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An Improved Expression for Information Quality of Basic Probability Assignment and Its Application in Fault Diagnosis

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ABSTRACT Information quality is widely used in many applications. However, how to measure information quality in basic probability assignment accurately is still an open issue. Generalized expression for information quality is an effective method to measure information quality in basic probability assignment. Nevertheless, the counter-intuitive results may be obtained when statements are of intersection. To address this issue, a new expression for information quality of basic probability assignment is proposed in this paper considering the frame of discernment and the influence of intersection among statements which can cause changes of uncertainty. Numerical examples are illustrated to demonstrate the effectiveness of the proposed method. In addition, an application in fault diagnosis is used to show the effectiveness of the proposed method.

INDEX TERMS Information quality, Gini entropy, Dempster-Shafer evidence theory, Basic probability assignment, Fault diagnosis

I. INTRODUCTION

The world is pervaded with uncertainties, and decisions must be made based on uncertain information. Dempster-Shafer evidence theory is an easy and effective framework for modeling uncertain information [1]–[5]. It is widely used in uncertainty reasoning and it has the ability to process many types of information in the real world to make accurate decision. There are also many other mathematical models to do uncertainty modeling like D numbers [6]–[10], Z numbers [11]–[16], fuzzy sets [17]–[20], intuitionistic fuzzy sets [21]–[25], pythagorean fuzzy sets [26]–[29], two-dimensional belief function [30], intuitionistic evidence sets [31], and so on.

Information quality is firstly proposed by Yager and Petry based on Gini entropy to measure the uncertainty for a probability distribution [32], [33]. It has been widely used in pattern classification [34], [35], decision making [36], [37] and so on [38]–[43], [43], [44]. Li *et al.* propose a generalized expression for information quality in basic probability assignment in Dempster-Shafer evidence theory [45], which makes information quality have greater scope of application.

Yager and Petry's method does not consider the length of each element and treat elements that have different length

equally, which is counter-intuitive. Li *et al.*'s method takes the length of each element into account, and fully considers the potential uncertainty created by non-single elements. However, when elements are of intersection, the previous methods do not take the effects of intersection of statements into account, counter-intuitive results may be obtained. To address this issue, an improved expression for information quality is proposed considering the length of frame of discernment (FOD) and the influence of intersection among statements. An exponential item is added to take the effects of intersection into account. The proposed expression can degenerate into generalized form of information quality and information quality proposed by Yager and Petry under certain conditions.

The rest of this paper is organised as follows. Section II introduces the preliminaries include Dempster-Shafer theory, information quality and generalized form of information quality. Improved expression for information quality in basic probability assignment will be proposed in Section III. In Section IV, numerical examples will be given to illustrate the effectiveness of proposed method. In Section V, an application in fault diagnosis is given to show the effectiveness

of proposed method. Finally, conclusions will be made in Section VI.

II. PRELIMINARIES

In this section, the preliminaries will be briefly introduced, including Dempster-Shafer evidence theory and information quality.

A. DEMPSTER-SHAFFER EVIDENCE THEORY

Dempster-Shafer evidence theory can represent the uncertainties effectively, and it is widely used in many fields like pattern classification [46]–[48], multi-criteria decision making [49]–[51] and so on [52]–[61]. The brief introduction to Dempster-Shafer evidence theory will be given as follows.

Definition 2.1: A set of hypothesis Θ is called the frame of discernment. It is defined as follows:

$$\Theta = \{h_1, h_2, \dots, h_n\} \quad (1)$$

The power set 2^Θ is defined as:

$$2^\Theta = \{\emptyset, \{h_1\}, \{h_1, h_2\}, \dots, \{h_1, h_2, \dots, h_n\}\} \quad (2)$$

where \emptyset is an empty set.

Definition 2.2: The discernment frame Θ must satisfy the following conditions:

- (1) $m(\emptyset) = 0$
- (2) $\sum_{A \in \Theta} m(A) = 1$

where $m : 2^\Theta \rightarrow [0, 1]$ is called the basic probability assignment (BPA). And m is also known as the mass function. For an BPA m on Θ , each subset A of Θ is called an focal element of m if $m(A) > 0$.

How to fuse different information is still an open issue. The basic combination rule is given as follows.

Definition 2.3: Suppose there are two independent BPAs, m_1, m_2 . The combination rule is used to fuse two BPAs. And the result of fused BPA is denoted by m . And it is given by:

$$\begin{cases} m(\emptyset) = 0 \\ m(A) = \frac{\sum_{B \cap C = A} m_1(B)m_2(C)}{1 - K} \end{cases} \quad (3)$$

where $K = \sum_{B \cap C = \emptyset} m_1(B)m_2(C)$.

The counter-intuitive results may be obtained when information is highly conflicted [62]. Hence, many methods are proposed to address this issue [63]–[67]. The weighted average method is proposed by Deng *et al.* based on Murphy's average method and Jousselme distance [68]–[70].

Definition 2.4: Given two mass functions m_1 and m_2 , the Jousselme distance between m_1 and m_2 is defined as:

$$d(m_1, m_2) = \sqrt{\frac{1}{2} (\vec{m}_1 - \vec{m}_2)^T \underline{\underline{D}} (\vec{m}_1 - \vec{m}_2)} \quad (4)$$

where \vec{m}_1 and \vec{m}_2 are respective belief functions in the notation of vector (and each size is $2^\Theta - 1$). And $\underline{\underline{D}}$ is an $|2^\Theta| \times |2^\Theta|$ matrix that each of its element is $\underline{\underline{D}}(s_1, s_2) = \frac{|s_1 \cap s_2|}{|s_1 \cup s_2|}$, $s_1, s_2 \in 2^\Theta$.

The similarity measure between m_1 and m_2 can be defined as

$$Sim(m_1, m_2) = 1 - d(m_1, m_2) \quad (5)$$

Suppose there are n mass functions. And we can construct a $2^n \times 2^n$ similarity matrix as follows:

$$A = \begin{bmatrix} 1 & S_{12} & \dots & S_{1n} \\ S_{21} & 1 & \dots & S_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1} & S_{n2} & \dots & 1 \end{bmatrix}_{n \times n}$$

And $S_{ij} = Sim(m_i, m_j)$, for $i = 1, 2, \dots, n, j = 1, 2, \dots, n$.

The support degree of one mass function m_i ($i = 1, 2, \dots, n$) can be defined as:

$$Sup(m_i) = \sum_{j=1}^n S_{ij} \quad (6)$$

And the credibility degree of one mass function m_i ($i = 1, 2, \dots, n$) can be denoted as:

$$Crd_i = \frac{Sup(m_i)}{\sum_{j=1}^n Sup(m_j)} \quad (7)$$

And the weighted mass function can be obtained by weighted average as follows:

$$w_i = Crd_i \quad (8)$$

$$m(A) = \sum_{i=1}^n w_i m_i(A) \quad (9)$$

where A is a focal element of m . Then weighted mass function can be used to fuse data in the later process.

B. INFORMATION QUALITY

Entropy is a measurement of the uncertainty of information. The larger the value of entropy, the bigger the uncertainty of information. Many methods for entropy can represent the uncertainty for information, like Shannon entropy [71], Tsallis entropy [72]–[74], Rényi entropy [75], [76], Deng entropy [77]–[88], generalized belief entropy [89] and so on [90]–[92].

Based on Gini entropy [33], information quality is proposed by Yager and Petry as another way to measure the degree of uncertainty of information [32]. The value of information quality is larger while the uncertainty of information is smaller. Information quality is defined as follows.

Definition 2.5: Given a probability function p_i , the information quality of p_i is defined as:

$$IQ_{p_i} = \sum_{j=1}^n |p_{ij}|^2 \quad (10)$$

When $p_{ij} = 1$, the defined information quality reaches its maximum value, and when all $p_{ij} = \frac{1}{n}$, which leads to most uncertainty of information, the value of information quality is smallest.

Inspired by the idea of Deng entropy [77], Li *et al.* propose generalized form of information quality in basic probability assignment [45], and it is defined as follows.

Definition 2.6: Given a basic probability allocation m_i , the generalized information quality of m_i is defined as follows.

$$IQ_{m_i} = \sum_{A \in X} \left(\frac{m_i(A)}{2^{|A|} - 1} \right)^2 \quad (11)$$

When all the statements are all single elements, the generalized form of information quality will degenerate into the form of information quality proposed by Yager and Petry.

III. PROPOSED METHOD

In this paper, we focus on the disadvantage of generalized form of information quality on the ignorance of the intersection between statements. In this section, we propose a new form of information quality based on generated form of information quality. The proposed form of information quality considering the length of FOD and the intersection between statements is given as follows.

$$IQ_{m_i} = \sum_{A \in X} \left(\frac{m_i(A)}{2^{|A|} - 1} \right)^2 e^{\sum_{\substack{B \in X \\ B \neq A}} \frac{|A \cap B|}{|X|}} \quad (12)$$

Where $|A|$ denotes the cardinality of proposition A , and $|A \cap B|$ denotes the cardinality of the intersection between proposition A and B . $|X|$ denotes the length of FOD. When all propositions have no intersection, the proposed form of information quality degenerates into Eq.(11). Then if belief is only assigned to single elements, the proposed form will degenerate into Eq.(10).

IV. NUMERICAL EXAMPLES

In this section, numerical examples will be given to demonstrate the efficiency of the proposed form of information quality.

Example 4.1: Suppose there are two basic probability allocations m_1 and m_2 . The frame of discernment $X = \{a, b, c, d\}$. $m_1(a, b) = 0.5, m_1(b, c) = 0.5, m_2(a, b) = 0.5, m_2(c, d) = 0.5$.

Intuitively, the information quality of m_1 is larger than m_2 because focal elements of m_1 are in intersection. Although the distributions of two mass functions are similar, m_1 has less information volume than m_2 as m_1 has less targets than m_2 . Hence, m_1 contains less information than m_2 and the value of information quality of m_1 is larger than m_2 .

With Yager and Petry's method [32], the information quality of m_1 and m_2 can be calculated as

$$\begin{aligned} IQ_{m_1} &= 0.5^2 + 0.5^2 \\ \log(IQ_{m_1}) &= -1.3863 \\ IQ_{m_2} &= 0.5^2 + 0.5^2 \\ \log(IQ_{m_2}) &= -1.3863 \end{aligned}$$

With the generalized form of information quality [45], the information quality of m_1 and m_2 can be calculated as

$$\begin{aligned} IQ_{m_1} &= \left(\frac{0.5}{2^2 - 1} \right)^2 + \left(\frac{0.5}{2^2 - 1} \right)^2 \\ \log(IQ_{m_1}) &= -2.8904 \\ IQ_{m_2} &= \left(\frac{0.5}{2^2 - 1} \right)^2 + \left(\frac{0.5}{2^2 - 1} \right)^2 \\ \log(IQ_{m_2}) &= -2.8904 \end{aligned}$$

With the proposed method, the information quality can be calculated as

$$\begin{aligned} IQ_{m_1} &= \left(\frac{0.5}{2^2 - 1} \right)^2 e^{\frac{1}{4}} + \left(\frac{0.5}{2^2 - 1} \right)^2 e^{\frac{1}{4}} \\ \log(IQ_{m_1}) &= -2.6404 \\ IQ_{m_2} &= \left(\frac{0.5}{2^2 - 1} \right)^2 e^{\frac{0}{4}} + \left(\frac{0.5}{2^2 - 1} \right)^2 e^{\frac{0}{4}} \\ \log(IQ_{m_2}) &= -2.8904 \end{aligned}$$

As we can see, information qualities of m_1 and m_2 are the same calculated by Yager and Petry method and Li *et al.*'s method. Counter-intuitive results are obtained. With the proposed method, information quality of m_1 is larger than m_2 , which is intuitive and reasonable. In this example, proposed method has better performance than existed methods.

Example 4.2: Suppose there are three basic probability allocations m_1, m_2 and m_3 . The frame of discernment $X = \{a, b, c, d, e, f\}$. $m_1(a, b) = 0.2, m_1(c, d) = 0.6, m_1(e, f) = 0.2, m_2(a, b) = 0.2, m_2(b, c) = 0.6, m_2(c, f) = 0.2, m_3(a, b) = 0.2, m_3(b, c) = 0.6, m_3(e, f) = 0.2$.

Intuitively, the information qualities of m_1, m_2 and m_3 are not the same. The information quality of m_1 is lowest while the information quality of m_2 is highest as elements in m_2 have largest intersection.

With Yager and Petry's method [32], the information quality can be obtained as follows.

$$\begin{aligned} IQ_{m_1} &= 0.2^2 + 0.6^2 + 0.2^2 \\ \log(IQ_{m_1}) &= -0.8210 \\ IQ_{m_2} &= 0.2^2 + 0.6^2 + 0.2^2 \\ \log(IQ_{m_2}) &= -0.8210 \\ IQ_{m_3} &= 0.2^2 + 0.6^2 + 0.2^2 \\ \log(IQ_{m_3}) &= -0.8210 \end{aligned}$$

With Li *et al.*'s method [45], the information quality can be obtained as follows.

$$\begin{aligned} IQ_{m_1} &= \left(\frac{0.2}{2^2 - 1} \right)^2 + \left(\frac{0.6}{2^2 - 1} \right)^2 + \left(\frac{0.2}{2^2 - 1} \right)^2 \\ \log(IQ_{m_1}) &= -3.0182 \\ IQ_{m_2} &= \left(\frac{0.2}{2^2 - 1} \right)^2 + \left(\frac{0.6}{2^2 - 1} \right)^2 + \left(\frac{0.2}{2^2 - 1} \right)^2 \\ \log(IQ_{m_2}) &= -3.0182 \end{aligned}$$

$$IQ_{m_3} = \left(\frac{0.2}{2^2-1}\right)^2 + \left(\frac{0.6}{2^2-1}\right)^2 + \left(\frac{0.2}{2^2-1}\right)^2$$

$$\log(IQ_{m_3}) = -3.0182$$

With proposed method, the information quality can be calculated as follows.

$$IQ_{m_1} = \left(\frac{0.2}{2^2-1}\right)^2 e^{\frac{0}{6}} + \left(\frac{0.6}{2^2-1}\right)^2 e^{\frac{0}{6}}$$

$$+ \left(\frac{0.2}{2^2-1}\right)^2 e^{\frac{0}{6}}$$

$$\log(IQ_{m_1}) = -3.0182$$

$$IQ_{m_2} = \left(\frac{0.2}{2^2-1}\right)^2 e^{\frac{1}{6}} + \left(\frac{0.6}{2^2-1}\right)^2 e^{\frac{2}{6}}$$

$$+ \left(\frac{0.2}{2^2-1}\right)^2 e^{\frac{1}{6}}$$

$$\log(IQ_{m_2}) = -2.7132$$

$$IQ_{m_3} = \left(\frac{0.2}{2^2-1}\right)^2 e^{\frac{1}{6}} + \left(\frac{0.6}{2^2-1}\right)^2 e^{\frac{1}{6}}$$

$$+ \left(\frac{0.2}{2^2-1}\right)^2 e^{\frac{0}{6}}$$

$$\log(IQ_{m_3}) = -2.8656$$

As we can see, information qualities of three mass functions are the same calculated by Yager and Petry method and Li *et al.*'s method. The obtained results are counter-intuitive. With the proposed method, information quality of m_2 is largest and information quality of m_1 is smallest. It is intuitive and reasonable.

Example 4.3: Suppose there is a basic probability allocation m , the FOD is $\Theta = \{1, 2, \dots, 14\}$. $m(6) = 0.05$, $m(3, 4, 5) = 0.05$, $m(\Theta) = 0.8$, $m(A) = 0.1$, A ranges from $\{1\}$ to $\{1, 2, \dots, 14\}$.

Figure 1 demonstrates the change of logarithmic value of different measures for information quality when A changes. Table 1 shows the logarithmic value of different measures for information quality in different cases. Values of Yager and Petry's method keep almost stable when A changes because the length of A do not affect the result, and it is not reasonable. Values calculated by proposed method is larger than Li *et al.*'s method, and it is reasonable because the effect of intersection is taken into account, and information quality is larger. Values calculated by two methods have same trends of change.

Example 4.4: Suppose there is a basic probability allocation m , the FOD is $\Theta = \{1, 2, \dots, 14\}$. $m(1) = 0.05$, $m(2) = 0.05, \dots, m(14) = 0.05$, $m(A) = 0.05$, A ranges from $\{1\}$ to $\{1, 2, \dots, 14\}$.

Figure 2 demonstrates the change of logarithmic value of different measures for information quality when A changes. Table 2 shows the logarithmic value of information quality calculated by proposed method in different cases. When the size of A becomes larger, information qualities calculated by Eq.(10) and Eq.(11) approach to a certain value. Because of

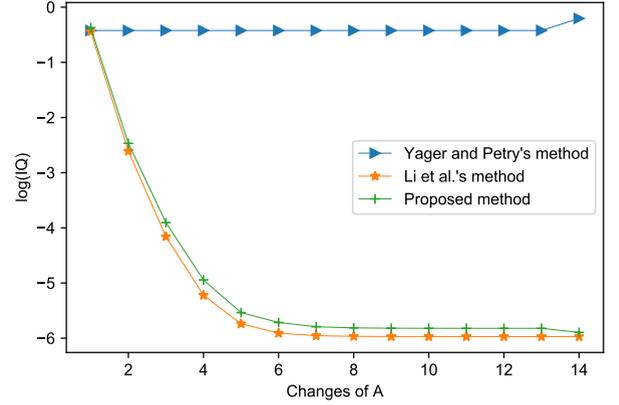


FIGURE 1: Different measurements of information quality with changes of A of BPA in Example 4.3

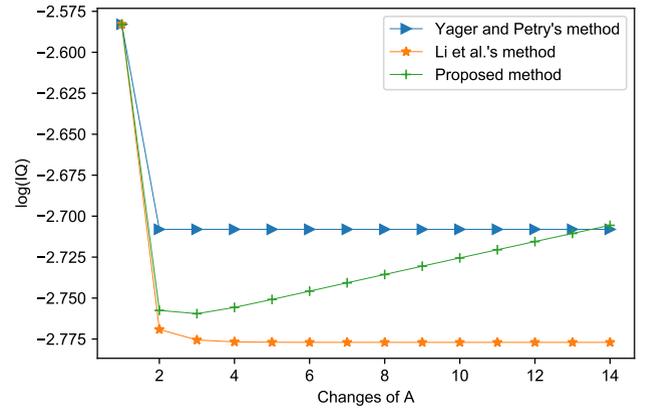


FIGURE 2: Different measurements of information quality with changes of A of BPA in Example 4.4

the effect of intersection elements, the information quality calculated by the proposed method is larger than it calculated by Eq.(10) and smaller than it calculated by Eq.(11) at most of times. It is reasonable that elements have more mutual information with changes of A , which leads to the increment of information quality.

TABLE 1: Logarithmic value of different measurements for information quality with changes of A of BPA

Cases	Yager's method [32]	Li et al.'s method [45]	Proposed method
A = {1}	-0.4231	-0.4423	-0.3709
A = {1, 2}	-0.4231	-2.6083	-2.4677
A = {1, 2, 3}	-0.4231	-4.1597	-3.9054
A = {1, 2, 3, 4}	-0.4231	-5.2222	-4.9442
A = {1, 2, ..., 5}	-0.4231	-5.7393	-5.5359
A = {1, 2, ..., 6}	-0.4231	-5.9100	-5.7164
A = {1, 2, ..., 7}	-0.4231	-5.9558	-5.7928
A = {1, 2, ..., 8}	-0.4231	-5.9674	-5.8140
A = {1, 2, ..., 9}	-0.4231	-5.9703	-5.8197
A = {1, 2, ..., 10}	-0.4231	-5.9710	-5.8212
A = {1, 2, ..., 11}	-0.4231	-5.9712	-5.8217
A = {1, 2, ..., 12}	-0.4231	-5.9712	-5.8218
A = {1, 2, ..., 13}	-0.4231	-5.9713	-5.8218
A = {1, 2, ..., 14}	-0.2046	-5.9713	-5.8968

TABLE 2: Logarithmic value of information quality with changes of A of BPA

Cases	Logarithmic value
A = {1}	-2.5829
A = {1, 2}	-2.7575
A = {1, 2, 3}	-2.7595
A = {1, 2, 3, 4}	-2.7560
A = {1, 2, ..., 5}	-2.7508
A = {1, 2, ..., 6}	-2.7458
A = {1, 2, ..., 7}	-2.7407
A = {1, 2, ..., 8}	-2.7356
A = {1, 2, ..., 9}	-2.7305
A = {1, 2, ..., 10}	-2.7255
A = {1, 2, ..., 11}	-2.7205
A = {1, 2, ..., 12}	-2.7155
A = {1, 2, ..., 13}	-2.7106
A = {1, 2, ..., 14}	-2.7056

V. APPLICATION

In this section, an application in fault diagnosis is investigated using proposed expression for information quality. The case study in [93] is recalled in this section. Based on method proposed by Yuan et al. [94], a new method using improved expression for information quality is proposed and the main steps of this method are shown in Figure 3.

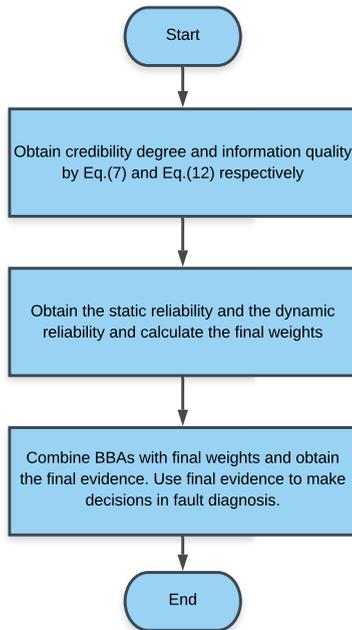


FIGURE 3: The flowchart of new method

In the example in [93], three fault types are called F_1 , F_2 and F_3 . The hypothesis set of faults is $\Theta = \{F_1, F_2, F_3\}$. Three sensors in the hypothesis set are independent. The results of fault diagnosis are called BOEs, denoted as E_1 , E_2 and E_3 . The BPAs of diagnosis results are shown in Table 3.

With the Dempster's combination rule in Eq.(3), the fused

result is $m(F_1) = 0.4519$, $m(F_2) = 0.5048$, $m(F_2, F_3) = 0.0336$, $m(\Theta) = 0.0096$. It is hard to judge which fault has been occurred because the values obtained by Dempster's combination rule are very close. The extreme values in m_2 influence the fused result.

To solve this problem, in [94], a fault diagnosis method is proposed. The reliability of each sensor is defined as the weight of a BOE. The weight of each BOE is defined as the product of a static reliability and a dynamic reliability, and it is defined as follows.

$$w(i) = w_s(i) \times w_d(i) \quad (13)$$

Where the static reliability $w_s(E_1) = 1$, $w_s(E_2) = 0.2040$, $w_s(E_3) = 1$, in this example. The dynamic reliability is defined with the use of information quality as follows.

$$w_d(i) = Crd(m_i) \times \frac{\frac{1}{IQ(m_i)}}{\max(\frac{1}{IQ(m_j)})}, j = 1, 2, 3 \quad (14)$$

In this example, the credibility degree and information quality can be obtained by Eq.(7) and Eq.(12). The values of them are:

$$Crd(m_1) = 1$$

$$Crd(m_2) = 0.5523$$

$$Crd(m_3) = 0.9660$$

$$IQ_{m_1} = 0.5280$$

$$IQ_{m_2} = 1.2116$$

$$IQ_{m_3} = 0.7071$$

The weight of each BOE can be calculated.

$$w(1) = w_s(1) \times Crd(m_1) \times \frac{\frac{1}{IQ(m_1)}}{\frac{1}{IQ(m_1)}}$$

$$= 1 \times 1 \times \frac{\frac{1}{0.5280}}{\frac{1}{0.5280}} = 0.5778$$

$$w(2) = w_s(2) \times Crd(m_2) \times \frac{\frac{1}{IQ(m_2)}}{\frac{1}{IQ(m_1)}}$$

$$= 0.2040 \times 0.5523 \times \frac{\frac{1}{1.2116}}{\frac{1}{0.5280}} = 0.0274$$

$$w(3) = w_s(3) \times Crd(m_3) \times \frac{\frac{1}{IQ(m_3)}}{\frac{1}{IQ(m_1)}}$$

$$= 1 \times 0.9660 \times \frac{\frac{1}{0.7071}}{\frac{1}{0.5280}} = 0.3947$$

The final fused result can be obtained by Eq.(3) and Eq.(9). $m(F_1) = 0.8996$, $m(F_2) = 0.0685$, $m(F_2, F_3) = 0.0245$, $m(\Theta) = 0.0074$. It is easy to tell and conclude that F_1 is the fault with the highest probability. The results with different methods are shown in Table 4. As Table 4 shows, the proposed method is compatible with Fan et al.'s method, Yuan et al.'s method and Zhou et al.'s method.

The application in fault diagnosis shows the effectiveness of the proposed form of information quality. Also, this application indicates a promising application prospect of this new form of information quality.

TABLE 3: BPAs of diagnosis result

Sensor report	F_1	F_2	F_2, F_3	Θ
$E_1 : m_1$	0.60	0.10	0.20	0.10
$E_2 : m_2$	0.05	0.80	0.05	0.10
$E_3 : m_3$	0.70	0.10	0.10	0.10

TABLE 4: Fault diagnosis results with different methods

Method	$m(F_1)$	$m(F_2)$	$m(F_2, F_3)$	$m(\Theta)$
Dempster's combination rule	0.4519	0.5048	0.0336	0.0096
Fan et al.'s method [93]	0.8119	0.1096	0.0526	0.0259
Yuan et al.'s method [94]	0.8948	0.0739	0.0241	0.0072
Zhou et al.'s method [81]	0.8951	0.0738	0.0240	0.0071
Proposed method	0.8996	0.0685	0.0245	0.0074

VI. CONCLUSION

In this paper, an improved expression for information quality is proposed considering the scale of the frame of discernment and the intersection between statements in basic probability of assignment, based on generalized form of information quality. Moreover, some numerical examples are illustrated to show the effectiveness of the proposed method. Results show that proposed method has better performance than the previous methods. In addition, an application in fault diagnosis is used to illustrate the effectiveness of the proposed method. In the future, we will further explore on other applications of this new form of information quality in basic probability assignment.

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The authors declare that there are no conflicts of interest.

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