

Use of overturning spectra in the performance evaluation of on-off control strategies for rocking objects

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ABSTRACT

Rocking of rigid bodies induced by seismic events triggers a number of complex dynamic phenomena such as impacts, sliding, uplift, which can potentially result in disastrous outcomes. Typical structures that present a significant seismic vulnerability with respect to overturning are water tanks, electrical and hospital equipment, statues and art objects. Several methods have been investigated in the past years to prevent the overturning or damage, such as rigid anchorages or base isolation devices.

This paper presents some numerical investigations about a novel on-off adaptive control strategy for rigid blocks subjected to rocking motion. In more detail, control algorithms were specifically conceived to regulate an adjustable stiffness of two restrainers placed at the lower corners of the block. The control's laws and the anchorage devices exhibited good performance when excited by simple one-sine pulse excitation, as reported by the authors in a previous study. The present work will instead investigate the performance and the robustness of the controlled system with respect to amplitude modulated harmonic excitations.

Keywords: *Rigid block, Rocking Motion, Adaptive Control, Non-Linear Response System, Seismic rocking*

1 INTRODUCTION

The study of the rocking motion of rigid objects interested many authors due to its intrinsic complexity and non-linearity. A thriving research area is the study of control strategies to minimise damage caused by the rocking motion. Typically, four different strategies are found in literature [1]: (i) lowering the centre of gravity of the rigid body; (ii) adjusting the base-to-height ratio proportions of the bodies; (iii) fixing the objects to the floor/wall and (iv) separating the objects from the ground using base isolation devices. Nonetheless, with respect to the significant amount of theoretical research on the response of free-standing blocks, there is a lack of studies about the response of anchored objects. The in-plane behaviour of a rigid block on a rigid plane anchored with elastic-brittle restraints was studied by Dimentberg et al. [2] and by Makris et al. [3]. In particular Dimentberg et al. investigated the behaviour of anchored blocks excited by white noise, while Makris et al. studied the response of them to pulse-type ground motions showing that there is a finite frequency range where the conclusions drawn by Dimentberg et al. do not hold concerning the response. The study of Makris et al. about pulse-type excitations reached the conclusion that, in general, anchored blocks survive to higher acceleration than free-standing blocks. However, it exists a frequency range where the opposite happens.

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This paper aims to expand a previous work of the same authors about a novel control strategy [4] for the dynamics of rigid blocks. The control strategies regulates the stiffness of the anchorages restraining two lower corners of a block. The control's laws and the anchorage devices exhibited good performance when excited by simple one-sine pulse excitation in the previous study. This article mainly investigates the performance of the control law when subject to quasi-harmonic excitation. At first, the analytical model used for computing the results is described. Successively, the two types of control laws are defined (i.e. a feedback and a feedback-feedforward control strategy). The performance of the two control strategies are assessed using an "ad hoc" utility index. Finally, the effect of amplitude modulated harmonic excitations and the influence of timing on the efficiency of the control laws are taken into consideration and quantified.

2 ANALYTICAL MODEL OF THE CONTROLLED BLOCK

The governing equations of rocking motion of unanchored (Figure 1a) and anchored (Figure 1b) rigid blocks are well known in literature. Housner [5] was the first to define the governing equations of the rocking motion of a rigid block subject to horizontal and vertical base excitations, taking into account the nonlinear effects. These equations were then modified by Makris et al. [6] in order to consider the presence of unilateral restraints placed at the corners of the rigid block.

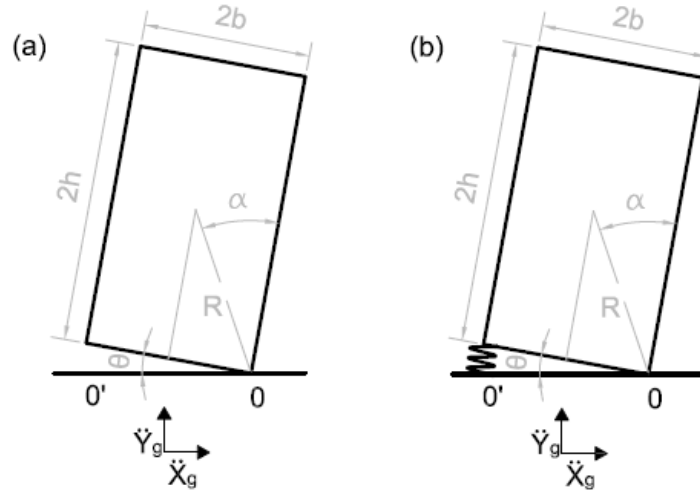


Figure 1 - A rigid rocking block, subjected to horizontal and vertical excitation
(a) unanchored (b) anchored

The semi-active restraint system is presented starting from the block model investigated by Dimentberg et al. [2], Zhang et al.[7] and Makris et al.[3]. The governing equations of rocking motion of a rigid block anchored with the semi-active restraint system can be derived from a rotational equilibrium about the two pivoting corners 0 and 0':

$$\ddot{\theta}(t) + \frac{1}{I_0} \{M_g(\theta) + M_{ex}(\theta) + M_{K,TOT}(\theta, K) + M_{D,TOT}(\theta)\} = 0 \quad (1)$$

where $M_g(\theta)$ and $M_{ex}(\theta)$ are the moment due to the block self-weight and the external excitation while $M_{K,TOT}(\theta, K)$ and $M_{D,TOT}(\theta)$ are the restoring moment exerted by the working spring and damper of the restraint system. The four moment terms read respectively:

$$M_g(\theta) = mgR \sin(\text{sgn}[\theta(t)]\alpha - \theta(t)) \quad (2)$$

$$M_{ex}(\theta) = m\ddot{X}_g(t)R\cos(\text{sgn}[\theta(t)]\alpha - \theta(t)) + m\ddot{Y}_g(t)R\sin(\text{sgn}[\theta(t)]\alpha - \theta(t)) \quad (3)$$

$$M_{K,TOT}(\theta, K) = M_K(\theta, K)f_{K,R}(\theta_{min}(t))f_{W,R}(\theta_{min}(t)) + M_K(\theta, K)f_{K,L}(\theta_{min}(t))f_{W,L}(\theta_{min}(t)) \quad (4)$$

$$M_{D,TOT}(\theta, K) = M_D(\theta, K)f_{D,R}(\theta_{min}(t))f_{W,R}(\theta_{min}(t)) + M_D(\theta, K)f_{D,L}(\theta_{min}(t))f_{W,L}(\theta_{min}(t)) \quad (5)$$

$M_K(\theta, K)$ and $M_D(\theta)$ are themselves defined as:

$$M_K(\theta, K) = \left[K(t) \cdot 4b \sin\left|\frac{\theta(t)}{2}\right| \cdot 2b \cos\left|\frac{\theta(t)}{2}\right| \right] \cdot = K(t) \cdot 4R^2 \sin^2\alpha \cdot \sin|\theta(t)| \quad (6)$$

$$M_D(\theta) = D \left(2b \cos\left|\frac{\theta(t)}{2}\right| \dot{\theta}(t) \right) \left(2b \cos\left|\frac{\theta(t)}{2}\right| \right) = D 4b^2 \cos^2\left|\frac{\theta(t)}{2}\right| \dot{\theta}(t) \quad (7)$$

Other unknown terms in Eqs. (4) and (5) are the fracture functions, i.e. the functions used to establish which of the two anchorages is working during the rocking motion. Assuming either zero or one values, these functions either nullify or activate the restoring moment contribution of the different elements of the restraint system. There are four fracture functions, one for each spring and damper elements. They require the knowledge of the block rotation associated to springs and dampers failure ($\theta_{K,u}$ and $\theta_{D,u}$ respectively). Consequently, the fracture functions of the right and left adjustable springs, named respectively $f_{K,R}(\theta_{min}(t))$ and $f_{K,L}(\theta_{max}(t))$, assume value 0 or 1 according to the following relationships:

$$f_{K,R}(\theta_{min}(t)) = \begin{cases} 1 & \text{if } \theta_{min}(\Delta t) > -\theta_{K,u} \\ 0 & \text{if } \theta_{min}(\Delta t) \leq -\theta_{K,u} \end{cases} \quad (8)$$

$$f_{K,L}(\theta_{max}(t)) = \begin{cases} 1 & \text{if } \theta_{max}(\Delta t) < \theta_{K,u} \\ 0 & \text{if } \theta_{max}(\Delta t) \geq \theta_{K,u} \end{cases} \quad (9)$$

where Δt is the time period elapsed from the beginning of the motion and the generic time t . Similarly, the fracture functions of the right and left damper devices, named respectively $f_{D,R}(\theta_{min}(t))$ and $f_{D,L}(\theta_{max}(t))$, are defined by the relationships:

$$f_{D,R}(\theta_{min}(t)) = \begin{cases} 1 & \text{if } \theta_{min}(\Delta t) > -\theta_{D,u} \\ 0 & \text{if } \theta_{min}(\Delta t) \leq -\theta_{D,u} \end{cases} \quad (10)$$

$$f_{D,L}(\theta_{max}(t)) = \begin{cases} 1 & \text{if } \theta_{max}(\Delta t) < \theta_{D,u} \\ 0 & \text{if } \theta_{max}(\Delta t) \geq \theta_{D,u} \end{cases} \quad (11)$$

The functions used to establish which of the two anchorages is working during the motion are two, like the positions of the restraint elements (the two corners of the body), and read:

$$f_{W,R}(\theta(t)) = \left(\frac{1 - \text{sgn}[\theta(t)]}{2} \right) \quad (12)$$

$$f_{w,L}(\theta(t)) = \left(\frac{1 + \text{sgn}[\theta(t)]}{2} \right) \quad (13)$$

where Eqs. (12) and (13) regulate the activation of the elements located at the right and left corner, respectively. An additional equation, connecting the angular velocities, immediately before and after the impact, must be coupled with Eq. (1) to determine the motion of a rigid block subjected a generic external excitation. Housner proposed for a free standing block a model to calculate the angular velocity of the rigid body immediately after the impact[5]. This model uses the following assumptions: (i) the impact is punctual; (ii) the impact time Δt_1 is very short; (iii) the block remains in the same position during the impact time. Under these assumptions, the relationship between the two angular velocities $\dot{\theta}(t_1^-)$ and $\dot{\theta}(t_1^+)$, immediately before and after the impact, respectively, can be derived from the conservation of the angular momentum and reads:

$$\frac{\dot{\theta}(t_1^+)}{\dot{\theta}(t_1^-)} = 1 - \frac{3}{2} \sin^2 \alpha = e \quad (14)$$

In the literature e is referred to as coefficient of restitution. Excluding bouncing motion, the value of e varies in the range $0 \div 1$. Indeed, negative values of e , for $h/b < 1/\sqrt{2}$ indicate that the angular velocity changes its sign after the impact, this resulting into bouncing of the block [8]. A unitary value for e corresponds to very slender blocks while 0 value refers to blocks with $h/b = 1/\sqrt{2}$. Several experimental tests have shown that the definition of e based only on body's critical angle does not describe the real impact mechanism [9][10][11][12]. Nevertheless the value provided by Eq. (14) reflects approximately the mean value of the e reported by experimental studies [12] [13].

The absence of a preload in the spring elements involves that the conditions to start and stop rocking motion of the anchored block are the same of the unanchored one. More specifically, the conditions to initiate a pure rocking motion are the same defined by Shenton et al. [14] and Tamiguchi [15]. In order to evaluate whether after the impact the object will rest or will undergo uplift on the other corner, the absolute values of overturning moment M_O and restoring moment M_R should be compared. The object will rest if the following inequality is satisfied:

$$|M_R| > |M_O| \quad (15)$$

3 CONTROL STRATEGIES

The restoring moment exerted by the semi-active restraint system, described by Eq. (6), varies over time. The time variant feature of $M_{K,TOT}$ must be ascribed to its dependence on the amplitude of the block rotation and on the value assumed at the generic time t by the adjustable stiffness, K . Two control strategies, based on a bang-bang control law, to set the value of K at the generic time t , have been investigated and compared.

3.1 Feedback control strategy

The first control strategy investigated, named CS01, is a feedback control and uses a bang-bang strategy to set the stiffness of the semi-active restraint system. The aim of this strategy is to preserve the positive effect of the elastic-brittle anchorages on the stability of rocking body and minimise/overcome the negative one. Comparing the behaviour of anchored bodies and free standing ones Makris et al.[3] and Dimitrakopoulos et al. [16] found that not only do the anchorages increase the minimum acceleration required to overturn a block, but also increase its tendency to overturn after the impact. The working anchorage, as a matter of fact, pulls the block at the upright

position and propels it at higher angular velocity in the other direction. As a consequence, the collapse occurs after the impact and it is characterized by lower acceleration levels.

In order to reduce/overcome the negative effect of the pull force and preserve the positive one, the first control strategy sets the value of K so that the magnitude of pull force is maximised when the block diverges from the upright position, and minimized in the opposite case. Consequently, the value of the adjustable stiffness K , at the generic time t , is set in accordance with the following law:

$$K(t) = \begin{cases} K_{max} & \text{if } \theta(t) \cdot \dot{\theta}(t) \geq 0 \\ K_{min} & \text{if } \theta(t) \cdot \dot{\theta}(t) < 0 \end{cases} \quad (16)$$

The data required by control strategy CS01 are the signs of the angular velocity $\dot{\theta}(t)$ and of the rotation $\theta(t)$. Figure 2 shows the value assumed by K in accordance with this control strategy.

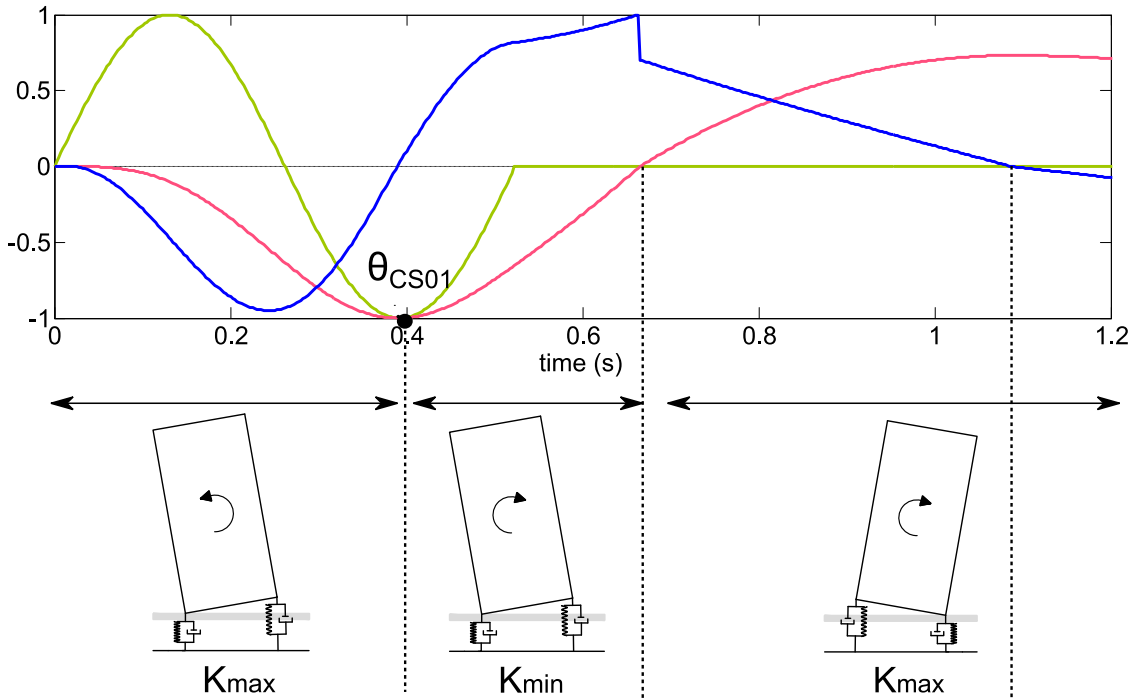


Figure 2. First control strategy. Normalized time-history of — ground motion, — rotation angle and — angular velocity of a block.

3.2 Feedback-feedforward control strategy

The second strategy under investigation, named CS02, is a case of feedback-feedforward control, i.e. where excitation is assumed to be available to the control algorithm. The stiffness of the semi-active anchorage is set using a bang-bang control law. The aim of the strategy is to adjust device parameters so as to reduce/nullify the magnitude of the destabilizing external action. As soon as the effects of the external force becomes zero and the body starts a free rocking motion, the working mode of control strategy CS02 switches to the working mode of control strategy CS01. Therefore, the stiffness of the anchorages is set using the following law:

$$K(t) = \begin{cases} \ddot{X}_g(t) = 0 & \begin{cases} K_{max} & \text{if } \theta(t) \cdot \dot{\theta}(t) \geq 0 \\ K_{min} & \text{if } \theta(t) \cdot \dot{\theta}(t) < 0 \end{cases} \\ \ddot{X}_g(t) \neq 0 & \begin{cases} K_{max} & \text{if } \ddot{X}_g(t) \cdot \theta(t) \leq 0 \\ K_{min} & \text{if } \ddot{X}_g(t) \cdot \theta(t) > 0 \end{cases} \end{cases} \quad (17)$$

As said, this control strategy requires, in addition to the signs of the angular velocity $\dot{\theta}(t)$ and of the rotation $\theta(t)$, also the direction of the external force. Figure 3 depicts the two working modes of the control strategy CS02. The block and the excitation are the same as in Figure 2.

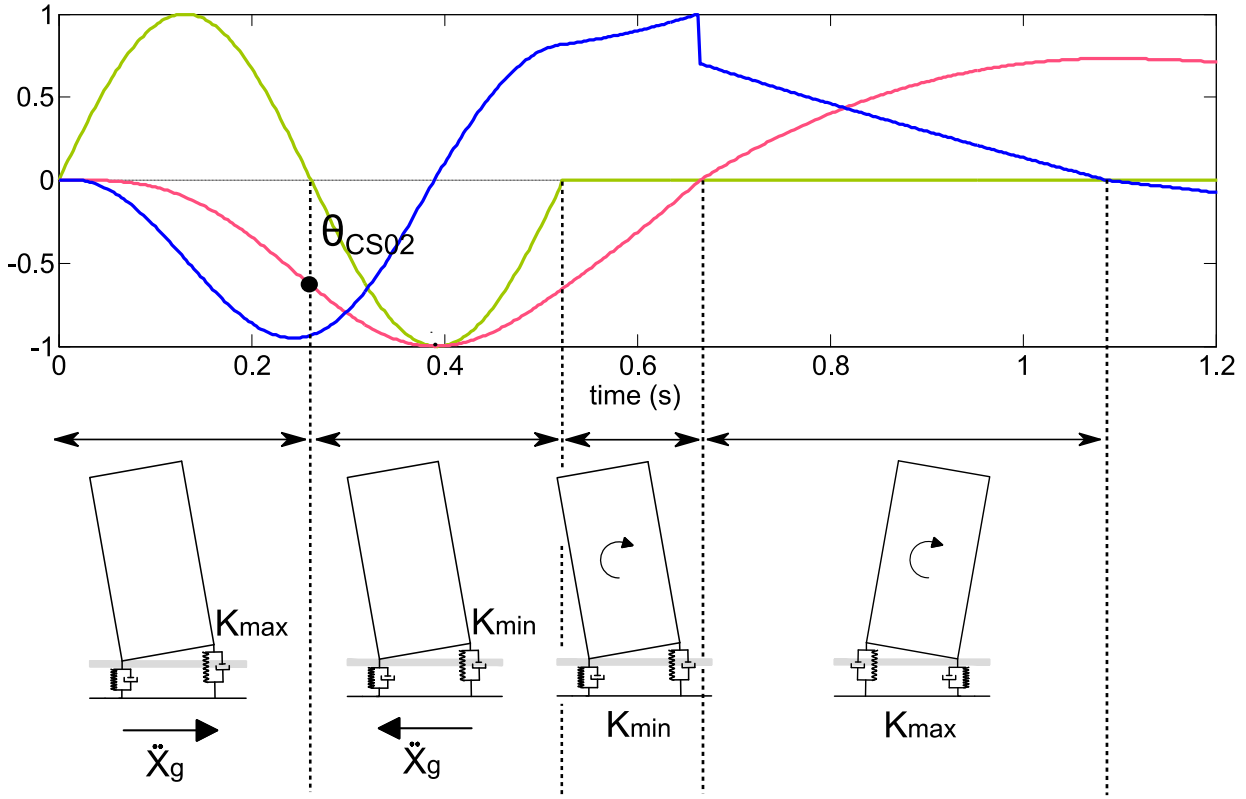


Figure 3 - Second control strategy. Normalized time-history of --- ground motion, --- rotation angle and --- angular velocity of a block.

4 UTILITY INDEX

The performance of the adjustable restraint system and of the control strategies to stabilise a rigid body has been evaluated through a utility index. The utility index is based on overturning spectra and requires the calculation, for each excitation frequency, of the minimum amplitude in acceleration, $a_{min,OVER}$, able to topple the block. The utility of a specific type of anchorage system can be defined at different excitation frequencies as percent increase of the $a_{min,OVER}$ with respect to a reference system (e.g. free standing block). In the same respect, such a definition also requires to fix a maximum acceleration amplitude, namely $a_{max,0}$, with respect to which the system must be protected. From a practical point of view, $a_{max,0}$ can be assumed equal to the acceleration of gravity. If the minimum overturning acceleration of the reference system and of the investigated one are denoted by $a_{min,REF}$ and $a_{min,SYS}$, respectively, the benefit in terms of overturning acceleration produced by the anchorage system can be quantified by:

$$\Delta a_{SYS}(\omega_p/p) = a_{min,SYS}^*(\omega_p/p) - a_{min,REF}^*(\omega_p/p) \quad (18)$$

where $a_{min,SYS}^*$ and $a_{min,REF}^*$ read, respectively:

$$a_{min,SYS}^*(\omega_p/p) = \begin{cases} a_{min,SYS}(\omega_p/p) & \text{if } a_{min,SYS}(\omega_p/p) < a_{max,0} \\ a_{max,0} & \text{if } a_{min,SYS}(\omega_p/p) \geq a_{max,0} \end{cases} \quad (19)$$

$$a_{min,REF}^*(\omega_p/p) = \begin{cases} a_{min,REF}(\omega_p/p) & \text{if } a_{min,REF}(\omega_p/p) < a_{max,0} \\ a_{max,0} & \text{if } a_{min,REF}(\omega_p/p) \geq a_{max,0} \end{cases} \quad (20)$$

The utility index J_{OS} is then defined as:

$$J_{OS} = 100 \left(\frac{1}{N^*} \sum_{n=1}^N \frac{\Delta a_{SYS}(\omega_p/p)}{\Delta a_{REF}(\omega_p/p)} \right) \quad (21)$$

with:

$$\Delta a_{REF}(\omega_p/p) = \begin{cases} a_{max,0} - a_{min,REF}^*(\omega_p/p) & \text{if } a_{min,SYS}^*(\omega_p/p) \geq a_{min,REF}^*(\omega_p/p) \\ a_{min,REF}^*(\omega_p/p) & \text{if } a_{min,SYS}^*(\omega_p/p) < a_{min,REF}^*(\omega_p/p) \end{cases} \quad (22)$$

where N^* is the number of investigated frequencies that present $a_{min,SYS}$ or $a_{min,REF}$ smaller than $a_{max,0}$. The aim of this assumption is to evaluate the mean utility of the control and therefore of the adaptive restraint system in the range of interest for seismic excitations. J_{OS} , expressed as percentage, can assume zero, positive or negative values. More specifically, a zero value implies no benefit in terms of block stability; a positive value reflects an increase of $a_{min,OVER}$ of the investigated system compared with the reference one; while a negative value of J_{OS} indicates a reduction of the block stability.

5 ROBUSTNESS OF THE CONTROL STRATEGY WITH RESPECT TO NOISE

In order to understand the importance of noise components for the performance of the adaptive restraint system, a benchmark block has been investigated ($B=0.40$ m, $H=1.60$ m, $B_y=0.47$ m, $R=0.82$ m, $\lambda=4$ and $p=3$ rad/s). The results presented in this study are limited to this type of block, even though the authors investigated several blocks with different B/H ratios. The block was subjected modulated harmonic excitations (see *Figure 4a*) of the following form:

$$\ddot{X}_g(t) = \frac{90t^2 \cdot (t - T_d)^6}{T_d^8} \cdot a_p \cdot \sin(\omega_p \cdot (t - T_d)) \quad (23)$$

where T_d is the duration of the intense part of the strong motion and is assumed equal to five times the period defined as $2\pi/\omega_p$. To this baseline harmonic excitation three different levels of noise, respectively set equal to 1%, 5% and 10% of the excitation energy, have been added (see *Figure 4b*).

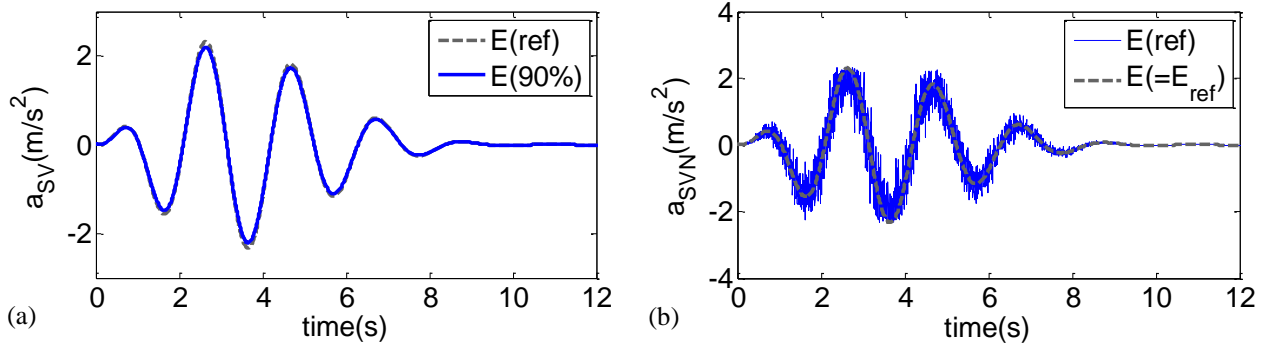


Figure 4 – (a) Baseline harmonic excitation with variable amplitude (b) amplitude-modulated harmonic excitation contaminated with noise (with correspondence in term of energy content and PGA value).

The harmonic excitations, with time-varying amplitude and contaminated with noise (SVN from this point on) were generated without altering the energy content and the Peak Ground Acceleration (PGA) value of the excitations, independently from the levels of noise. Indeed, in accordance with previous studies, the values assumed by these two quantities are known to be crucial for the body stability[5] [8][17][18][19].

5.1 Influence of high frequency components on the stability of a free-standing block

The degradation of the control strategies due to noise components was investigated, studying the dynamic behaviour of a free standing (FS) block and controlled one (CS01 and CS02), in terms of minimum acceleration required for the body overturning. For instance, Figure 5(a) depicts the lower threshold of the unsafe region for amplitude-modulated harmonic excitations contaminated with two different noise realisations, whilst Figure 5(b) compares the block rotation time-history evaluated for the main frequency and amplitude, labelled with “Ex01” in Figure 5(a).

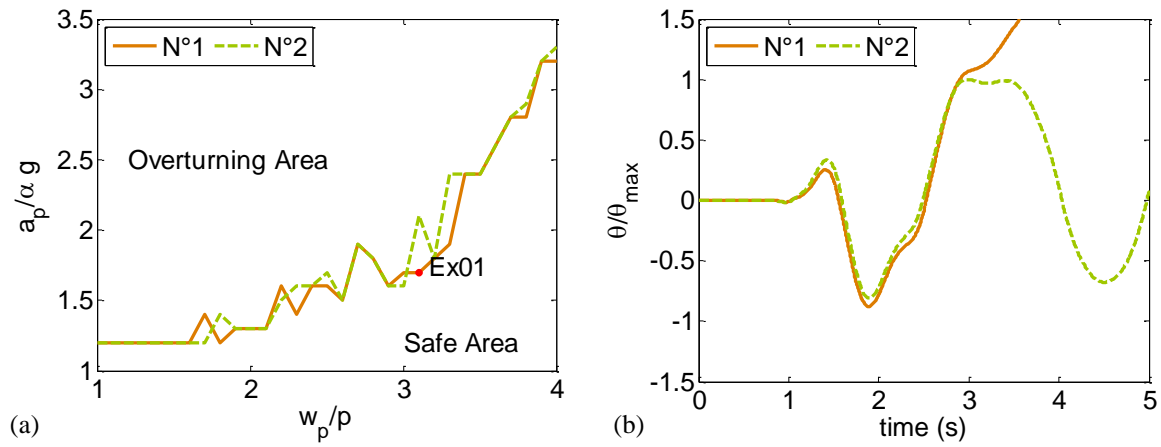


Figure 5 - (a) Lower threshold of the overturning area evaluated for two different realisations of Gaussian white noise (N°1 and N°2); (b) Comparison of BLC06 rotation time-history for excitation “Ex01” in Figure 5(a).

To overcome the variability of the minimum acceleration, an appropriate number of noise realisations must be considered. Increasing progressively the number of noise realisations it was observed that the minimum number of contaminated signals to be taken into account to define a stable $a_{min,OVER}$ was in the range of 40. In fact, as shown in Figure 6(a) for a FS block subjected to contaminated-amplitude-modulated harmonic excitations, the $a_{min,OVER}$ tends to converge to a stable threshold for the aforementioned numbers of noise realisations. Therefore, from this point on the $a_{min,OVER}$ of SVN excitations will be assumed to coincide with the $a_{min,OVER}$ determined considering 40 noise realizations. Figure 6(b-c-d) reflects the different sensitivity of the uncontrolled and controlled blocks to noise (R001, R005, R010). In more detail, the discrepancy between the $a_{min,OVER}$ of the contaminated in and pure amplitude-modulated harmonic excitations decreases when moving from the uncontrolled free standing block to the controlled one. In fact, in the presence of noise $a_{min,OVER}$ of the controlled block becomes more smoothed and spikes tend to disappear. Furthermore, reducing the level of noise, the $a_{min,OVER}$ of the free standing block decreases while the $a_{min,OVER}$ of the controlled block increases. The different trends of $a_{min,OVER}$ with the level of noise value can be appreciated also by the values of the utility index J_{OS} reported in Table 1. The value of J_{OS} of the free standing block decreases with increasing the level of noise, whilst the opposite happens for the controlled body. Reducing the level of noise, the discrepancy in the performances of the two control strategies decreases. In spite of this, the block controlled

through the feedback strategy (CS01) results to be more stable than the block controlled by the feedback-feedforward strategy (CS02).

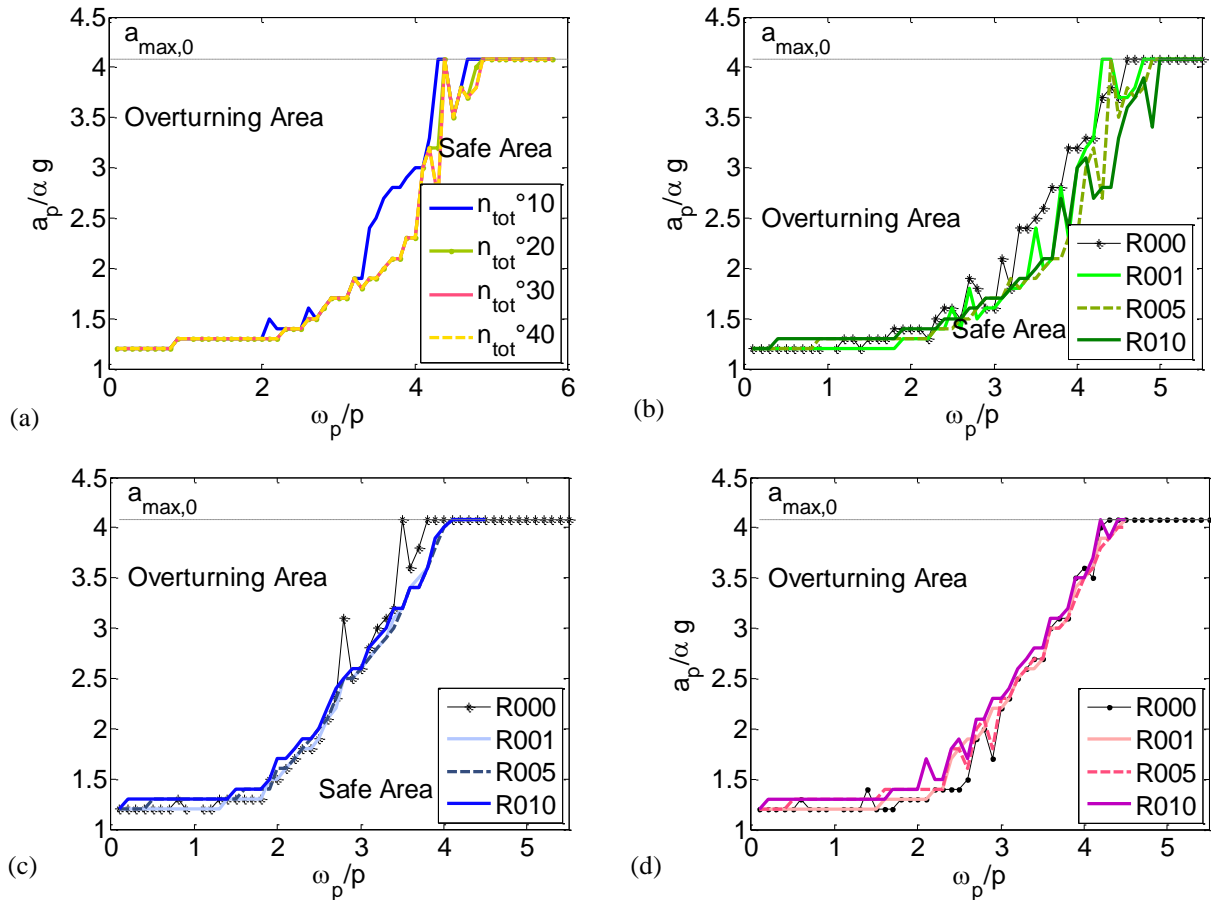


Figure 6 - Lower threshold of the overturning regions of BLC06 evaluated for (a) increasing number of noise realisations (b) FS block; (c) block controlled, CS01 strategy; (d) block controlled, CS02 strategy.

Table 1: Utility index J_{OS} associated to different noise levels (R001, R005, R010). The two columns report the percent obtained by assuming as a reference threshold respectively: the noise free one (R000); and the threshold of the FS system with the same noise level.

Control strategy	J_{OS} with ref. no noise			J_{OS} with ref. FS with noise		
	R001	R005	R010	R001	R005	R010
FS	-2.01%	-5.43%	-6.28%	-	-	-
CS01	-2.88%	-1.61%	-0.26%	+28.02%	+30.51%	+32.44%
CS02	+1.18%	+2.13%	+8.09%	+16.55%	+19.65%	+23.52%

5.2 Influence of timing on the stability of the controlled block

The thresholds of the overturning regions can be affected by the sampling ratio, whose influence was studied in the range 2.5 ms to 0.20 s, where it was found that the feedback strategy CS01 was still the more stable, in fact appreciable difference was found only for the largest sampling time of 0.20 s. As shown in Figure 7, control strategy CS02 deteriorates its performance for sampling time as low as 0.10 s.

Successively, simulations regarded the joint effect of sampling and delay (e.g. due to actuation). The influence of the delay, $\Delta\tau$, on the threshold for the two control strategies is shown in

Figure 8. The graphs in Figure 8 show that realistic values for the actuation delay, in the range of 0.06 ms, do not affect significantly the performance of the control, except for some local effects.

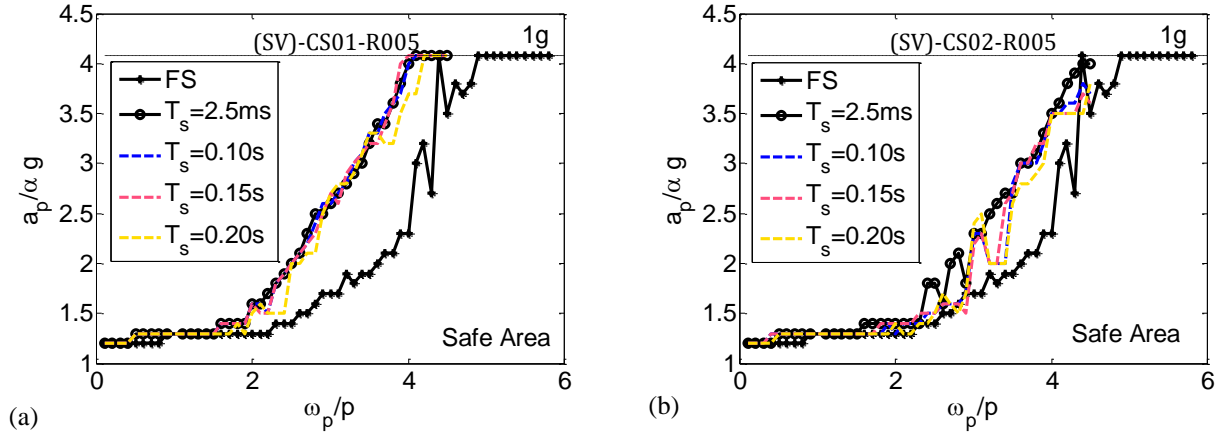


Figure 7 - Overlapping of the lower thresholds of the overturning spectra of the block controlled through the control strategy CS01 (a) and CS02(b) subjected to harmonic excitations with amplitude modulation and different value of T_s and noise RMS of 5%.

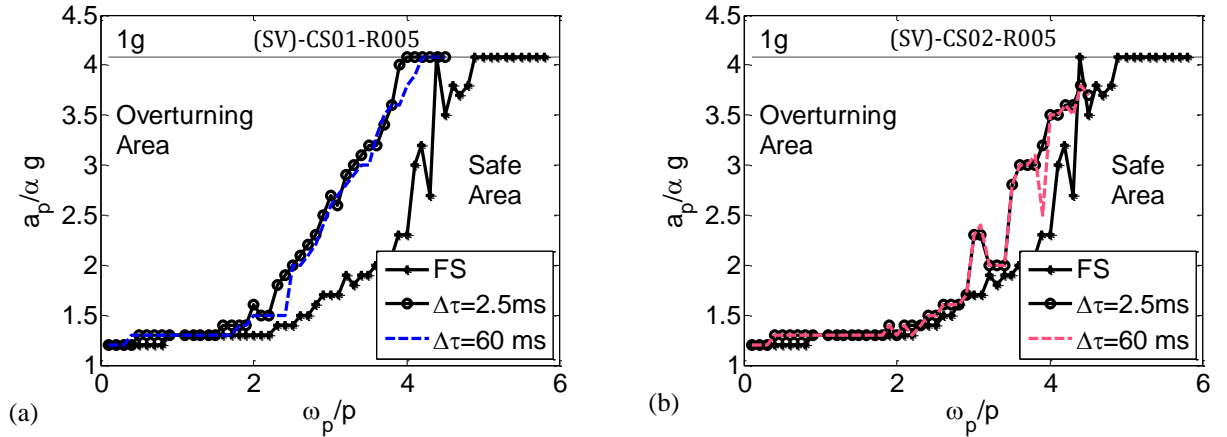


Figure 8 - Overlapping of the lower thresholds of the overturning spectra of the block controlled through the control strategy CS01 (a) and CS02(b) subjected to harmonic excitations with amplitude modulation and different value of $\Delta\tau$ and noise RMS of 5%. T_s is assumed constant and equal to 100 ms.

6 CONCLUSIONS

This paper has presented a few numerical investigations about the performance and the robustness of on-off adaptive control strategies against rocking motion. The efficiency of this semi-active control was tested with respect to quasi-harmonic excitations. The influence of noise on the stability of a free-standing and of a controlled block was quantified. In particular, a feedback control law (CS01) demonstrated to be more robust than a feedback-feedforward one (CS02). In fact, whilst the utility index for law CS01 was seen to be in the range +28% to +32% for noise up to the 10% of the harmonic input signal, for law CS02 the same index fell in the range +16% to +23%. For what concerns the influence of sampling rate, it was found that the feedback control strategy is less affected by sampling than the feedback-feedforward one. Finally, the effect of action actuation delay was considered for two values: 2.5 ms and 60 ms. It was concluded that both strategies perform well with respect to time-delay effects, as the threshold between overturning area and safe area remained substantially the same.

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