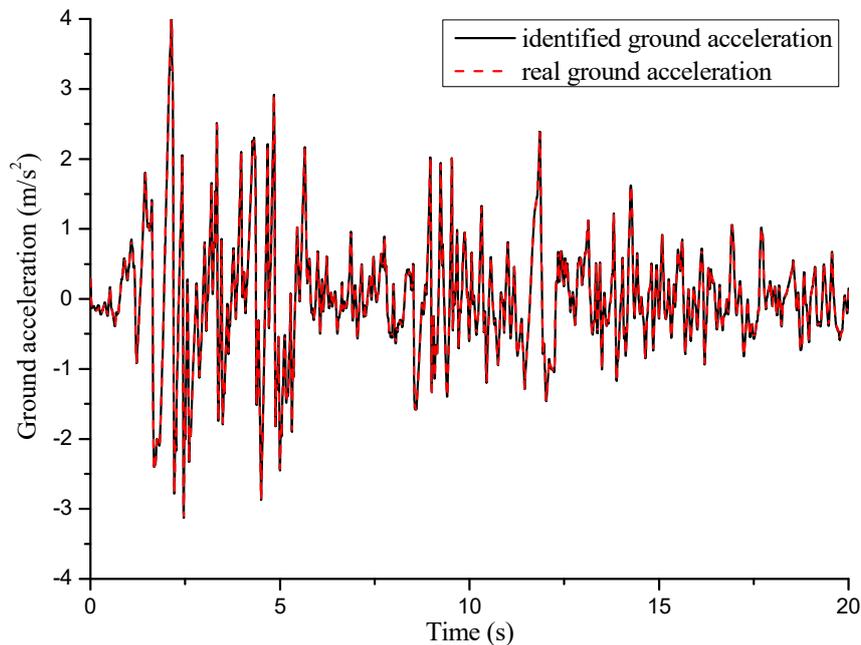


Figure 4: Identified results of unknown earthquake-induced ground accelerations

4 CONCLUDING REMARKS

An efficient algorithm for simultaneous identification of structural parameters and unknown excitations of a building structure has been presented in this paper. The feasibility and accuracy of the proposed algorithm have been assessed through numerical examples. The results clearly demonstrate that the proposed algorithm can simultaneously identify unknown excitations and structural parameters efficiently and accurately.

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