

# Optimising valve placement and pressure control for multi-feed sectors in water supply networks using outer approximation

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## ABSTRACT

*In this paper, we investigate the problem of optimising both the location and control settings of pressure reducing valves in multi-feed sectors in water supply networks. A simplification of the problem formulation is presented with respect to previously published research by reducing the degree of nonlinearity of optimisation constraints. The resulting optimisation problem is a nonconvex mixed integer nonlinear program. We then apply an outer approximation algorithm for the generation of good quality solutions of the considered design-for-control problem. We evaluate the presented method on a large-scale operational water supply network in the UK.*

**Keywords:** Pressure management, multi-feed sectors, valve placement, optimisation

## 1 Introduction

The pressure control of complex multi-feed water supply network benefits from the reduction in leakage and bursts [1] without sacrificing the redundancy in connectivity and the system resilience observed in single feed pressure controlled sectors (e.g. single-feed District Metered Areas). Multiple serviceability objectives that are controlled via the outcome delivery incentives introduced by Ofwat, the water regulator in the UK, can be successfully achieved when a multi-feed pressure control is implemented [4]. Here we consider the design-for-pressure control that is implemented with multiple diaphragm actuated pressure control valves. These network controllers can be operated to reduce pressure at their downstream node or they can be closed, allowing a range of pressure differentials across a pipe. In this paper, we consider the problem of optimising both the locations of the valves and their control setting, simultaneously. The mathematical formulation for optimal valve placement and operation considers pressure heads and flow rates as continuous decision variables, while binary variables are introduced to model the placement of valves. Mass and energy conservations laws are enforced as optimisation constraints. In particular, energy conservation laws involve frictional head loss terms which result in nonconvex constraints. These constraints were modelled in previous literature using highly nonlinear polynomial expressions [5]–[7]. In this manuscript, we introduce a simplified problem formulation that reduces the degree of nonlinearity of the constraints. Nonetheless, the optimisation problem results in a nonconvex mixed integer nonlinear program (MINLP) that is difficult to solve. Global solution approaches for nonconvex MINLPs are computationally expensive [8] and can easily become impractical when the size of the problem grows. In comparison, the application of convex optimisation techniques generally requires less computational effort but it does not guarantee global optimality for nonconvex MINLPs. We investigate the application of outer approximation techniques for the solution of the MINLP considered here. In particular, we implement the OA/ER algorithm from [9]. As mentioned above, our MINLP is nonconvex, thus OA/ER is applied as heuristic. Using an operational network model as case study, we show that the use of OA/ER can enable convergence to a (local) solution in large scale networks.

## 2 Problem formulation

We model a water distribution network with  $n_0$  water sources (eg. reservoirs or tanks),  $n_n$  nodes and  $n_p$  pipes, as a graph  $(V, E)$ , with  $n_n + n_0$  vertices and  $n_p$  links. Moreover, we consider control over a typical 24 hour horizon and we include in the formulation  $n_t = 24$  different demand scenarios. For each time step  $t \in [1, \dots, n_t]$ , we define the vectors of unknown hydraulic heads and flows  $h^t \in \mathbb{R}^{n_n}$  and  $q^t \in \mathbb{R}^{n_p}$ , respectively. In addition, we have vectors of elevation  $e \in \mathbb{R}^{n_n}$  and demands  $d^t \in \mathbb{R}^{n_n}$ ,  $t \in \{1, \dots, n_t\}$ . Hydraulic heads at the water sources are known and denoted by  $h_{0i}^t$  for each  $i = 1, \dots, n_0$  and  $t \in \{1, \dots, n_t\}$ . The friction head loss across a pipe  $j$  at time  $t$  can be represented by either the Hazen-Williams (HW) or Darcy-Weisbach (DW) formulae. For both DW and HW models, authors in [10] propose the use of a quadratic approximation for head losses minimising the relative errors. Once such quadratic approximation is identified, this can be represented as  $\phi_j(q_j^t) := (a_j |q_j^t| + b_j)q_j^t$ . Let  $\Phi(q^t) := [\phi_1(q_1^t), \dots, \phi_{n_p}(q_{n_p}^t)]^T$ , for each  $t \in \{1, \dots, n_t\}$ . In addition, we have the node-edge incidence matrices  $A_{12}^T \in \mathbb{R}^{n_n \times n_p}$  and  $A_{10}^T \in \mathbb{R}^{n_0 \times n_p}$  for the  $n_n$  nodes and the  $n_0$  water sources, respectively. Finally, we introduce the vectors of unknown binary variables  $v^+ \in \{0, 1\}^{n_p}$  and  $v^- \in \{0, 1\}^{n_p}$ , where for each  $k = 1, \dots, n_p$  we have:

- $v_j^+ = 1 \Leftrightarrow$  there is a valve on link  $j$  in the assigned positive flow direction
- $v_j^- = 1 \Leftrightarrow$  there is a valve on link  $j$  in the assigned negative flow direction
- $v_j^+ = v_j^- = 0 \Leftrightarrow$  no valve is placed on link  $j$
- $v_j^+ + v_j^- \leq 1$

As an example application, here we consider the minimisation of pressure driven leakage. The average zone pressure (AZP), which is an indicator for the level of leakage in the zone of the network being controlled [1], is used as the objective function to be minimised. AZP is defined as:

$$\frac{1}{n_t W} \sum_{t=1}^{n_t} \sum_{i=1}^{n_n} w_i (h_i^t - e_i) \quad (1)$$

Where  $w_i = \sum_{j \in I(i)} L_j / 2$  and  $I(i)$  is the set of indices for links incident at node  $i$ , counted only once.

Moreover, we have a normalization factor  $W = \sum_{i=1}^{n_n} w_i$ . The optimization problem for optimal valve placement and operation is formulated as a nonconvex Mixed Integer Nonlinear Program (MINLP):

$$\begin{aligned}
\min \quad & \frac{1}{n_l W} \sum_{t=1}^{n_l} \sum_{i=1}^{n_n} w_i (h_i^t - e_i) \\
\text{s.t.} \quad & \Phi(q^t) + A_{12} h^t + \eta^t + A_{10} h_0^t = 0, \quad t = 1, \dots, n_l, \\
& A_{12}^T q^t - d^t = 0, \quad t = 1, \dots, n_l, \\
& \eta^t - M v^+ \leq 0, \quad t = 1, \dots, n_l, \\
& -\eta^t - M v^- \leq 0, \quad t = 1, \dots, n_l, \\
& -q^t + Q^{\max} v^+ \leq q^{\max}, \quad t = 1, \dots, n_l, \\
& q^t + Q^{\max} v^- \leq q^{\max}, \quad t = 1, \dots, n_l, \\
& h^t \leq h_{\max}^t, \quad t = 1, \dots, n_l, \\
& -h^t \leq -h_{\min}^t, \quad t = 1, \dots, n_l, \\
& v^+ + v^- \leq \mathbf{e}, \sum_{k=1}^{n_p} (v_k^+ + v_k^-) = n_v, v^+, v^- \in \{0, 1\}^{n_p}
\end{aligned} \tag{2}$$

In the formulation of Problem (2)  $M \in \mathbb{R}^{n_p \times n_p}$  is a diagonal matrix of sufficiently big positive numbers, while  $h_{\min}^t$  and  $h_{\max}^t$  represent maximum and minimum allowed hydraulic heads at network nodes, respectively. Moreover,  $q^{\max} \in \mathbb{R}^{n_p}$  is a vector of maximum allowed flows and  $Q^{\max} := \text{diag}(q^{\max})$ . Finally, we have set  $\mathbf{e} := [1, \dots, 1]^T \in \mathbb{R}^{n_p}$ . For a detailed description of optimisation constraints, see [11]. We observe that the formulation in Problem (2) is simplified with respect to the one considered in [5]–[7]. In fact, the formulation used in previous literature includes more constraints with higher degree of nonlinearity involving all continuous unknowns.

Let  $V := \left\{ (v^+, v^-) \in \{0, 1\}^{n_p} \times \{0, 1\}^{n_p} \mid v^+ + v^- \leq \mathbf{e}, \sum_{k=1}^{n_p} (v_k^+ + v_k^-) = n_v \right\}$ . Problem (2) can be rewritten as

$$\begin{aligned}
\min \quad & f(x) \\
\text{s.t.} \quad & c(x) = 0, \\
& x \in X(v^+, v^-), \\
& (v^+, v^-) \in V.
\end{aligned} \tag{3}$$

Where  $c(\cdot)$  represents the nonlinear equality constraints in Problem (2), while the compact polyhedral  $X(v^+, v^-)$  is defined by the remaining linear constraints in Problem (2). We observe that, given binary choices  $(\hat{v}^+, \hat{v}^-) \in V$ , the control settings of the corresponding valves can be optimised by solving the following nonconvex nonlinear program (NLP):

$$\begin{aligned}
\min \quad & f(x) \\
\text{s.t.} \quad & c(x) = 0, \\
& x \in X(\hat{v}^+, \hat{v}^-).
\end{aligned} \tag{4}$$

In the following, we sometime refer to Problem (5) as  $NLP(\hat{v}^+, \hat{v}^-)$ .

### 3 Outer Approximation

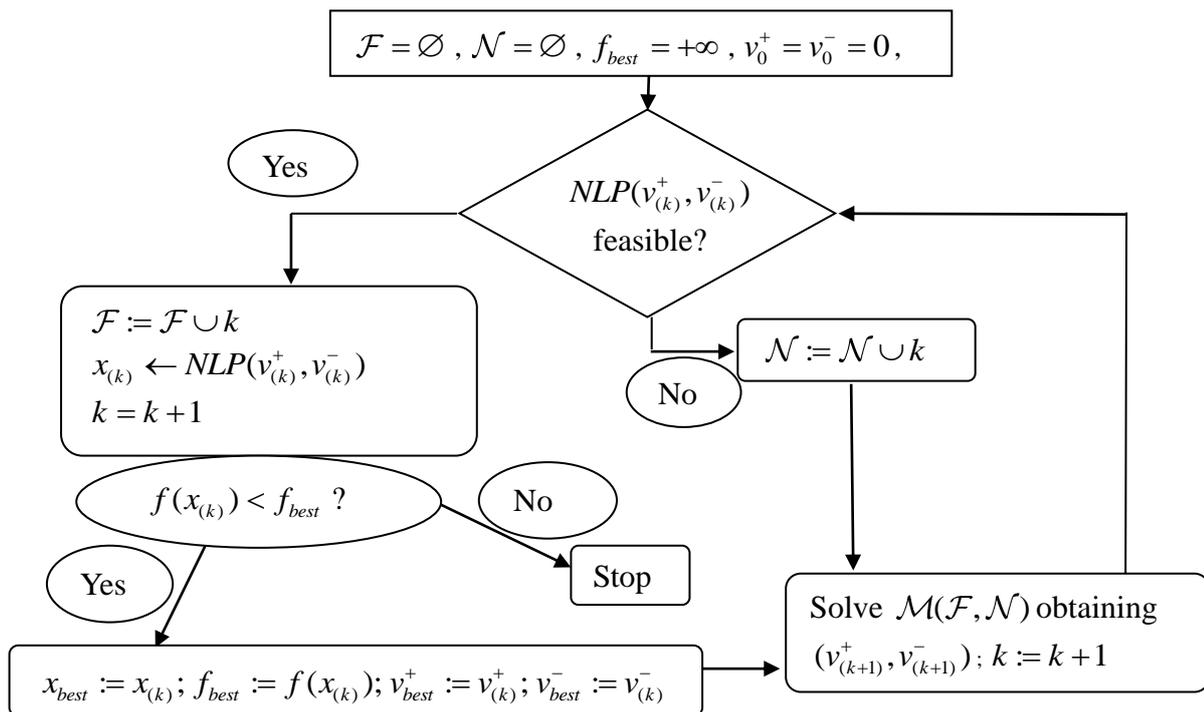
In order to solve Problem (4), we implement the Outer Approximation with Equality Relaxation (OA/ER) algorithm [9]. The method relies on the solution of an alternating sequence of NLPs and linear relaxations of the original MINLP. We observe that Problem (4) is nonconvex, as it involves nonconvex equality constraints. Therefore, OA/ER is applied as heuristic, it will not provide any global optimality guarantee. Nonetheless, the application of OA/ER to nonconvex problems often produces solutions close to the global optimum [12]. Given a sequence  $(v_{(k)}^+, v_{(k)}^-)_{k \in \mathcal{F} \cup \mathcal{N}} \subseteq V$ , we introduce the following definitions:

- $k \in \mathcal{F} \Leftrightarrow NLP(v_{(k)}^+, v_{(k)}^-)$  is feasible. In this case,  $x_{(k)}$  is a (local) solution of  $NLP(v_{(k)}^+, v_{(k)}^-)$  while  $\lambda_{(k)}$  is the corresponding Lagrangian multiplier for  $c(\cdot)$ .
- $k \in \mathcal{N} \Leftrightarrow NLP(v_{(k)}^+, v_{(k)}^-)$  is not feasible.

We define the following linear relaxation of the original MINLP (4):

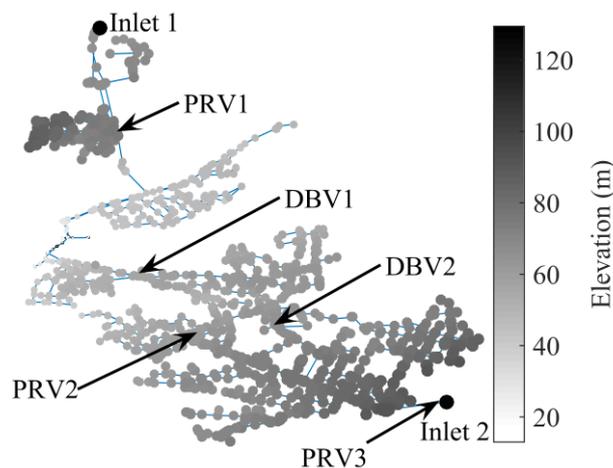
$$\begin{aligned}
 & \min \quad \mu \\
 & \text{s.t.} \quad f_{(k)} + \nabla f_{(k)}^T (x - x_{(k)}) \leq \mu, \\
 & \quad S_{(k)} J_{(k)} (x - x_{(k)}) \leq 0, \quad \forall k \in \mathcal{F} \\
 & \quad x \in X(v^+, v^-), \\
 & \quad (v_{(k)}^+)^T v^+ + (v_{(k)}^-)^T v^- \leq n_v - 1, \quad \forall k \in \mathcal{F} \cup \mathcal{N} \\
 & \quad (v^+, v^-) \in V.
 \end{aligned} \tag{5}$$

We refer to the mixed inter linear program (MILP) in (6) as  $\mathcal{M}(\mathcal{F}, \mathcal{N})$ . The OA/ER algorithm is summarised in the following flow chart:



## 4 Case study

We consider the Smart Water Network Demonstrator operated by Bristol Water, InfraSense Labs at Imperial College London and Cla-Val presented in [13]; and we refer to the model as BWFLnet. This water supply network consists of 2374 nodes, 2434 pipes and 2 inlets (with fixed known hydraulic heads); its graph and elevation map is presented in Figure 1. BWFLnet is composed of three interconnected District Metered Areas (DMAs) and it is currently operated with a dynamic adaptive topology [13]. Two originally closed boundary valves (BVs) between the DMAs have been replaced by two dynamic boundary valves (DBVs) that are closed at night hours and open during diurnal network operation. Three pressure reducing valves (PRVs) are optimally operated in order to minimise AZP. The network model and control options have been expanded from the model presented in [13]. The quadratic approximation proposed in [10] is used to model friction. In this study, the commercial solver GUROBI [14] is applied for the solution of all the MILPs considered by OA/ER. In addition, the solution of each NLP corresponding to a specific choice of valve locations is performed by the solver IPOPT [15].



*Figure 1 BWFLnet network model*

In the present formulation we consider 24 different demand conditions, one for each hour of the day. We model the existing PRVs as smooth pipes. Moreover, the operation of the two DBVs is embedded within the optimisation constraints by varying the coefficients of the quadratic head loss approximation to model the valves opening and closing. Details on the operation of the DBVs have been provided by the valves' manufacturer. The links corresponding to the DBVs are excluded from the set of possible valve locations. In addition, for all the nodes, we set a minimum hydraulic pressure head requirement of 18 m. We formulate Problem (4) for the optimal placement and operation of 3 control valves, addressing the minimisation of AZP. The number of continuous variables, binary variables and constraints is reported in Table 1. The OA/ER method outlined in the previous section was applied to the considered case study. Convergence was achieved after two iterations requiring a CPU time of 8198 s. The optimal locations on  $V_1^*$ ,  $V_2^*$ ,  $V_3^*$  are presented in Figure 2 and corresponds to an AZP of 36.72 m.

Table 1 Problem size characteristics for BWFLnet

No. cont. var.	No. bin var.	No. lin. const.	No. nonlin. const.
221808	4864	407027	58412

The case study is a real-world operational water network where the control valves are updated every 15 minutes, i.e. the daily operation considers 96 different demand conditions. Therefore, in order to assess the effectiveness of the optimal valve locations in Figure 2 for the pressure management of BWFLnet, we solve Problem (5) for fixed valve locations to  $V_1^*$ ,  $V_2^*$ ,  $V_3^*$  and number of multiple demand conditions  $nl = 96$ , addressing the minimisation of average zone pressure. The optimal AZP for 96 different time steps is equal to 36.72m and it is identical to the AZP for the computed optimal solution when only 24 demand conditions were considered.

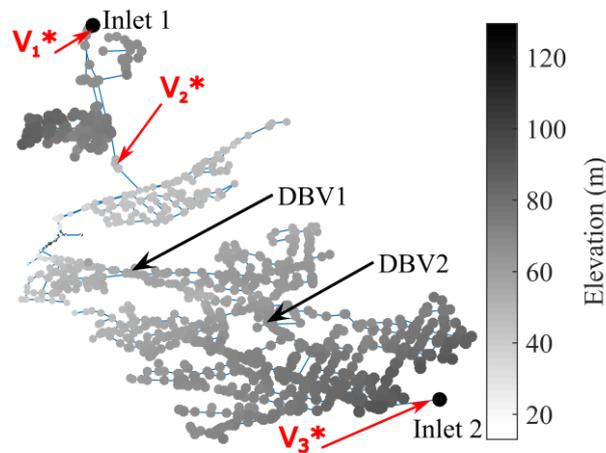


Figure 2 BWFLnet with new valve locations

In the following, we further investigate the quality of the optimised valve controls for  $V_1^*$ ,  $V_2^*$ ,  $V_3^*$  when 96 different demand scenarios are considered. We evaluate the effects of the quadratic approximation for head losses on the operation of the optimally placed control valves. When optimal pressure settings are derived, these are implemented within a null space hydraulic solver [16] to simulate network pressures and flows, while valves are optimally operated – for further details see also Appendix 2 in [17]. The optimisation solution compares with hydraulically simulated pressures and flows as follows:

- Maximum difference on flows: 0.32 1/s
- Maximum difference on pressures: 1.09 m
- 95.37% of pressures are within 0.5 m difference
- Average daily AZP (optimisation): 36.72 m
- Average daily AZP (simulation): 36.71 m

Given the level of uncertainty that characterizes the modeling of real-world water networks, we conclude that the use of a quadratic approximation do no significantly affect the quality of the solution. For a detailed error analysis on the use of quadratic approximations for friction head losses see [17]. We compare the performance of  $V_1^*$ ,  $V_2^*$ ,  $V_3^*$  with the three PRVs currently installed in

BWFLnet, that are optimally operated to minimise AZP – see Figure 1. In order to do so, we solve Problem (5) for fixed valve locations to PRV1, PRV2, PRV3 and number of multiple demand conditions  $n_i=96$ , where the minimisation of average zone pressure is the objective. A comparison between AZP profiles corresponding to the optimal operation of  $V_1^*$ ,  $V_2^*$ ,  $V_3^*$  and PRV1, PRV2, PRV3 is reported in Figure 3. The optimally located  $V_1^*$ ,  $V_2^*$ ,  $V_3^*$  result in a slightly lower AZP than the current valves' locations.

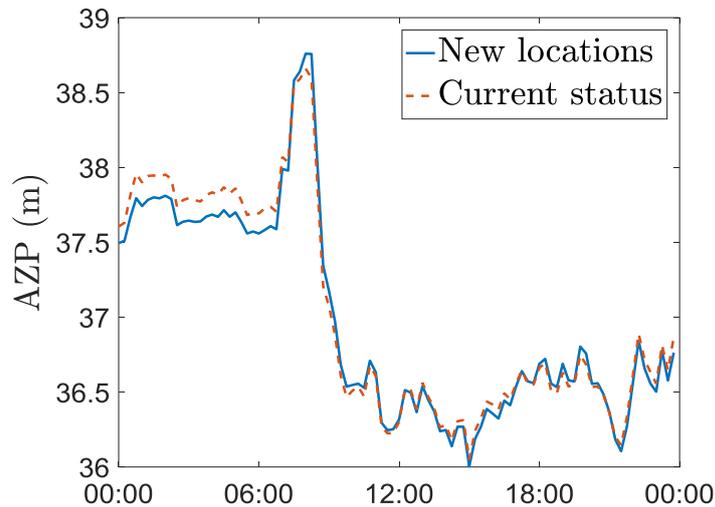


Figure 3 Comparison of AZP profiles

## 5 Conclusion

We have presented a new mathematical formulation of the optimal valve placement and pressure optimisation problem for multi-feed sectors (DMAs) in water supply networks with a lower degree of nonlinearity in comparison to previously published research. We have adapted state-of-the-art mathematical programming techniques to the resulting nonconvex MINLP, where the minimisation of average zone pressure is the objective. In particular, we have implemented an outer approximation algorithm (OA/ER) for the solution of the considered problem. OA/ER relies on the solution of an alternating sequence of nonlinear programs (NLPs) and mixed integer linear programs (MILPs). Since the problem formulation involves nonconvex equality constraints, OA/ER cannot provide a global optimality guarantee. Nevertheless, OA/ER is shown to converge to a good quality hydraulically feasible solution for a large operational network model with multiple demand conditions.

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