

Online Supplement for:
Operating with an Incomplete Checklist

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This online supplement contains the proofs for the propositions in the main text.

A Proof of Proposition 1: Optimality of Myopic Policy for Homogeneous, Parallel Actions

If we follow the myopic policy, the decision maker will execute if and only if

$$\frac{j}{n} \leq \frac{\mu}{\lambda j + \mu} d + \frac{\lambda j}{\lambda j + \mu} \frac{j-1}{n} \tag{A.1}$$

We show that this problem has the monotone condition introduced in Chapter 3.5 of Chow et al. (1971) (see also Chapter 5 of Ferguson (2004)). The monotone condition states that if the myopic policy recommends to stop in the current state (i.e., condition (A.1) holds), then the myopic policy will recommend to stop in any future state. If the monotone condition applies, then the myopic policy is optimal. Rearranging (A.1), yields that the myopic condition holds if

$$j \leq \frac{\mu d n}{\lambda + \mu} \tag{A.2}$$

If condition (A.2) applies for some state j , it will hold for all future states because all future states are less than j . Consequently, condition (A.1) applies and we have the monotone condition.

B Proof of Proposition 2 : Optimality of Myopic Policy for Parallel Actions

As with Appendix A, we show the problem satisfies the monotone property of Chow et al. (1971). This requires showing that if

$$f(x) \leq \frac{\mu}{\sum_{k: x(k)=1} \lambda_k + \mu} d + \sum_{i: x(i)=1} \frac{\lambda_i f(x^{-i})}{\sum_{k: x(k)=1} \lambda_k + \mu}, \quad (\text{B.1})$$

then condition (B.1) will also hold for any future state y descending from state x . That is condition (B.1) will hold for any state $y < x$, where the inequality is componentwise. Rearranging (B.1) yields

$$\sum_{i: x(i)=1} \lambda_i (f(x) - f(x^{-i})) + \mu f(x) \leq \mu d. \quad (\text{B.2})$$

We now show that if condition (B.2) holds for x it will hold for any $y < x$. First note the right-hand-side of (B.2) is constant. We next argue that the left-hand-side of (B.2) is smaller for $y < x$. Clearly $\mu f(x) > \mu f(y)$ by the first condition of Proposition 2. Next we will focus on the summation term:

$$\sum_{i: x(i)=1} \lambda_i (f(x) - f(x^{-i})) \quad (\text{B.3})$$

For $y < x$ there will be fewer terms in the sum in (B.3), which will decrease the value on the left-hand-side of (B.2). Finally for an index i such that $y(i) = x(i) = 1$, the term in the summation of (B.3) will be smaller for y than x because of the second condition of Proposition 2 (the marginally decreasing effect). Consequently because all the terms on left-hand-side of (B.2) are smaller for y than x , condition (B.2) and hence condition (B.1) also hold for state y . We have our monotone property and the optimality of the myopic policy. Corollary 1 follows immediately from the preceding discussion.

C Proof of Proposition 3: Optimality of Myopic Policy for Sequential Actions

The analysis is very similar to Appendix B, so we will only sketch the idea. To show the monotone property of Chow et al. (1971), we have to show that if

$$\lambda_j (f(j) - f(j-1)) + \mu f(j) \leq \mu d. \tag{C.1}$$

holds for any j , it will hold for any $i < j$. This follows immediately because $f(\cdot)$ is increasing and convex and λ_j are non-decreasing. This implies the left-hand-side increases with j . Consequently there exists some threshold τ^* such that it is optimal to execute for $j \leq \tau^*$ and wait for $j > \tau^*$.

Finally to prove Corollary 8, we compare Proposition 2 to Corollary 7. We must show that

$$\left\lfloor \frac{\mu}{\lambda + \mu} dn \right\rfloor \leq \lfloor \max(0, dn - \lambda/\mu) \rfloor \tag{C.2}$$

First if $dn < \lambda/\mu + 1$ both sides of (C.2) equal 0. Next assume $dn \geq \lambda/\mu + 1$. Basic algebra reveals this condition is equivalent to $\frac{\mu}{\lambda + \mu} dn \leq dn - \lambda/\mu$, which completes the proof.

D Proof of Proposition 4: Optimality of Myopic Policy for Mixed Actions

The steps are nearly identical to those in Appendices B–C. To show the monotone property of Chow et al. (1971), we have to show that if

$$\lambda(j_p + I(j_s > 0)) \left(f\left(\frac{j_p + j_s}{n_p + n_s}\right) - f\left(\frac{j_p + j_s - 1}{n_p + n_s}\right) \right) + \mu f\left(\frac{j_p + j_s}{n_p + n_s}\right) \leq \mu d. \tag{D.1}$$

holds for any (j_p, j_s) , it will also hold for any state $(\tilde{j}_p, \tilde{j}_s)$, such that $\tilde{j}_p \leq j_p$ and $\tilde{j}_s \leq j_s$. This follows because $f(\cdot)$ is increasing and convex.

E Proof of Propositions 5: Optimality of Myopic Policy with Action Costs

Proceeding in a similar fashion to Appendices B–D, the equivalent condition to (B.2) is

$$\sum_{i: x^{(i)}=1} \lambda_i (f(x) - (f(x^{-i}) + \kappa_i)) + \mu f(x) \leq \mu d. \quad (\text{E.1})$$

To show this scenario satisfies the monotone property of Chow et al. (1971), requires proving that if condition (E.1) is satisfied for x , it will also be satisfied for any $y < x$. The $\mu f(x)$ term is smaller for any $y < x$ by the first condition of Proposition 2, which also applies for Proposition 5. The summation term in (E.1) is smaller for any state $y < x$ by the same logic presented in Appendix B, provided all summands are positive. The new condition in Proposition 5 provides that guarantee. Because all the terms on left-hand-side of (E.1) are smaller for $y < x$, condition (E.1) also holds for any $y < x$.

F Proof of Proposition 6: Optimality of Myopic Policy with Rushed Execution

If $f(x) \geq d(x)$, the myopic policy will recommend to wait and hence the operator should optimally wait in state x . This follows by inspection of (7) and condition 1 of Proposition 2, which also applies for Proposition 6. For the remainder of the proof we assume $f(x) < d(x)$, and show the myopic policy also produces optimal results in this case.

The equivalent condition to (B.2) for this scenario is

$$\sum_{i: x^{(i)}=1} \lambda_i (f(x) - f(x^{-i})) - \mu(d(x) - f(x)) \leq 0. \quad (\text{F.1})$$

The summation term is smaller for any $y < x$ by the same logic presented in Appendix B. The term $(d(x) - f(x))$ is positive by assumption and increases for $y < x$ by the new condition of Proposition 6. Thus the entire left-hand-side of (F.1) decreases for $y < x$, and hence condition (F.1) will also hold for $y < x$.

G Proof of Proposition 7: Optimality of Myopic Policy with Dependence

The myopic policy recommends to stop if

$$\frac{j}{n} \leq \left(p_U \frac{\mu_U}{\lambda_U j + \mu_U} + (1 - p_U) \frac{\mu_R}{\lambda_R j + \mu_R} \right) d + \left(p_U \frac{\lambda_U j}{\lambda_U j + \mu_U} + (1 - p_U) \frac{\lambda_R j}{\lambda_R j + \mu_R} \right) \frac{j - 1}{n}. \quad (\text{G.1})$$

To utilize the monotone property of Chow et al. (1971), we need to show that if (G.1) holds for a given j , it will hold for all $i < j$. For notational simplicity let us define the probability the window closes as

$$q(j, p_U) \equiv p_U \frac{\mu_U}{\lambda_U j + \mu_U} + (1 - p_U) \frac{\mu_R}{\lambda_R j + \mu_R} \quad (\text{G.2})$$

Rearranging (G.1) yields

$$\frac{1}{nq(j, p_U)} + \frac{j - 1}{n} \leq d \quad (\text{G.3})$$

To complete the proof, we show that the left-hand side of (G.3) is nondecreasing in j , regardless of how p_U varies between 0 and 1. By induction it suffices to show the following condition holds:

$$\frac{1}{nq(j, p_U)} + \frac{j-1}{n} \geq \frac{1}{nq(j-1, \tilde{p}_U)} + \frac{j-2}{n} \quad \forall 1 \leq j \leq n, \quad p_U, \tilde{p}_U \in [0, 1] \quad (\text{G.4})$$

Rearranging (G.4) yields the equivalent condition

$$\frac{1}{q(j-1, \tilde{p}_U)} - \frac{1}{q(j, p_U)} \leq 1 \quad \forall 1 \leq j \leq n, \quad p_U, \tilde{p}_U \in [0, 1]. \quad (\text{G.5})$$

To prove condition (G.5) we start by rearranging terms in the assumption of Proposition 7:

$$j \left(\frac{\lambda_R}{\mu_R} - \frac{\lambda_U}{\mu_U} \right) - \frac{\lambda_R}{\mu_R} < 1 \quad \forall j \leq n.$$

Further algebra yields

$$\frac{1}{\frac{\mu_R}{\lambda_R(j-1) + \mu_R}} - \frac{1}{\frac{\mu_U}{\lambda_U j + \mu_U}} < 1 \quad \forall j \leq n. \quad (\text{G.6})$$

By the assumption $\frac{\mu_U}{\lambda_U} > \frac{\mu_R}{\lambda_R}$ and inspection of (G.2)

$$\frac{\mu_R}{\lambda_R(j-1) + \mu_R} \leq q(j-1, p_U) \quad \forall 1 \leq j \leq n, \quad p_U \in [0, 1] \quad (\text{G.7})$$

$$\frac{\mu_U}{\lambda_U j + \mu_U} \geq q(j, p_U) \quad \forall 1 \leq j \leq n, \quad p_U \in [0, 1] \quad (\text{G.8})$$

Combining conditions (G.6)–(G.8) implies condition (G.5) and completes the proof.

H Proof of Propositions 8: Optimality of Myopic Policy for General Distributions

As with the rest of the proofs, we rely on monotone property of Chow et al. (1971). The myopic policy recommends to stop if the following condition holds:

$$\frac{j}{n} \leq r_G(t, \Delta t) \Delta t \times d + (1 - r_G(t, \Delta t) \Delta t) \sum_{k=0}^j \binom{j}{k} (r_H(t, \Delta t) \Delta t)^k (1 - r_H(t, \Delta t) \Delta t)^{j-k} \frac{j-k}{n}. \quad (\text{H.1})$$

We need to show that if (H.1) holds for a given t , it will hold for all future time $\hat{t} > t$. The summation term is an expectation of a binomial, and thus condition (H.1) simplifies significantly

$$\frac{j}{n} \leq r_G(t, \Delta t) \Delta t \times d + \frac{j}{n} (1 - r_G(t, \Delta t) \Delta t) (1 - r_H(t, \Delta t) \Delta t). \quad (\text{H.2})$$

The first condition of Proposition 8 forbids both $r_G(t, \Delta t) = 0$ and $r_H(t, \Delta t) = 0$ simultaneously. If $r_G(t, \Delta t) = 0$ and $r_H(t, \Delta t) > 0$, then condition (H.2) does not hold. Therefore the myopic policy recommends to wait, and hence the optimal policy should wait. For the remainder of the proof we assume $r_G(t, \Delta t) > 0$. Further manipulation of (H.2) yields

$$\frac{j}{n} \left(1 + \frac{r_H(t, \Delta t)}{r_G(t, \Delta t)} - r_H(t, \Delta t) \Delta t \right) \leq d \quad (\text{H.3})$$

As j can only decrease as time increases, condition (H.3) will hold for all time greater than t if the term in parentheses does not increase in time. Recall that Δt is a fixed parameter; only j and t vary over time. The third condition of Proposition 8 guarantees the second term in parentheses does not increase in t . The second condition of Proposition 8 guarantees that last term in parentheses is non-increasing in time. Thus for any fixed Δt , we have shown the myopic policy is optimal and the operator should execute the operation if condition (H.3) holds. Rearranging (H.3) produces the stopping expression in Proposition 8.