

# PASSIVE VIBRATION SUPPRESSION USING MULTIPLE INERTER-BASED DEVICES FOR A MULTI-STOREY BUILDING STRUCTURE

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**ABSTRACT.** This paper investigates the application of multiple passive absorbers incorporating inerter(s), spring(s) and damper(s) to suppress vibrations in a multi-story structure. Different from the one terminal mass element, the inerter was proposed as a two-terminal element, with the property that the applied force is proportional to the relative acceleration across its terminals. The device can be configured to include gearing, hence allowing a far higher inertance than device mass. The advantage of using a TID as a suppression device mounted at the bottom of a multi-storey building has been identified. In this paper, a five-storey building model with two TIDs subjected to the base excitation is studied. Both of the these devices are located at the bottom. The criterion selected for the optimisation is the minimisation of the maximum relative displacement of the building. Furthermore, the resulting structural responses are compared to the case where a single TID is used at the bottom, to show the potential benefits that arise from using multiple devices. In addition, we show that the resulting optimal inertance needed for each device is smaller compared to the case where a single TID is used between the ground and the first floor, which makes the suppression system easier to manufacture.

**KEYWORDS:** Multiple passive vibration suppression, relative displacement, base excitation, inerter

## 1 INTRODUCTION

Mitigating seismic response of a structure is very important in civil engineering. Tuned mass dampers (TMDs) proposed by Frahm [1] in 1909 has been widely accepted as an effective passive control device. The classical method of choosing the damping ratio is based on the tuning method proposed by Den-Hartog [2]. In [3], dual tuned mass dampers (2TMD) was proposed for harmonically forced oscillation of the structure and it was shown that 2TMD are more effective than a single TMD. Multiple tuned mass dampers (MTMD) has been studied in [4, 5], which conclude that MTMD can have a better performance than a single TMD with the same mass.

The inerter, introduced by Smith in [6], is a two terminal device with the property that the force through the terminals is proportional to the relative acceleration between them. With the gearing included in the device, the inerter can have a far higher inertance than its mass. It offers many practical possibilities for passive mechanical control, such as vehicle suspension [7, 18], and railway vehicle suspension [11, 10] have been identified. The results showed that the performance of the systems can be significantly

improved with the use of inerters. The inerter has been successfully deployed in Formula One racing in 2005, under the name of J-damper [12].

The applications of the inerter to the civil engineering have been studied in [8, 14, 13, 15, 9, 16]. Ikago *et al.* [13] presented the tuned viscous damper (TVMD) as the control device of a single degree of freedom system. In 2013, Lazar *et al.* [9] proposed a tuned inerter damper (TID) by substituting the mass of the widely used TMD with an inerter. It showed that the performance with a TID mounted between the structure and the ground can be better than that with a TMD at the top. A new device cooperating with inerter, named the TMDI, has been proposed in [16]. In [17], several optimal inerter-based devices were proposed with respect to the inerter's size and the brace stiffness. All these applications are focus on using one inerter-based control device at the bottom of the building structure. In [14], TVMDs were mounted between every storey in a multi-storey building and Takewaki *et al.* [15] investigated the earthquake response reduction with inerter-like devices known as inertial dampers used between several stories of a building. The effectiveness of the multiple inerter-based devices for suppressing vibrations of a building has not been fully studied.

In this paper, two tuned inerter dampers (2TID) mounted at the bottom of a five storey building subjected to the base excitation is considered. The relative displacements of the storeys to that of the base is chosen as the performance index and using this, an optimisation objective function is proposed. The optimum parameters of the 2TID can then be obtained with respect to the total inertance. For the sake of comparison, a single TID located at the bottom with the same total inertance is also considered.

This paper is structured as follows. In Section 2, an idealised building model is considered for performance analysis. We also propose the objective function and the optimisation approach used in this paper. In Section 3, two kinds of the 2TID configuration have been introduced and the optimum parameters of them and a single TID are both obtained with respect to the total inertance. The comparison of the structural response with these three configurations has also been given. Finally, conclusions are drawn in Section 4.

## 2 BUILDING MODEL AND OBJECTIVE FUNCTION

In this section, an idealised five storey building model with control device mounted at the bottom is introduced and the dynamic equations have been derived in the Laplace domain. Then we introduce an objective function with the relative displacement chosen as the performance index.

### 2.1 The building model

Considering a five-storey building model shown in Figure 1, with equivalent floor masses  $m$  and identical inter-storey structural elasticities, represented by stiffness  $k$ . Since the structural damping is relatively small compared with that of the absorber, it is taken to be zero in this work. The suppression system is attended between the first storey and the ground because it generates a force based on the relative velocity and so this location has been shown to be optimum for a TID [9]. Figure 1 gives a schematic representation of the building, where  $f_d$  represents the force generated by the suppression device. In this paper, we fix the parameters of the five storey building model as  $m = 1 \text{ kNs}^2/\text{m}$  and

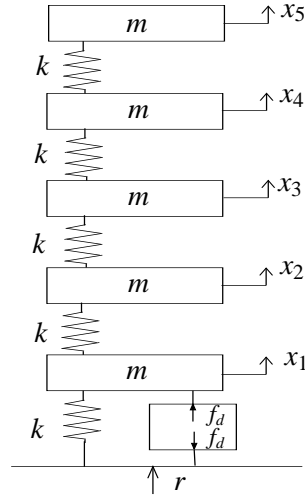


Figure 1: Schematic representation of an idealised building and lower floor suppression device.

$k = 1500 \text{ kN/m}$ . The parameters for the building model are the same as the one used in [9] and these numerical values were selected for convenience while retaining realistic natural frequencies and noting the parameters scale linearly.

The equations of motion for the system in Figure 1 are written in absolute coordinates as

$$\begin{aligned} m\ddot{x}_1 + 2kx_1 &= kx_2 + kr + f_d, \\ m\ddot{x}_2 + 2kx_2 &= kx_1 + kx_3, \\ m\ddot{x}_3 + 2kx_3 &= kx_2 + kx_4, \\ m\ddot{x}_4 + 2kx_4 &= kx_3 + kx_5, \\ m\ddot{x}_5 + kx_5 &= kx_4. \end{aligned}$$

where in the Laplace domain  $f_d(s) = sY(s)(R(s) - X_1(s))$  with  $Y(s)$  is the transfer function of the control system from force to the relative velocity and  $f_d(s)$ ,  $X_i(s)$ ,  $R(s)$  being the Laplace transforms of  $f_d(t)$ ,  $x_i(t)$ ,  $r(t)$ , respectively. By defining the relative displacement  $z_i = x_i - r$ , ( $i = 1, 2, 3$ ), the steady-static equation of motion with respect to the relative displacement, in matrix form, in the Laplace domain is

$$\begin{bmatrix} m & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & m \end{bmatrix} s^2 Z + \begin{bmatrix} 2k + sY(s) & -k & 0 & 0 & 0 \\ -k & 2k & -k & 0 & 0 \\ 0 & -k & 2k & -k & 0 \\ 0 & 0 & -k & 2k & -k \\ 0 & 0 & 0 & -k & k \end{bmatrix} Z = - \begin{bmatrix} m \\ m \\ m \\ m \\ m \end{bmatrix} s^2 R$$

where  $Z = X - R$  represents the vector of relative storey displacements in the Laplace domain.

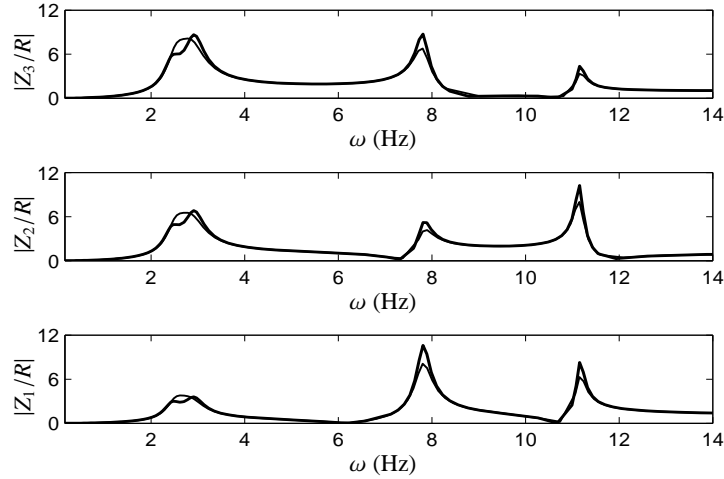


Figure 2: Displacement comparison: optimised TID using (1) with  $k_d = 142.7$  kN/m,  $c_d = 3.263$  kNs/m (thin line) and TID proposed in [9] with  $k_d = 138.6$  kN/m,  $c_d = 2.5$  kNs/m (thick line).

## 2.2 Objective function and optimisation approach

There are many design criteria for a vibration absorber, such as the absolute displacement and the absolute acceleration. This paper considers the relative displacements of each building storey to that of the base as the performance index. The objective function is defined as

$$J_\infty = \max \left( \left\| T_{R \rightarrow Z_i}(j\omega) \right\|_\infty \right), \quad i = 1, \dots, n \quad (1)$$

where  $T_{R \rightarrow Z_i}$  denotes the transfer function from  $R$  to  $Z_i$ ,  $\left\| T_{R \rightarrow Z_i}(j\omega) \right\|_\infty$  is the standard  $H_\infty$ -norm, which represents the maximum magnitude of  $T_{R \rightarrow Z_i}$  across all frequencies. Although researchers often consider the frequency range about the first fundamental frequency in cost functions, particularly, for those based on comfort, they commonly apply a weight distributions in the frequency domain. Here, it is the method that is important, so we select a simple unity weighting. In our research,  $J_\infty$  represents the biggest  $H_\infty$ -norm among all the stories. The optimisation problem herein is to search the optimal parameters of the control device by minimising the objective function  $J_\infty$ . It should be noted that we use the patternsearch and fminsearch tool box in MATLAB, which is quite sensitive to the setting of the initial values. However, if the initial values are given properly, the optimisation are more accurate and faster than those global optimum design methodologies, such as the genetic algorithm.

For MIMO systems, the design of the absorber is normally carried out by investigating the fundamental mode of response, with initial tuning based on the assumption that the natural frequencies are well separated, hence the contributions from higher mode will be ignored. In reality, the modal cross coupling has a deleterious effect on the tuning in some cases. Hence, we propose objective function (1) to avert this problem. To show the potential advantage of the objective function (1), we optimise the building

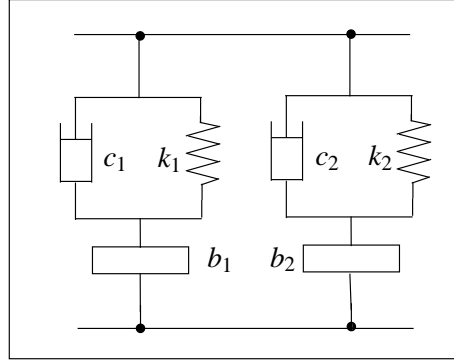


Figure 3: The two tuned inerter dampers (2TID).

model used in [9] with the same configuration TID. The inerter's size is fixed as  $b = 499$  kg, which is as the same as that in [9]. Then we obtain  $k_d = 142.7$  kN/m,  $c_d = 3.263$  kNs/m by optimising  $J_\infty$  with the Matlab implementation of patternsearch algorithm. The authors of [9] chose the values of spring and damper as  $k_d = 138.6$  kN/m,  $c_d = 2.5$  kNs/m based on Den Hartog tuning method [2]. The displacement responses of the three storeys in Figure 1 with a TID using these two set of values have been shown in Figure 2. It can be seen that with objective function (1), the TID device results in much smaller displacements of all the three floors in the vicinity of the second and third fundamental frequencies. Although the displacement of the first storey in the first fundamental frequency obtained from the objective function  $J_\infty$  is a slightly bigger, the max response displacement of the first fundamental frequency is smaller (for the third storey) comparing with the results in [9].

### 3 OPTIMISATION RESULTS

In this section, a 2TID control device is considered, see Figure 3, with the transfer function

$$Y(s) = \frac{b_1 b_2 (c_1 + c_2) s^4 + (b_1 b_2 k_1 + b_1 b_2 k_2 + b_1 c_1 c_2 + b_2 c_1 c_2) s^3 + (c_1 k_2 + c_2 k_1) (b_1 + b_2) s^2 + k_1 k_2 (b_1 + b_2) s}{(b_1 s^2 + c_1 s + k_1) (b_2 s^2 + c_2 s + k_2)}$$

It is well known that for a TMD, the mass ratio needs to be limited. 10% of the whole mass is often seen as a generous upper bound, since adding large mass to the original system will add significant structural loading. Although the inerter can have a high inertence with a much lower mass because of its gearing, its size will be larger with a higher inertence. Hence, we limit the size of inerter of our absorber. Here, we choose the total inertence  $b = b_1 + b_2 \in [100 \text{ kg}, 6000 \text{ kg}]$ . The 2TID control device is optimised for two cases. One is for the case where the 2TID respectively focuses on the first and second natural frequencies of the original building model, since adding the restricted amount of inertence will not change the natural frequency significantly and this case is denoted as 2TID<sub>1</sub>. The first and second frequency of the original building shown in Figure 1 are 1.75 Hz and 5.12 Hz, respectively. The other case, denoted as 2TID<sub>2</sub> is to optimise the 2TID device with no frequency constraint. In order to compare the multiple inerter based device with the single controller, a single TID has also been taken

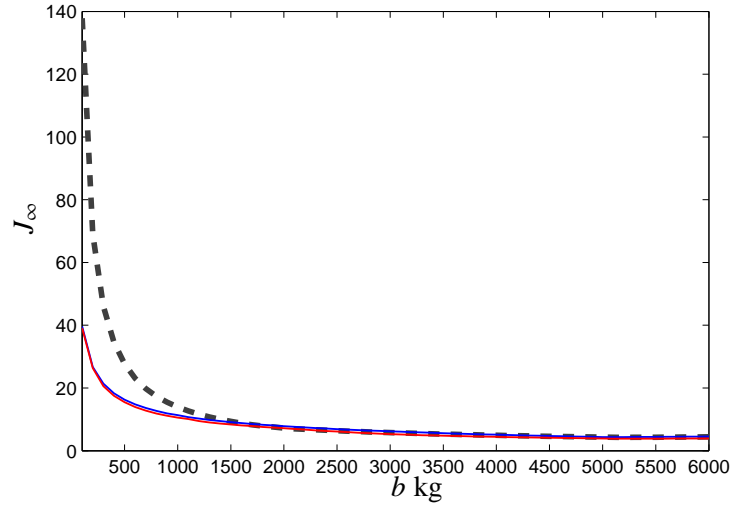


Figure 4: The optimum results: 2TID<sub>1</sub> (blue), 2TID<sub>2</sub> (red) and one TID (black dashed), where  $b$  is the total inertia.

into consideration and the inertia of the TID is from 100 kg to 6000 kg, as well. The optimum results of the objective function  $J_\infty$  for the three configurations (2TID<sub>1</sub>, 2TID<sub>2</sub> and a single TID) with the whole range of the inerter's size have been shown in Figure 4.

It can be seen that the optimum results of the 2TID<sub>1</sub> and 2TID<sub>2</sub> for the two cases are close to each other with respect to the inerter's size although 2TID<sub>2</sub> performs slightly better than the 2TID<sub>1</sub>. It can be checked that the highest variation between these two cases is 14.7% when  $b = 3300$  kg with 2TID<sub>1</sub> achieving  $J_\infty = 5.83$  and 2TID<sub>2</sub> achieving  $J_\infty = 4.97$ . It can also be seen that when the value of  $b$  is small, the 2TID<sub>1</sub> and 2TID<sub>2</sub> perform much better than a single TID and the difference of the objective function  $J_\infty$  between them and the single TID decreases as the total inertia increasing. The 2TID<sub>1</sub> and 2TID<sub>2</sub> are more effective than the single TID when  $b \in [100 \text{ kg}, 1500 \text{ kg}]$ . The optimal inertia proportion  $u = b_1/b$  of the 2TID<sub>1</sub>, 2TID<sub>2</sub> has been shown in Figure 5(a). The optimum values of stiffness and damping of the 2TID<sub>1</sub>, 2TID<sub>2</sub> and the single TID have been shown in Figure 5(b), (c) respectively.

Table 1:  $J_\infty$  optimisation with the TID, 2TID<sub>1</sub> and 2TID<sub>2</sub> when  $b = 500$  kg.

layout	$J_\infty$	element values ( $u = b_1/b$ , $c$ (kNs/m), $k$ (kN/m))
TID	27.7	$c = 1.9$ , $k = 61.8$
2TID <sub>1</sub>	16.3	$u = 0.89$ , $c_1 = 0.76$ , $c_2 = 1.11$
2TID <sub>2</sub>	15.5	$u = 0.88$ , $c_1 = 0.71$ , $k_1 = 54.0$ , $c_2 = 1.1$ , $k_2 = 78.1$

Table 2:  $J_\infty$  optimisation with the TID, 2TID<sub>1</sub> and 2TID<sub>2</sub> when  $b = 2500$  kg.

layout	$J_\infty$	element values ( $u = b_1/b$ , $c$ (kNs/m), $k$ (kN/m))
TID	6.37	$c = 11.8$ , $k = 320.7$
2TID <sub>1</sub>	6.9	$u = 1$ , $c_1 = 13.1$ , $c_2 = -$
2TID <sub>2</sub>	6.11	$u = 0.65$ , $c_1 = 7.79$ , $k_1 = 236.9$ , $c_2 = 3.17$ , $k_2 = 74.6$

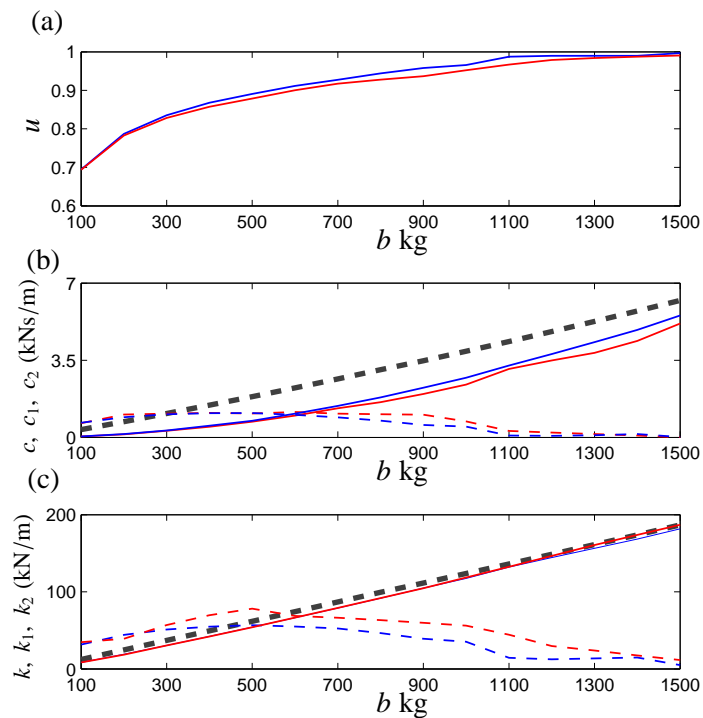


Figure 5: The optimum parameters:(a) the optimum inertance proportion (blue for 2TID<sub>1</sub>, red for 2TID<sub>2</sub>), (b) the optimum damping, (c) the optimum stiffness with one TID (black), 2TID<sub>1</sub> (blue solid for  $c_1$ ,  $k_1$ , blue dashed for  $c_2$ ,  $k_2$ ) and 2TID<sub>2</sub> (red solid for  $c_1$ ,  $k_1$ , red dashed for  $c_2$ ,  $k_2$ ).

We have compared the optimum configurations (2TID<sub>1</sub>, 2TID<sub>2</sub> and a single TID) with two set of  $b$  values. For  $b = 500$  kg, the optimum results and optimal parameter settings for these configurations are illustrated in Table 1. Comparing with the single TID, 41.2% improvement of the value of  $J_\infty$  can be obtained with the layout 2TID<sub>1</sub> and 44.0% improvement with 2TID<sub>2</sub>. The biggest infinity norm of the building model occurs at the fifth storey, hence we show the comparison of the displacement of the

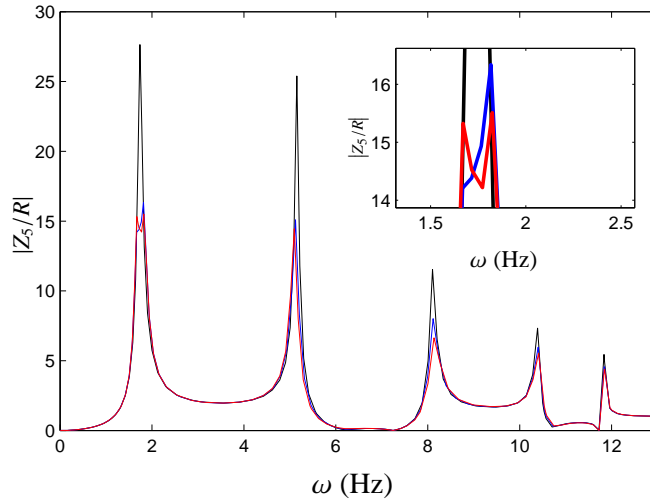


Figure 6: Displacement response comparison: a single TID (black), 2TID<sub>1</sub> (blue) and 2TID<sub>2</sub> (red) when  $b = 500$  kg.

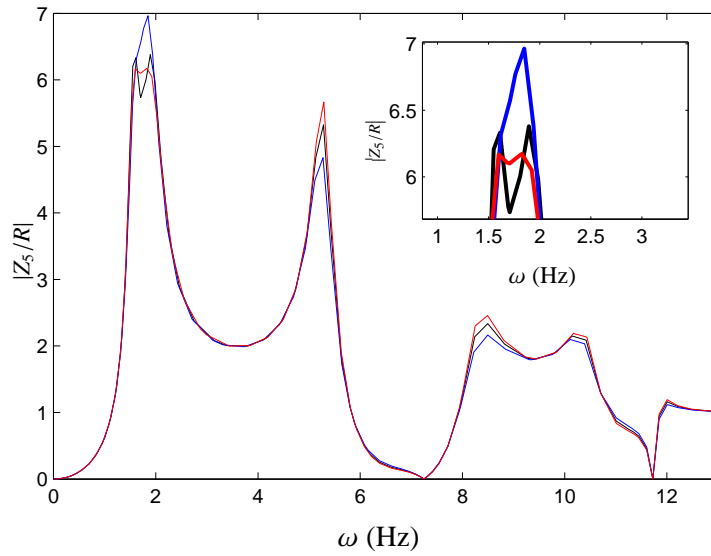


Figure 7: Displacement response comparison: a single TID (black), 2TID<sub>1</sub> (blue) and 2TID<sub>2</sub> (red) when  $b = 2500$  kg.



highest floor between these configurations in Figure 6. The displacement around the first fundamental frequency has been shown as the subfigure of Figure 6. It can also be seen from the figure that the  $2TID_1$  and  $2TID_2$  perform much better than the TID across all the frequencies and the  $2TID_2$  achieve a slightly better response than the  $2TID_1$ . Furthermore, since the total inertance of the  $2TID_1$  and  $2TID_2$  configuration is the same as that of a single TID, the inertance of each TID included in the  $2TID_1$  and  $2TID_2$  configuration is smaller than that of the single TID and this makes the absorber easier to manufacture.

When  $b = 2500$  kg, it can be seen from Figure 4 that the three configurations have similar performance and the Table 2 shows the optimal results and element values of them. Comparing with the value of  $J_\infty$  of the single TID,  $2TID_1$  has a degradation of 8.3% and  $2TID_2$  achieve an improvement of 4.1%. The comparison of the fifth storey displacement response with these three configurations has been shown in Figure 7, and it also shows  $2TID_2$  performs slightly better than a single TID, however,  $2TID_1$  can not provide better performance.

#### 4 CONCLUSIONS

This paper has studied the performance benefits of using multiple vibration suppression device with inerters. An idealised five-storey building model was considered for the analyses and the performance index was chosen as the maximum relative displacement of the stories relative to the base. Two different  $2TID$  configurations including two parallel TIDs are introduced as the proposed vibration suppression system, one is the fixed frequency configuration  $2TID_1$  and the other is  $2TID_2$  with no frequency constraint. The optimisation was carried out with a specific range of inerter's size in total because of the importance of mass ratio and the manufacture problem. For sake of comparison, a single TID as the suppression device was also investigated. A comparison between these three configurations have also been investigated to show the effectiveness of the multiple inerter-based devices. From the building considered here, we find that the optimum results of the two kinds of  $2TID$  configuration are similar with  $2TID_2$  performing slightly better than  $2TID_1$ . In addition, all of them can achieve much smaller value of  $J_\infty$  than a single TID when the inertance is smaller than 1500 kg.

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