FRACTALITY IN WATER DISTRIBUTION NETWORKS

Kegong Diao[1], David Butler[2], Bogumil Ulanicki[3]

[1,3] De Montfort University, The Gateway, Leicester LE1 9BH
[2] University of Exeter, North Park Road, Exeter EX4 4QF

[1]*kegong.diao@dmu.ac.uk*

ABSTRACT

Fractals have been identified as a common feature of many natural and artificial systems that exhibit similar patterning at different scales. Understanding fractals is a critical aspect of decoding complex systems, as the pattern of such large systems can be revealed by identifying only a small part of the system. Furthermore, identify existing features of such systems can start at the large scale with the fewest details of the system under scrutiny before doing a more detailed analysis at finer scales. Such a process provides an efficient and reliable way of analysing and managing information of big data systems.

This study revealed the fractality in water distribution networks (WDNs) based on research on fractals in complex networks. Specifically, we explored the existence of fractal patterns in six real world WDNs of different complexities (e.g. from a network with only 21 pipes to a network with 2465 pipes). The box-covering algorithm has been applied, which is the most widely used method to distinguish between fractal or non-fractal networks. The WDNs are first mapped into undirected graphs. Next, the method partitions the nodes into boxes of size l_B*, i.e. the maximal distance between nodes within each box is at most* $l_B - 1$*. By varying the box sizes, different minimum numbers of boxes* N_B *required to cover the entire network can be identified. A network is fractal if the regression line for* $\log(N_B)$ *and* $\log(l_B)$ *is linear.*

The results demonstrate the existence of fractal patterns in all case study WDNs, as linear regression lines with coefficient of determination over 0.95 ($R^2 > 0.95$ *) are obtained in all analyses. As further verification, the self-similarity on multiscales is revealed, i.e. the similarity in patterns of component criticality. Based on the fractal patterns, a systematic method is also developed for more efficient identification of critical pipes in WDNs, e.g. reducing the computational load by 61% in the case study.*

Keywords: Complexity, Criticality, Fractality, Water distribution networks

1 BACKGROUND

Fractals have been identified as one of the most general features of many natural and artificial systems that exhibit similar patterning at different scales [1,2,3]. Understanding fractals is a critical aspect of decoding complex systems, as the pattern of such large systems (with enormous components) can be revealed by identifying only a small part of the system (e.g. as shown in Fig. 1(A)). Furthermore, identify existing features of such systems can start at the large scale with the fewest details of the system under scrutiny before doing a more detailed analysis at finer scales (e.g. as shown in Fig. 1(B)). Such a process provides an efficient and reliable way of analysing and managing information of big data systems.

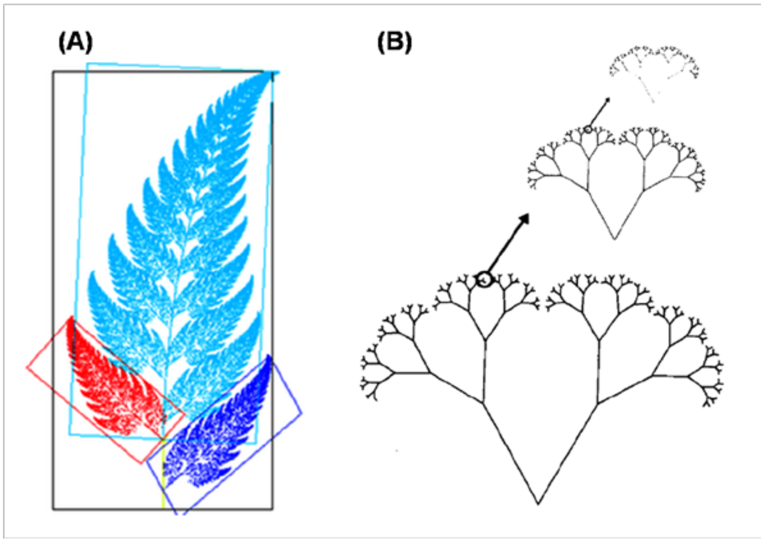

Figure 1. The self-similarity of fractal systems (reproduced from [4])

The fractality of complex systems has become an emerging topic in complex network science. A number of algorithms have been developed to identify whether the studied complex network is fractal or non-fractal [3, 5, 6, 7]. However, the practical application of fractality to analysis of specific problems in specific networks has not yet been comprehensively explored. In the field of water distribution network analysis, a few studies have investigated the application of fractal-based methods. For example, Qi et al (2014) developed a fractal self-growth model to generate more cost efficient design strategy for the WDN planning of one northern city in China [8]. Similarly, Zeng and Li (2014) generated several WDNs using basic fractal patterns, and investigated the reliability of the WDNs [9]. Kowalski et al (2015) developed a method for strategic placement of water quality and pressure sensors based on the fractal properties in the topology of WDNs [10]. Di Nardo et al (2017) applied fractal and complex network metrics to assess the WDNs' resilience [11]. The method can quantify the impact of pipe failure on the WDNs without using hydraulic simulations, and consequently avoid tedious simulation of vast number of pipe failure scenarios. Although the researches in [8-11] made substantial contributions in studying the fractality of WDNs, no systematic studies have been carried out to identify the existence of fractal patterns in variety of WDNs. For this reason, this study aims to explore the fractality of various WDNs with different configurations and complexity, and also develop a fractal-based criticality analysis method to more efficiently identify all the critical pipes in WDNs than the traditional enumeration method.

2 METHODS

A network is fractal if there is a power-law relation between the number of boxes needed to cover the network (N_B) and the box size (l_B) [12]:

$$N_B(l_B) = N_0 l_B^{-d_B}$$

where $N_B(l_B)$ is the number of boxes at size l_B, see as Fig. 2; N_0 the number of vertices in the original network (Fig. 2) [3, 12]; and d_B the fractal dimension, i.e. a scaling factor specifying how a pattern's detail changes with the scale at which it is considered [13]. In another word, A network is fractal if the regression line for $\log(N_B)$ and $\log(l_B)$ is linear (Fig. 3), and the slope of the line is denoted as fraction dimension d_B.

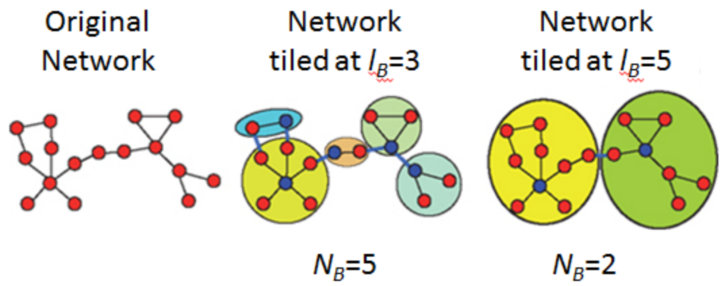

Figure 2. Procedure of the box-covering algorithm (reproduced from [2])

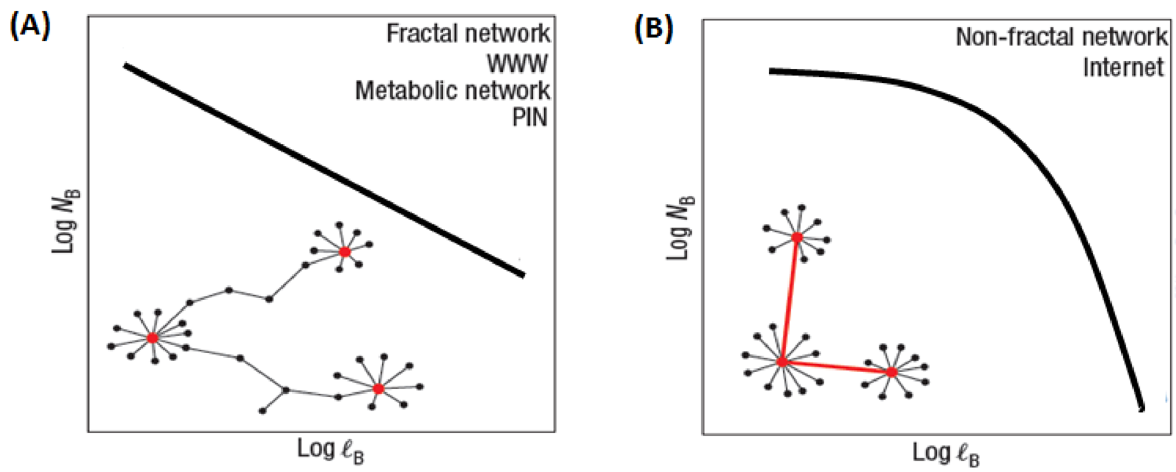

Figure 3. The regression lines of (A) fractal networks and (B) non-fractal networks (reproduced from [2])

To explore the fractality of WDNs, the box-covering algorithm [2] named as Maximum-Excluded-Mass-Burning (MEMB) (http://lev.ccny.cuny.edu/hmakse/soft_data.html) has been applied, which is the most widely used method to distinguish between fractal or non-fractal networks. The method is implemented as follow:

2.1 Mapping WDN into graphs

The WDNs are first mapped into undirected graphs in which the vertices represent the consumers, sources, and tanks and the edges represent the connecting pipes, pumps, and valves [14]. The graph is stored as a TXT file, in which each row refers to a link and the first and second column refers to the two ends of the link respectively. The file is used as the input for the box-covering algorithm.

2.2 Fractality Identification by WDN Clustering

Next, the network topology is partitioned by grouping vertices into boxes (i.e. clusters) of size l_B, i.e. the maximal distance between nodes within each box is at most $l_B - 1$. By varying the box sizes, different minimum numbers of boxes N_B required to cover the entire network can be identified (Fig. 1). Specifically, this procedure is implemented as follows [15]:

1) Specify the box size l_B (e.g. $l_B = 3$).
2) Mark all nodes as uncovered and non-centres (i.e. start with no box).
3) Calculate the mass [i.e. the number of uncovered nodes within a shortest-path distance less than $(l_B - 1)/2$] for all non-centre nodes.
4) Select the node *p* with the maximum mass as the next centre of a box.
5) Mark all the nodes with shortest-path distance less than $(l_B - 1)/2$ from *p* as covered (i.e. all the nodes belong to the box in which node *p* is the centre).
6) Repeat steps 3) – 5) until all nodes are either covered or centres.
7) Record the number of boxes N_B needed.
8) Repeat step 1) – 7) by varying l_B to obtain a number of corresponding N_B.
9) Plot $\log(N_B)$ against $\log(l_B)$ and determine the fraction dimension d_B.

3 CASE STUDIES

This study explores the existence of fractal patterns in six real world WDNs of different complexities (e.g. from a network with only 21 pipes to a network with 2465 pipes) as shown in Figure 4. The Alpine network has been used for vulnerability analysis [16, 17], and all the others are benchmarks available at 'http://emps.exeter.ac.uk/engineering/research/cws/downloads/benchmarks/'. No modifications were made to the properties of the networks.

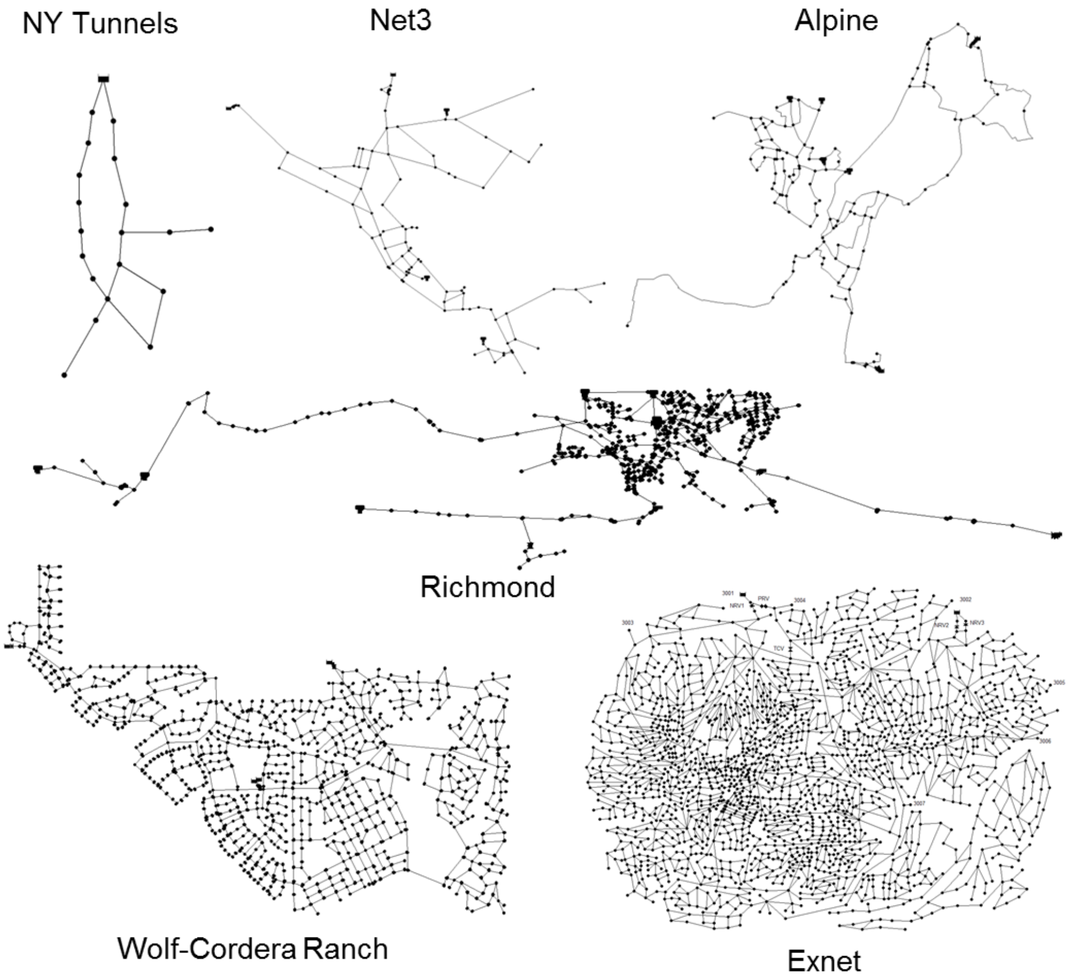

Figure 4. Case study water distribution networks

4 RESULTS AND DISCUSSIONS

The results in Fig. 5 demonstrate the existence of fractal patterns in all case study WDNs, as linear regression lines with coefficient of determination [18] over 0.95 ($R^2 > 0.95$) are obtained in all analyses. Hence, some properties of WDNs should have similar patterns both in different clusters/boxes (i.e. self-similarity) and at different scales. To further verify these facts, the criticality of pipes in Exnet is studied as an example. In this study, the criticality of a pipe is measured by the percentage of water supply shortage resulting from loss of the pipe. Hydraulic simulation in EPANET [Rossman, 2000] is used to model the loss of a pipe by changing the status of the pipe to completely close throughout the simulation period, and estimate the subsequent shortage of water supply (more details of the estimation is available in [17]). Figure 6(A) plots the criticality of all Exnet pipes for the whole network following a descending order. Similarly, Figure 6(B) and (C) plot the criticality of pipes in each cluster (i.e. $N_B = 37$ in this case) and that of interconnections among clusters, respectively. Moreover, Figure 6(D) plots the criticality of all clusters, measured by the water supply shortage resulting from loss of all interconnections between a cluster and the other clusters (i.e. isolate a whole cluster from the system).

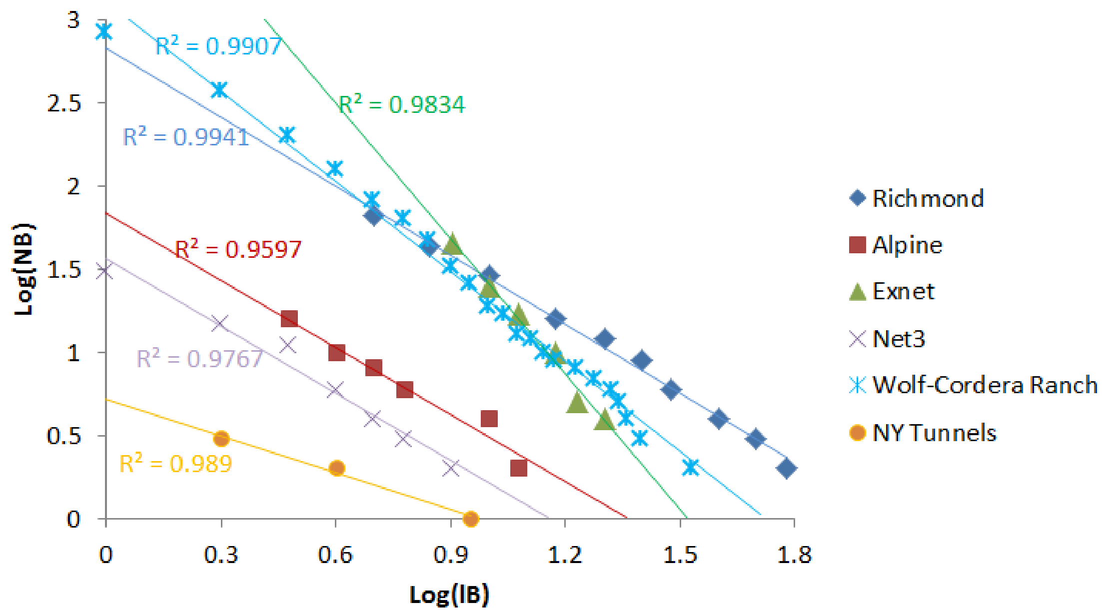

Figure 5. The regression lines of case study water distribution networks

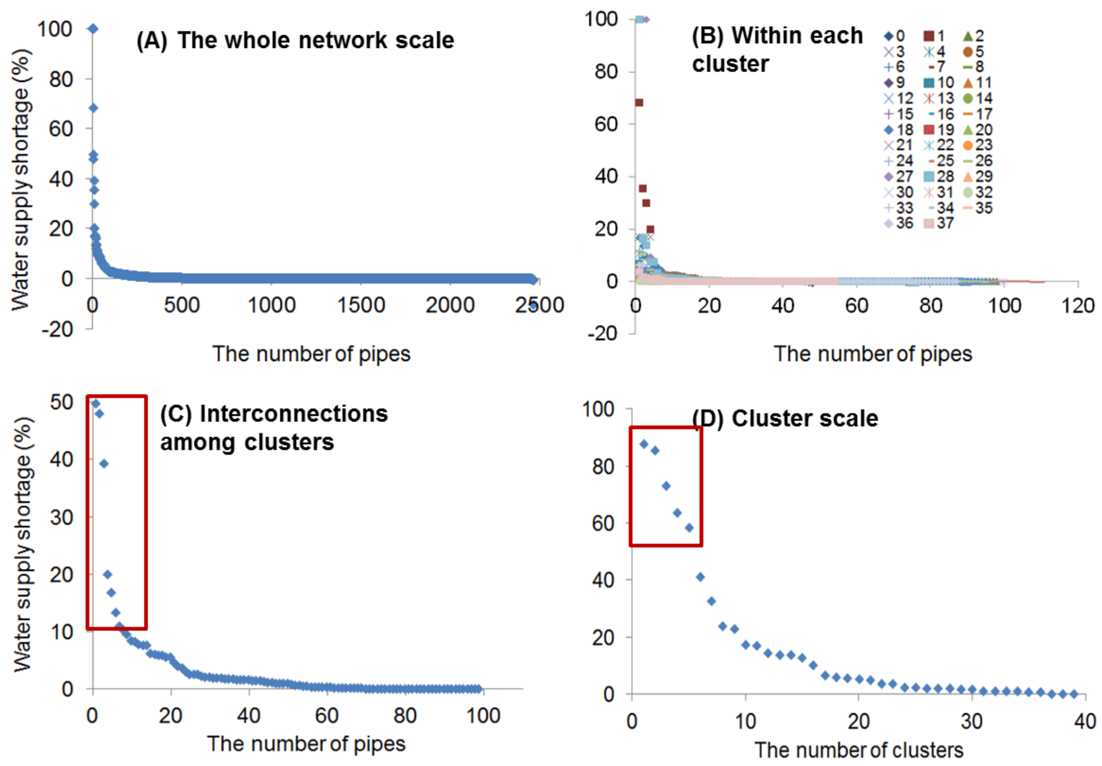

Figure 6. The patterns of criticality in Exnet

As Figure 6 shows, all the patterns of criticality are similar to each other and identically follow a long tail distribution [19], which reveals the self-similarity and multiscale similarity in the fractal WDN. As for self-similarity, pipes in different clusters [Fig. 6(B)] have very similar patterns of criticality [Fig. 6(A)]. Further, the patterns are also extremely similar to that of all pipes in the whole network. Hence, the patterns of WDNs can be revealed by identifying only a small part of the system (e.g. one cluster). As for multiscale similarity, the criticality of all clusters (i.e. at the cluster scale, Fig. 6(D)) also shows a similar pattern to that of all pipes at the whole network scale (Fig. 6(A)). Therefore, identifying existing features of WDNs can start at the large scale with the fewest details (e.g. the cluster scale) of the system under scrutiny before doing a more detailed analysis at finer scales (e.g. the whole network scale). Such a process provides an efficient and reliable way of analysing and managing information of big data systems. For instance, to identify all the critical pipes in WDNs, it is possible to first identify the critical clusters and eliminate the uncritical ones to improve efficiency. This example is introduced in the APPLICATION section to further demonstrate the practical application of WDN fractality.

5 APPLICATION OF FRACTALITY

This section is to demonstrate how to use the fractality of criticality described above to identify all the critical pipes in WDNs. The procedure is summarized below and further explained using Exnet as an example:

1) Set a threshold of criticality for the critical pipes. For instance, a pipe is regarded as critical if its criticality is equal or larger than 10% (Loss of the pipe will cause at least 10% of water supply shortage).

2) Analyze the WDN at cluster scale:

2.1) Calculate the criticality of all clusters (e.g. 37 for the Exnet) and the interconnections among clusters (e.g. 106 for the Exnet).

2.2) Plot the criticality of all clusters following a descending order [e.g. Fig. 6(D)] and select the clusters with significant higher criticality as critical clusters, e.g. the clusters in the rectangular in Fig. 6(D).

2.3) Select all the interconnections with their criticality above the threshold identified in step 1) as critical pipes (in the rectangular in Fig. 6(C)), and select all clusters connected by those critical interconnections as critical clusters (if not already selected in step 2)), see as Fig. 7.

3) Analyze the WDN at whole network scale (the finest scale):

Calculate the criticality of all the pipes within critical clusters and identify the critical pipes above the threshold.

Compared with the traditional enumeration method, the fractal approach above avoids analyzing the system completely based on the finest scale which requires calculating the criticality for each pipe (i.e. 2465 calculations for Exnet). By eliminating uncritical clusters at the cluster scale, the fractal method can reduce the total number of calculations by about 61% (i.e. only requires 953 calculations including 37 for clusters, 106 for interconnection among clusters, and 810 for pipes within critical clusters, respectively). However, as Fig. 7 shows, the fractal method is as reliable as the traditional method since all the critical pipes falls in the identified critical clusters and critical interconnections.

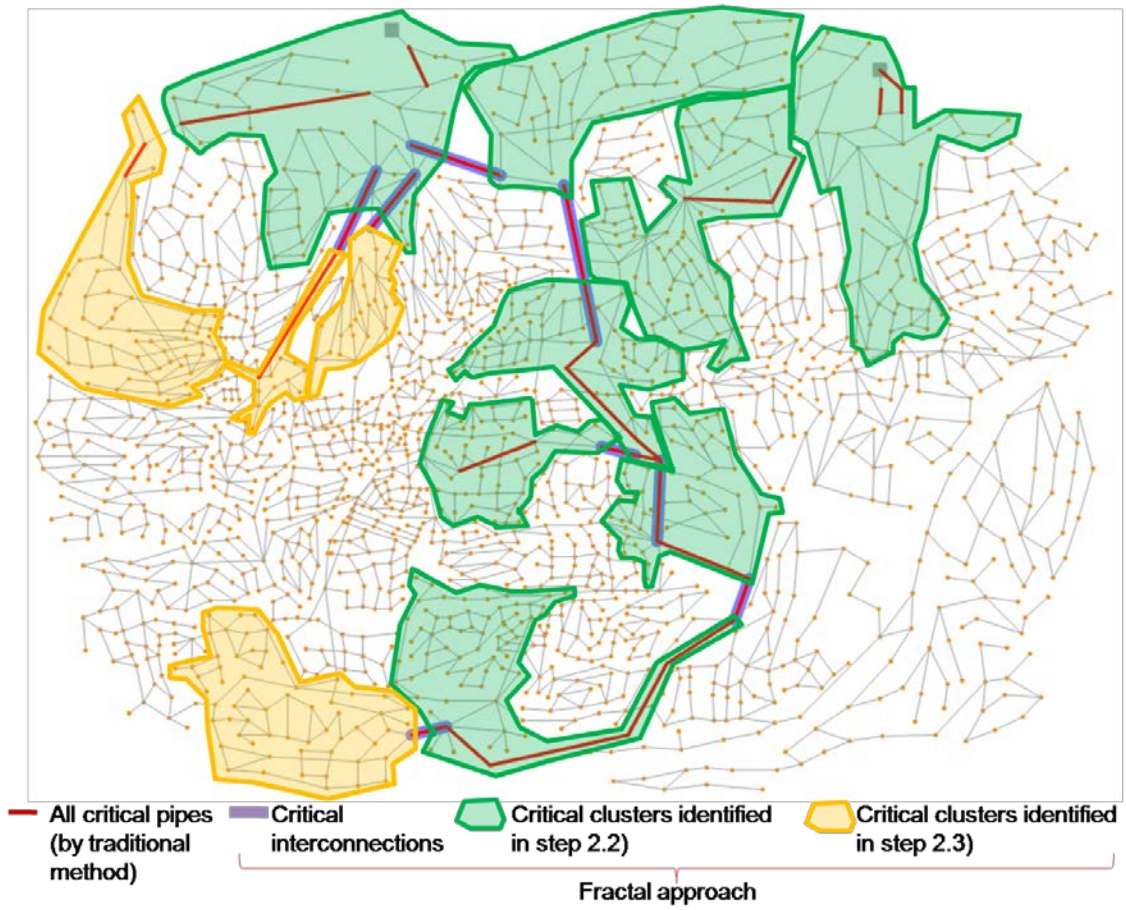

Figure 7. Verification of the fractal method by comparing the results with the traditional method

6 CONCLUSIONS

Toward deepening insights on the complexity of water distribution networks (WDNs), this study investigated the fractality in WDNs by using the box-covering algorithm in complex network science. Six real world WDNs of different complexities were studied and the results show that:

- All the case study networks are fractal regardless of the complexity of the systems.
- For the property studied (i.e. pipe criticality), similar patterns are observed both at different scales and at different parts of the system.

- All the critical pipes in a WDN can be identified much more efficiently using the fractal patterns, e.g. 61% reduction of the computational load.

Hence, fractals may also be a property of WDNs and provide a new perspective for understanding WDNs' complexity. Eventually, fractal-based analytical tools can be developed to realise reliable and efficient analyses of extremely complex problems in WDN design, operational control, maintenance, sustainable development planning, etc. Future studies will analyse further real-world WDNs with various topological and behavioural properties considered, and apply fractal in variety of specific water distribution system analyses.

References

[1] B. B. Mandelbrot, *The Fractal Geometry of Nature*. W.H.Freeman & Co Ltd, 1982.

[2] C. Song, S. Havlin and H. A. Makse, "Origins of fractality in the growth of complex networks", *Nature Physics*, vol. 2, pp. 275-281, 2006.

[3] C. Song, S. Havlin and H. A. Makse, "Self-similarity of complex networks", *Nature*, *433*, pp. 392-395, 2005.

[4] L. A. Lipsitz and A. L Goldberger, "Loss of 'complexity' and aging. Potential applications of fractals and chaos theory to senescence", *JAMA*, vol. 267(13), pp. 1806-1809, 1992.

[5] K. I. Goh, G. Salvi, B. Kahng and D. Kim, "Skeleton and fractal scaling in complex networks", *Phys.Rev.Lett*, vol. 96, 018701, 2006.

[6] J. S. Kim, K. I. Goh, G. Salvi, E. Oh, B. Kahng and D. Kim, "Fractality in complex networks: critical and supercritical skeletons", *Phys.Rev.E*, vol. 75, 016110, 2007.

[7] F. Radicchi, J. J. Ramasco, A. Barrat and S. Fortunato, "Complex Networks renormalization: flows and fixed points", *Phys.Rev.Lett*. vol. 101, 148701, 2008.

[8] S. Qi, J. Ye, J. Gao, W. Wu, J. Wang, Z. Zang, L. Chen, T. Shi and L. Zhou, "Fractal-based Planning of Urban Water Distribution System in China", *Procedia Engineering*, vol. 89, pp. 886-892, 2014.

[9] F. Zeng and K. Li, "Reliability Analysis of Fractal Water Distribution Networks Based on Basic Patterns", *Proceedings of the 2013 Georgia Water Resources Conference, held April 10–11, 2013, at the University of Georgia*.

[10] D. Kowalski, B. Kowalska, and M. Kwietniewski, "Monitoring of water distribution system effectiveness using fractal geometry", *Bull. Pol. Ac.: Tech*., vol. 63 (1), pp. 155–161, 2015.

[11] A. Di Nardo, M. Di Natale, C. Giudicianni, R. Greco and G. F. Santonastaso, "Complex network and fractal theory for the assessment of water distribution network resilience to pipe failures", Water Science and Technology: Water Supply, Vol. 17(4), DOI: 10.2166/ws.2017.124, 2017.

[12] L. K.Gallos, M. Sigman and H. A.Makse, "The conundrum of functional brain networks: small-world efficiency or fractal modularity", *Frontiers in Physiology*, vol. 3(123), pp. 1-9, 2012.

[13] *Fractal Dimensions and Complexity* [Online], Available: https://imagej.nih.gov/ij/plugins/fraclac/FLHelp/FractalDimension.htm, date accessed Feb 2017.

[14] L. Perelman and A. Ostfeld, "Topological clustering for water distribution systems analysis," *Environmental Modelling & Software*, vol. 26(7), pp. 969-972, 2011.

[15] C. Song, L. K. Gallos, S. Havlin and H. A. Makse, "How to calculate the fractal dimension of a complex network: the box covering algorithm", *J. Stat. Mech.*, 2007.

[16] R. Sitzenfrei, M. Mair, M. Möderl and W. Rauch, "Cascade vulnerability for risk analysis of water infrastructure", *Water Sci Technol.*, vol. 64(9), pp. 1885–1891, 2011.

[17] K. G. Diao, R. Farmani, G. T. Fu, M. Astaraie-Imani, S. Ward and D. Butler, "Clustering analysis of water distribution systems: identifying critical components and community impacts", *Water Science & Technology*, vol. 70(11), pp. 1764–1773, 2014.

[18] L. Magee, "R2 measures based on Wald and likelihood ratio joint significance tests". *The American Statistician*, vol. 44, pp. 250-253, 1990.

[19] Alpheus Bingham and Dwayne Spradlin, *The Long Tail of Expertise. Pearson Education. p. 5*, ISBN 9780132823135, 2011.